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Special Issue

Optimizing Production Mix Involving Linear Programming with Fuzzy Resources and Fuzzy Constraints

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ABSTRACT

In this paper, Fuzzy Linear Programming (FLP) was used to model the production processes at a university-based bakery for optimal decisions in the daily productions of the bakery. Using the production data of five products from the bakery, a fuzzy linear programme was developed to help make decisions when fuzzy resources were involved. As usual, classical linear programme gave only one feasible solution. However, while it was established that solving the linear programme from the production mix when fuzzy constraints were introduced (Verdegay's model) gave a more robust and alternative set of decisions than the classical model, it was equally established that a better result could be obtained when all the constraints and the objective function were fuzzy (Werners' Model).

1. INTRODUCTION

Linear Programming (LP) has its origin during the second world war (1939-1945) when a balance between man and material (resources) had to be maintained. Hence, LP was developed in order to plan expenditures and returns to reduce costs of the army and increase losses imposed on the enemy. During that time, Marshall K. Wood worked on the allocation of the resources for the United States and methods were developed to allocate resources in such a way that will minimize or maximize the desired objective of the problems as the case may require. As time went on, economists formulated classical economic problems, transportation problems and assignment problems. Then the simplex method of solving linear programmes was introduced by George B. Dantzig [1] which, for the first time, efficiently tackled the LP problem in most cases. Many industries found the use of LPvaluable, hence, accelerating its development.

LP is the mathematical technique which involves the use of limited resources in an optimal manner. LP problems requiring such judicious use of resources are called optimization problems. In such classical optimization problems, a solution which is feasible is the goal.

It is also important to note that optimization has gone beyond allocating resources. Various optimization methods have also been developed to solve problems that occur in other physical sciences. Some of such can be found in [6,5,12,15,17,18] just to mention a few.

Most of the traditional tools for formal modeling, reasoning and computing are precise in nature, that is, they are the yes-or-no type and not more-or-less type. Unfortunately, many real life problems encountered from time to time suggest the need for more robust mathematical tools. Take, for an instance, if an investor hopes to make profit "around" an amount using "not more" than "around certain quantity" of raw materials, the usual LP method cannot model the situation properly. This is as a result of the vague language "around" which introduces uncertainty into the problem. Also, to be able to develop model which classifies goods and services as being "expensive" or "affordable" and the like, it is necessary for the existence of a mathematical modeling of vague knowledge to capture elements that cannot be precisely said to be in a set or not but are in between.

Mathematical study of vagueness of this sort (fuzzy sets) began with a professor of electrical engineering, Lofti A. Zadeh, in the University of California at Berkeley, when he published the first paper *Fuzzy Sets* in 1965 [20]. In it, he noted that "The notion of a fuzzy set provides a convenient point of departure for the construction

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of a conceptual frame-work which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Essentially, such a framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables."

Fuzzy set theory provides a mathematical framework in which vague conceptual phenomena can be rigorously studied. It can also be considered as a modeling language well suited for situations in which fuzzy linguistic variables such as "much," "very much," "high," "very high" and the like occur. Zadeh and others continued to develop the fuzzy logic at that time. The idea of fuzzy sets and fuzzy logic, though were not well accepted then, have now been extended to many areas of discipline and studies.

Extension of fuzziness to LP has led to Fuzzy Linear Programming (FLP) in which some of the parameters constitute fuzzy set(s). One case is to have a crisp objective but fuzzy constraints. Another case is to have a fuzzy objective but crisp constraints. However, it is also possible to have a FLP in which both the constraints and the objective are fuzzy. Precisely, in this paper, an approach used by [4], crisp objective but fuzzy constraints, is improved on.

2. PRELIMINARIES

Definition 1. [20] A fuzzy set A in X is a set of ordered pairs $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x)$ is the grade of membership of $x \in A$ and $\mu_A : X \to [0, 1]$.

Definition 2. [20] The support of a fuzzy set A, $S(A) = \{x \in A : \mu_A(x) > 0\}$.

Definition 3. ([20] A fuzzy set A is empty if and only if $\mu_A(x) = 0, \forall x \in X$.

Definition 4. ([20] Two fuzzy sets A and B are equal if and only if $\mu_A(x) = \mu_B(x), \forall x \in X$.

Definition 5. [20] A fuzzy set A is contained in a fuzzy set B, written as $A \subseteq B$, if and only if $\mu_A(x) \leqslant \mu_B(x), \forall x \in X$.

Definition 6. [20] The intersection of two fuzzy sets A and B is denoted by $A \cap B$ and is defined as the largest fuzzy set contained in both A and B. The membership function of $A \cap B$ is given by $\mu_A(x) \wedge \mu_B(x) = min\{\mu_A(x), \mu_B(x), \forall x \in X\}$.

Definition 7. [20] The union of A and B, denoted by $A \cup B$, is defined as the smallest fuzzy set containing both A and B. The membership function of $A \cup B$ is given by $\mu_A(x) \vee \mu_B(x) = max\{\mu_A(x), \mu_B(x), \forall x \in X\}$.

Definition 8. [20] If A is a fuzzy subset of X, then an α-level set of A is a nonfuzzy set A_α which comprises all elements of X whose grade of membership is greater than or equal to α. It is denoted by $A_\alpha = \{x \in X : \mu_A(x) \geqslant \alpha, \ \forall\, x \in X\}$.

3. LINEAR PROGRAMMING

A LP is an optimization problem in which the objective function and constraints are given as mathematical functions and functional relationships. Linear programmes are of the form

$$\begin{aligned}&\text{Optimize} \quad z = f(x_1 x_2 x_3 \dots, x_n)\\ &\text{Subject to}\end{aligned}$$

$$\left.\begin{array}{l} g_1(x_1, x_2, x_3, ..., x_n) \\ g_2(x_1, x_2, x_3, ..., x_n) \\ . \\ . \\ . \\ g_m(x_1, x_2, x_3, ..., x_n) \end{array}\right\} \begin{array}{c}\leqslant \\ = \\ \geqslant\end{array} \left\{\begin{array}{l} b_1 \\ b_2 \\ . \\ . \\ . \\ b_n \end{array}\right. \tag{1}$$

where $b_1, b_2, b_3, \cdots, b_m$ are real numbers and $x_1, x_2, x_3, \cdots, x_n \geqslant 0$.

The programme attempts to find the best solution, called an ***optimal solution***, for a problem under consideration. It seeks to find values of the decision variables that optimize (that is, maximize or minimize) an objective function among a set of values.

Solving LP by simplex method according to [1] involves using matrix procedures to solve standard LP of the form

$$\begin{aligned}&\text{Optimize} && z = C^T X\\ &\text{Subject to} && AX = B\\ & && X \geqslant 0\end{aligned} \tag{2}$$

where

$$C = \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ . \\ c_n \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_n \end{pmatrix} \geqslant 0 \tag{3}$$

and

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \tag{4}$$

In what follows, X_0 denotes the matrix of slack variables, C_0 designates the cost matrix associated with X_0. For minimization programs and maximization programmes, the simplex method utilizes Tables 1 and 2 respectively.

4. METHODOLOGY: FLP

In FLP, the fuzziness of available resources is characterized by the membership function over the tolerance range. Then, the membership function of the optimal solutions is constructed by the convolution of the membership function of the constraints. The general

Table 1 | Simplex table for minimization programmes.

	X^T C^T	
$\mathbf{X_0 C_0}$	**A**	**B**
	$C^T - C_0^T A$	$-C_0^T B$

Table 2 Simplex table for maximization programmes.

	X^T C^T	
X_0C_0	A	B
	$C_0^TA - C^T$	C_0^TB

model of linear programmes with fuzzy resources is

$$\begin{aligned} \text{Maximize} \quad & z = C^TX \\ \text{Subject to} \quad & (Ax)_i \leqslant \tilde{b}_i \\ & i = 1, 2, \cdots, m \\ & x_i \in X \\ & x_i \geqslant 0 \end{aligned} \tag{5}$$

where $\tilde{b}_i \in [b_i, b_i + p_i]$ are the fuzzy resources and z is the objective function. $(Ax)_i \leqslant \tilde{b}_i$ is equivalent to $(Ax)_i \leqslant (b_i + \theta p_i)$, where θ is in [0, 1], given that the tolerance p_i and the actual resources b_i for each fuzzy constraint are known.

Verdegay's Approach—A Nonsymmetric Model: Verdegay in [19] considered that the membership functions of the fuzzy constraints in (5) are

$$\mu_i(x) = \begin{cases} 1, & \text{if } (Ax)_i \leqslant b_i \\ 1 - \dfrac{(Ax_i) - b_i}{p_i}, & \text{if } b_i < (Ax)_i < b_i + p_i \\ 0, & \text{if } (Ax)_i \geqslant b_i + p_i \end{cases} \tag{6}$$

Furthermore, if these μ_is are continuous and monotonic functions, and trade-off between them are allowed, then (5) is equivalent to

$$\begin{aligned} \text{Maximise} \quad & z = C^TX \\ \text{Subject to} \quad & x \in X_\alpha \end{aligned} \tag{7}$$

where $X_\alpha = \{x | \mu_i(x) \geqslant \alpha, x \geqslant 0\}$ is the α-cut of X, for each $\alpha \in [0, 1]$. The α-cut concept is based on the works of [13] and [11].

In the membership functions, the degree of satisfaction of the constraints is depending on whether $(Ax)_i \leqslant b_i$ (ith constraint is absolutely satisfied when $\mu_i(x) = 1$) or $(Ax)_i \geqslant b_i + p_i$, where p_i is the maximum tolerance from b_i (in which case the degree of satisfaction approaches 0 as $\tilde{b}_i$ approaches $b_i + p_i$). Thus, $(Ax)_i \in (b_i, b_i + p_i)$ means that the membership functions are monotonically increasing. Hence, the more resources consumed, the less satisfaction the decision-maker feels.

Now, the membership function of (6) is substituted into (7) to get a parametric programme.

$$\begin{aligned} \text{Maximize} \quad & z = C^TX \\ \text{Subject to} \quad & (Ax)_i \leqslant b_i + (1 - \alpha)p_i \\ & x \in X \geqslant 0 \\ & \alpha \in [0, 1] \end{aligned} \tag{8}$$

where $\alpha = 1 - \theta$. Note that for each α, there is an optimal solution, so the solution with α grade of membership is actually fuzzy.

But, further than having a FLP with fuzzy constraints, the objective function can altogether be made fuzzy and the results of these two can be compared.

Werners' Approach—A Nonsymmetric Model [16]: Given the linear programme with fuzzy resources

$$\begin{aligned} \text{Maximize} \quad & z = C^TX \\ \text{Subject to} \quad & (Ax)_i \leqslant \tilde{b}_i \\ & i = 1, 2, \cdots, m \\ & x_i \in X \\ & x_i \geqslant 0 \end{aligned} \tag{9}$$

where $\tilde{b}_i$, are in $[b_i, b_i + p_i]$. Assume that tolerance p_i for each fuzzy constraint is known, then, $(Ax)_i \leqslant \tilde{b}_i$ is equivalent to $(Ax)_i \leqslant (b_i + \theta p_i)$, where θ is in $[0, 1]$.

Werners proposed that the objective function of Equation (9) should be fuzzy because of fuzzy total resources or fuzzy inequality constraints. Let it be assumed that the tolerances p_is for the fuzzy resources are available and given. Then solving Equation (9), Werners first defined z^0 and z^1 as follows:

$$\begin{aligned} z^0 = \text{maximize} \quad & cx \\ \text{subject to} \quad & (Ax)_i \leqslant b_i \\ & x \geqslant 0 \end{aligned} \tag{10}$$

$$\begin{aligned} z^1 = \text{maximize} \quad & cx \\ \text{subject to} \quad & (Ax)_i \leqslant b_i + p_i \\ & x \geqslant 0 \end{aligned} \tag{11}$$

Thus, a continuously nondecreasing linear membership function can be constructed for the objective function by use of z^0 and z^1. Since the optimal solution will be in between z^0 and z^1, the satisfaction of the optimal solution will increase when its value increases. The membership function $\mu_0(x)$ of the objective function is

$$\mu_0(x) = \begin{cases} 1, & \text{if } cx < z^1 \\ 1 - \dfrac{z^1 - cx}{z^1 - z^0}, & \text{if } z^0 \leqslant cx \leqslant z^1 \\ 0, & \text{if } cx > z^0 \end{cases} \tag{12}$$

With the above membership function, the max-min operator can be used to obtain optimal decision. Then, Equation (9) can be solved by solving

$$\begin{aligned} \text{maximize} \quad & \alpha \\ \text{subject to} \quad & \mu_0(x) \geqslant \alpha \\ & \mu_i(x) \geqslant \alpha \\ & \alpha \in [0, 1] \\ & x \geqslant 0 \end{aligned} \tag{13}$$

5. RESULTS AND DISCUSSION

The data used in this study was collected from an institution-based bakery. The production data of five different kinds of bread stated below was used for the purpose of this project. The linear programme was solved, using the Simplex method to get the crisp optimal solution.

Furthermore, fuzzy elements were introduced to the resources and the new Fuzzy linear programme was solved using the **Verdegay's Approach—A Nonsymmetric Model**, where only constraints were fuzzy. Then, the **Werners' Approach—A Nonsymmetric Model**, where both the objective function and the constraints were fuzzy was also applied to the data. The results of the Classical Linear Programme and these Fuzzy linear Programmes were compared.

Products Considered: Let UC represent *Unit of Currency*

i. $70UC$ bread

ii. $100UC$ bread

iii. Chocolate bread

iv. Coconut bread

v. White bread

Note also, $f_1(x)$, $f_2(x)$, $f_3(x)$ and $f_4(x)$ will denote the inputs flour, sugar, salt and butter respectively. P is the profit function and P^* the optimal profit.

From the data collected, the bakery made a profit of $24.88UC$ on a unit of $70UC$ bread, $35.18UC$ on a unit of $100UC$ bread, $119.39UC$ on a unit of chocolate bread, $126.84UC$ on a unit of coconut bread and $90.84UC$ on a unit of white bread. The bakery used $200kg$ of flour with tolerance level of $50kg$, $16kg$ of sugar, $3.85kg$ of salt with tolerance level of $0.8kg$, and $1kg$ of butter per production time. The data used are given in Table 3, where all inputs are multiplied by 10^{-3}.

5.1. Obtaining Crisp Solution

The associated linear programme is

$$\begin{aligned} \text{Maximize } & P(x_1,x_2,x_3,x_4,x_5) = 24.88x_1 + 35.18x_2 \\ & \quad +119.39x_3 + 126.84x_4 + 90.84x_5 \\ \text{Subject to } & f_1(x) = (126x_1 + 174x_2 + 420x_3 \\ & \quad +420x_4 + 420x_5)10^{-3} \leqslant 200 \\ & f_2(x) = (7.5x_1 + 10x_2 + 59x_3 + 25x_4 \\ & \quad +25x_5)10^{-3} \leqslant 16 \\ & f_3(x) = (2x_1 + 2.8x_2 + 5.5x_3 + 6.7x_4 \\ & \quad +6.7x_5)10^{-3} \leqslant 3.85 \\ & f_4(x) = (4.2x_3 + 4.2x_4)10^{-3} \leqslant 1 \end{aligned} \tag{14}$$

where $x_1, x_2, x_3, x_4, x_5 \geqslant 0$, x_1 is the $70UC$ bread, x_2 is the $100UC$ bread, x_3 is the chocolate bread, x_4 is the coconut bread and x_5 is the white bread.

From the simplex table we have the crisp optimal solution

$$\begin{aligned} x^* &= \left(0,0,0,\frac{5000}{21},\frac{5000}{21}\right) \\ &= (0,0,0,238.095,238.095) \end{aligned}$$

Table 3 The input data for the Bakery problem.

Bread Kind	Flour	Sugar	Salt	Butter	Unit Profit
$70UC$	126	7.5	2.0	0.0	24.88
$100UC$	174	10.0	2.8	0.0	35.18
Chocolate	420	59.0	5.5	4.2	119.39
Coconut	420	25.0	6.7	4.2	126.84
White	420	25.0	6.7	0.0	90.84
Available resources	$\leqslant 200$	$\leqslant 16$	$\leqslant 3.85$	$\leqslant 1$	Maximize

and

$$P^* = \frac{362800}{7}UC = 51,828.57UC$$

The actual resources used are $200kg$, $11.905kg$, $3.191kg$ and $1kg$ of flour, sugar, salt and butter respectively.

5.2. Obtaining Fuzzy Solution When Only Some Constraints are Fuzzy

Given that the flour used has a tolerance of $50kg$ and the salt has a tolerance of $0.8kg$, the following are the membership functions for flour and salt, using the Verdegay's Approach—A Nonsymmetric Model. Figures 1 and 2 for flour,

$$\mu_1(x) = \begin{cases} 1, & \text{if } f_1(x) \leqslant 200 \\ 1 - \dfrac{f_1(x) - 200}{50}, & \text{if } 200 < f_1(x) < 250 \\ 0, & \text{if } f_1(x) \geqslant 250 \end{cases} \tag{15}$$

For salt,

$$\mu_3(x) = \begin{cases} 1, & \text{if } f_3(x) \leqslant 3.85 \\ 1 - \dfrac{f_3(x) - 3.85}{0.8}, & \text{if } 3.85 < f_3(x) < 4.65 \\ 0, & \text{if } f_3(x) \geqslant 4.65 \end{cases} \tag{16}$$

The fuzzy linear programme of (14) is

$$\begin{aligned} \text{Maximize } & P(x_1,x_2,x_3,x_4,x_5) = 24.88x_1 + 35.18x_2 \\ & \quad +119.39x_3 + 126.84x_4 + 90.84x_5 \\ \text{Subject to } & f_1(x) = (126x_1 + 174x_2 + 420x_3 + 420x_4 \\ & \quad +420x_5)10^{-3} \leqslant 200 + 50(1-\alpha) \\ & f_2(x) = (7.5x_1 + 10x_2 + 59x_3 + 25x_4 \\ & \quad +25x_5)10^{-3} \leqslant 16 \\ & f_3(x) = (2x_1 + 2.8x_2 + 5.5x_3 + 6.7x_4 \\ & \quad +6.7x_5)10^{-3} \leqslant 3.85 + 0.8(1-\alpha) \\ & f_4(x) = (4.2x_3 + 4.2x_4)10^{-3} \leqslant 1 \end{aligned} \tag{17}$$

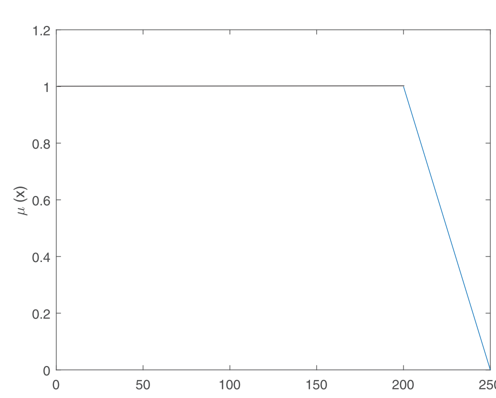

Figure 1 The membership function of flour constraint.

where $x_1, x_2, x_3, x_4, x_5 \geqslant 0$ and $\alpha \in [0,1]$. Setting $\theta = 1-\alpha$ (θ gives the level set for each of the membership function), the following fuzzy programming problem is obtained

$$\begin{aligned} \text{Maximize } & P(x_1,x_2,x_3,x_4,x_5) = 24.88x_1 + 35.18x_2 \\ & \quad +119.39x_3 + 126.84x_4 + 90.84x_5 \\ \text{Subject to } & f_1(x) = (126x_1 + 174x_2 + 420x_3 + 420x_4 \\ & \quad +420x_5)10^{-3} \leqslant 200 + 50\theta \\ & f_2(x) = (7.5x_1 + 10x_2 + 59x_3 + 25x_4 \\ & \quad +25x_5)10^{-3} \leqslant 16 \\ & f_3(x) = (2x_1 + 2.8x_2 + 5.5x_3 + 6.7x_4 \\ & \quad +6.7x_5)10^{-3} \leqslant 3.85 + 0.8\theta \\ & f_4(x) = (4.2x_3 + 4.2x_4)10^{-3} \leqslant 1 \end{aligned} \tag{18}$$

where $x_1, x_2, x_3, x_4, x_5 \geqslant 0$ and $\theta \in [0,1]$ is a parameter of the level set. Using the parametric technique and the simplex method, the optimal solution is then

$$\begin{aligned} x^* &= \left(0,0,0,\frac{5000}{21},\frac{5000}{21} + \frac{2500}{21}\theta\right) \\ &= (0,0,0,238.095,238.095 + 119.048\theta) \end{aligned}$$

and

$$P^* = \frac{362800}{7} + \frac{75700}{7}\theta = 51828.571 + 10814.286\theta$$

The solutions of the FLP (18) is given in the Table 4 below.

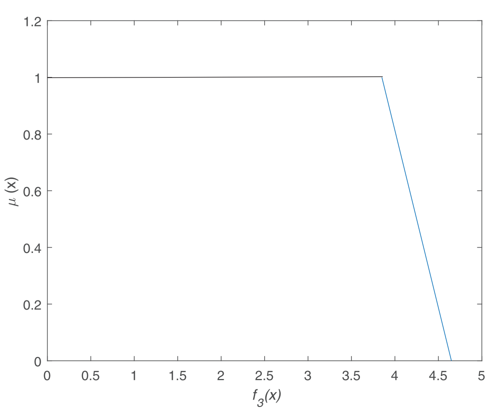

Figure 2 The membership function of salt constraint.

Table 4 The solutions of the fuzzy liner programing (FLP) (18).

θ	P^*	f_1	f_2	f_3	f_4
0.00	51828.52	200.00	11.91	3.19	1.00
0.10	52910.00	205.00	12.20	3.27	1.00
0.20	53991.43	210.00	12.50	3.35	1.00
0.30	55072.66	215.00	12.80	3.43	1.00
0.40	56154.29	220.00	13.06	3.51	1.00
0.50	57235.71	225.00	13.39	3.59	1.00
0.60	58317.14	230.00	13.69	3.67	1.00
0.70	59398.57	235.00	13.99	3.75	1.00
0.80	60480.00	240.00	14.29	3.83	1.00
0.90	61561.43	245.00	14.58	3.91	1.00
1.00	62642.86	250.00	14.88	3.99	1.00

5.3. Obtaining Fuzzy Solution When Both the Objective Function and All Constraints are Fuzzy

From the crisp LP problem in (14), it can be seen that 16kg of sugar and 3.85kg of salt will have too many ideal resources. So, given that 12kg of sugar and 3.45kg of salt are used with tolerances 50kg, 4kg, 0.4kg and 0.2kg for flour, sugar, salt and butter respectively. Then, the problem becomes

$$\begin{aligned} \text{Maximize } & P(x_1,x_2,x_3,x_4,x_5) = 24.88x_1 + 35.18x_2 \\ & \quad +119.39x_3 + 126.84x_4 + 90.84x_5 \\ \text{Subject to } & f_1(x) = (126x_1 + 174x_2 + 420x_3 + 420x_4 \\ & \quad +420x_5)10^{-3} \leqslant \widetilde{200} \\ & f_2(x) = (7.5x_1 + 10x_2 + 59x_3 + 25x_4 \\ & \quad +25x_5)10^{-3} \leqslant \widetilde{12} \\ & f_3(x) = (2x_1 + 2.8x_2 + 5.5x_3 + 46.7x_4 \\ & \quad +6.7x_5)10^{-3} \leqslant \widetilde{3.45} \\ & f_4(x) = (4.2x_3 + 4.2x_4)10^{-3} \leqslant \tilde{1} \end{aligned} \tag{19}$$

where $x_1, x_2, x_3, x_4, x_5 \geqslant 0$ and the membership function $\mu_i(x)$ for the ith fuzzy constraints are

$$\mu_1(x) = \begin{cases} 1, & \text{if } f_1(x) \leqslant 200 \\ 1 - \dfrac{f_1(x) - 200}{50}, & \text{if } 200 < f_1(x) < 250 \\ 0, & \text{if } f_1(x) \geqslant 250 \end{cases} \tag{20}$$

$$\mu_2(x) = \begin{cases} 1, & \text{if } f_2(x) \leqslant 12 \\ 1 - \dfrac{f_2(x) - 12}{4}, & \text{if } 12 < f_2(x) < 16 \\ 0, & \text{if } f_2(x) \geqslant 16 \end{cases} \tag{21}$$

$$\mu_3(x) = \begin{cases} 1, & \text{if } f_3(x) \leqslant 3.45 \\ 1 - \dfrac{f_3(x) - 3.45}{0.4}, & \text{if } 3.45 < f_3(x) < 3.85 \\ 0, & \text{if } f_3(x) \geqslant 3.85 \end{cases} \tag{22}$$

$$\mu_4(x) = \begin{cases} 1, & \text{if } f_4(x) \leqslant 1 \\ 1 - \dfrac{f_4(x) - 1}{0.2}, & \text{if } 1 < f_4(x) < 1.2 \\ 0, & \text{if } f_4(x) \geqslant 1.2 \end{cases} \tag{23}$$

Setting $\theta = 1 - \alpha$, $\theta \in [0,1]$ is a parameter. Using the parametric technique and the simplex method, we get,

$$x^* = \left(0,0,0,\frac{5000}{21} + \frac{1000}{21}\theta, \frac{5000}{21} + \frac{1500}{21}\theta\right)$$

and

$$P^* = \frac{362800}{7} + \frac{87700}{7}\theta$$

Then, solving Equation (11),

$$P^0 = P^*(\theta = 0) = 51828.57$$

$$P^1 = P^*(\theta = 1) = 64357.14$$

where θ is the parameter in the new parametric programming problem. The symmetric LP problem can then be formulated as follows:

$$\begin{aligned} &\text{Maximize} && \alpha \\ &\text{Subject to} && \mu_0(x) \geqslant \alpha \\ &&& \mu_i(x) \geqslant \alpha, i = 1,2,3,4 \\ &&& \alpha \in [0,1] \end{aligned} \tag{24}$$

where $x \geqslant 0$ and the membership function μ_0 of the fuzzy objective $\mu_0(x)$ is defined as

$$\begin{cases} 1, & \text{if } P \geqslant 64357.14 \\ 1 - \dfrac{64357.14 - P}{64357.14 - 51828.57}, & \text{if } 51828.57 < P < 64357.14 \\ 0, & \text{if } P \leqslant 51828.57 \end{cases} \tag{25}$$

The problem is actually equivalent to

$$\begin{aligned} &\text{Minimize } \theta \\ &\text{Subject to } 24.88x_1 + 35.18x_2 + 119.39x_3 + 126.84x_4 \\ &\quad +90.84x_5 \geqslant 64357.14 - 12528.57 \\ &\quad f_1(x) = (126x_1 + 174x_2 + 420x_3 + 420x_4 \\ &\quad +420x_5)10^{-3} \leqslant 200 + 50\theta \\ &\quad f_2(x) = (7.5x_1 + 10x_2 + 59x_3 + 25x_4 \\ &\quad +25x_5)10^{-3} \leqslant 12 + 4\theta \\ &\quad f_3(x) = (2x_1 + 2.8x_2 + 5.5x_3 + 6.7x_4 \\ &\quad +6.7x_5)10^{-3} \leqslant 3.45 + 0.4\theta \\ &\quad f_4(x) = (4.2x_3 + 4.2x_4)10^{-3} \leqslant 1 + 0.2\theta \end{aligned} \tag{26}$$

where $x_1, x_2, x_3, x_4, x_5 \geqslant 0$ and $\theta = 1 - \alpha \in [0,1]$. The solution is

$$x^* = (0,0,0,238.095 + 47.62\theta, 238.095 + 71.43\theta).$$

6. COMPARISON OF RESULTS

As can be seen from Section 5.1, using the classical LP produced one feasible point

$$\begin{aligned} x^* &= \left(0,0,0,\frac{5000}{21}, \frac{5000}{21} + \frac{5000}{21}\theta\right) \\ &= (0,0,0,238.095, 238.095 + 119.048\theta) \end{aligned}$$

with the optimal profit

$$\begin{aligned} P^* &= \frac{362800}{7} + \frac{75700}{7}\theta \\ &= 51828.571 + 10814.286\theta \end{aligned}$$

The result of Verdegay's model can be seen in Table 4, giving various optimal points inclusive of the one obtained by classical LP. More importantly, the result of Werner's model can be seen in Table 5. It also gave as many optimal solutions as the Verdegay's model but, with as much resources as used under Verdegay's model, it gives greater profit margin.

7. CONCLUSIONS

The study has revealed that, using FLP gives a more robust information on making decision. More importantly, it has also shown that, with almost the same resources, modeling production mix with both the objective function and the constraints being fuzzy yields a better result and can enhance better performance and production output more than crisp modeling and modeling with only the resources being fuzzy.

Table 5 The solutions of the fuzzy linear programing (FLP) (26).

θ	P^*	f_1	f_2	f_3	f_4
0.00	51828.52	200.00	11.91	3.19	1.00
0.10	53081.40	205.00	12.20	3.27	1.02
0.20	54334.28	210.00	12.50	3.35	1.04
0.30	55587.17	215.00	12.80	3.43	1.06
0.40	56840.05	220.00	13.06	3.51	1.08
0.50	58092.93	225.00	13.39	3.59	1.10
0.60	59345.81	230.00	13.69	3.67	1.12
0.70	60598.70	235.00	13.99	3.75	1.14
0.80	61851.58	240.00	14.29	3.83	1.16
0.90	63104.46	245.00	14.58	3.91	1.18
1.00	64357.34	250.00	14.88	3.99	1.20

CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest.

AUTHORS' CONTRIBUTIONS

All the authors have contributed fairly well.

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