

# Mathematical Communication on Solution of Divergence Problems Viewed from the Type of Geometric Analogy Reasoning Students

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**Abstract**-One of the most effective strategies to enhance student's geometric understanding in mathematical communication is by giving assignments based on analogy reasoning. Geometric analogical reasoning (GAR) is a reasoning skill used in completing geometry case by applying visual, concept or problem-solving approach. Completion of assignments with GAR increases quality when combined with divergence problems question that required by students to be well communicated. This study aims to analyze differences in student's mathematical communication on geometric problems with divergence problem model that is reviewed based on GAR type. There was six students were involved in this research. This study was conducted using qualitative approach with observation and in-depth interview as data collection methods. The data was analyzed by followed systematic procedure includes: data reduction, data display, data interpretation, and conclusion. Based on the analysis show that there are three different types of GAR in mathematic students, namely the visual analogy, the analogy of structure or conceptual, and the analogy in the way of completion as well as three different types of communication. The type of communication observed has a pattern that is congruent with the three types of GAR that emerge, namely explanation through images or visuals, through concepts and through the inclusion of examples. Thus, it is suggested, (1) students must be able to make questions in the form of divergence problems. (2) Students must have the ability to communicate divergence problems solutions according to the type of GAR they have.

**Keywords:** *mathematical communication, divergence problems, GAR*

## I. INTRODUCTION

Problem solving in mathematics, especially in geometry topic requires good mathematical communication skills. The communication was composed by two distinguish skill, a mathematic

written communication skill occurs when students work on a written task sheet (Santos & Semana, 2015), then mathematic verbal communication skill occurs when students deliver the completion of an assignment sheet to the other students (Putra, Widyawati, Asyhari, & Putra, 2018). High developed communication skill is depicted by more detailed, completed, and systematic information delivered by student. It means high developed communication make problem-solving more communicative. The students' mathematical communication skill needs to be developed by variety strategies, such as giving homework task that has similar problem and solutions from learning class session, then increase task complexity and requires holistic solutions. In other research, one indicator of mathematical ability is the ability to perform analogy reasoning. So, by giving tiered assignments may contribute increase student logical and analogical skill in mathematic problems (Rohrer, Dedrick, & Stershic, 2015).

Analogy reasoning frequently involves in both problem-solving and theorem proofing of geometry topic. According to (Magdas, 2015), analogy reasoning on geometry is expressed in constructed analogy argument. Problem-solving and theorem proofing in Geometry topic involve certain skills of geometric analogy reasoning (GAR) (Lovett & Forbus, 2017). GAR is a reasoning skill and ability in geometric problem solving that characterized analogical pattern skill. As the result, mathematical communication will be different for problems with the same solution, similar, or somewhat like the problem that has been given.

Generally, solving the problems related to geometry space is necessary to begin by drawing a 3D space. A precision picture can help to speed up the analogical reasoning that enhances problem solving (Fava, 2017; Loc & Uyen, 2015; Tunteler, Pronk, & Resing, 2008). Reasoning geometry analogy is needed because lines and spot are needed as an assistance to obtain certain patterns. By repeating some cases and problem-solving method that similar to the previous problems, although not always every problem has exact same patterns the mathematic student will be help to increase their skill in problem solving. Mathematic student as a prospective

teacher need to be trained to communicate their understanding and problem solving skill based on the GAR's types of their own (Lauermann & König, 2016).

Student's ability in GAR composes by encoding, inferring, mapping, and applying are important to be enhance simultaneously to break gaps down between learning needs and daily life. As a teacher, difference analysis is needed on how to communicate the essential topics of mathematics by applying divergence problems approach in terms of the type of GAR.

Divergence problems approach can be implemented in mathematical cases by composing different level of quests, that openly answered by the student (Nogueira & Veiga, 2014). It means the students are able to answer by their own perspective regarding their understanding and knowledge achievement on GAR. In further way, giving free opportunities to student to answer openly may reveal their ways of mathematical communication skill. By those explanations this research aims to study form of the divergence problems and analyzing the GAR of mathematic student and its relationship

#### A. *Geometric Analogy Reasoning (GAR)*

Geometry Space is a topic in mathematic learning delivered at every education level, from elementary to high school with different material depths in every level. Each level requires different reasoning, so giving the right and appropriate assignments is very important (Ayllon, Gomez, & Ballesta-Claver, 2016; Psycharis & Kallia, 2017) In the initial stages of geometry learning, student reasoning can be built through practice exercises that are similar to the examples that have been explained, then in the next stage, the complexity of the exercises and how to solve them. improved in a more varied way.

Meanwhile, to increase GAR capability there are various strategies that can be used. The GAR of the student can be developed through applying innovative and creative learning process and technology, which is assisted by computer application programs, such as Geogebra (Arbain & Shukor, 2015; Botana et al., 2015) or Geometer's Sketchpad (Dhayanti, Johar, & Zubainur, 2018; Khairiree, 2015; Tieng & Eu, 2014). The similarity cases was frequently used by student in this case can be similar in whole or only similar for some aspects.

Students who are trained to solve problems by applying patterns of reasoning based on similarity or similarity can be trained through three things. The three things include comparing two different things in terms of similarities and differences, looking for related relationships, and find conclusions based on similarities to determine a solution. Reasoning that has an analogy pattern in GAR needs to be supported by the ability to communicate with problem solving, according to the type or type of reasoning analogy.

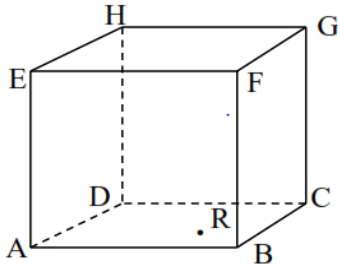
#### B. *Divergence Problems for Revealing the Growth of Geometric Analogy Reasoning*

Divergence problems or divergent tasks is cases or assignments that gives more than one alternative solution for student to solve (Yang, Tseng, & Wang, 2017). Based on the characteristic, divergence problems can be grouped as divergence problems with different correct answers (DP 1) and divergence problems with many ways or strategies in the solution (DP 2). Furthermore, mathematic lecturers need to give assignments or problems that answered using solution by applying analogy reasoning, so students can be trained to improve their GAR.

To understand the topic, the following sample depict variation of divergent problem with the type is DP 1. This case was used to assess GAR skill of the subject in this research.

Case 1 : DP 1.

Look at the picture of the ABCD-EFGH cube below.

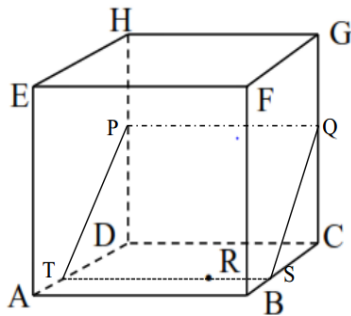


Is known:  
 ABCD-EFGH cube.  
 Point R is located in the ABCD plane.  
 Point P on the DH rib and Q point on the CG rib.

Describe:  
 Plane  $\alpha$  through points P, Q, and R.  
 Describe: Draw the  $\alpha$  plane in various positions, based on several possible positions of the P and Q points that you specify.

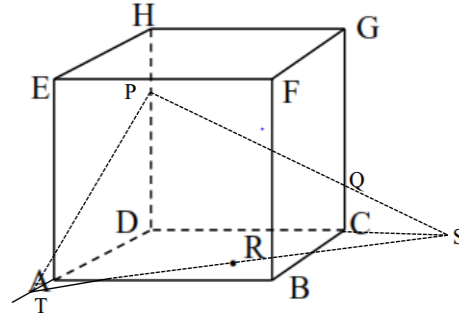
The DP 1 problem has different correct answers because it is a type 1 divergence problem. Some alternative correct answers can be found based on analogy reasoning. Analytical reasoning: (1) Through 2 points one-line can be made through both points. (2) Through two parallel or intersecting lines, a plane can be made.

Answer 1: If  $PD = QC$ .



When  $PD > QC$ .  
 Connect PQ, then the PQ line will cut DC in S.  
 Connect SR, then the SR line will cut DA in T.  
 Connect PT.  
 Then the plane  $\alpha =$  PST triangle plane

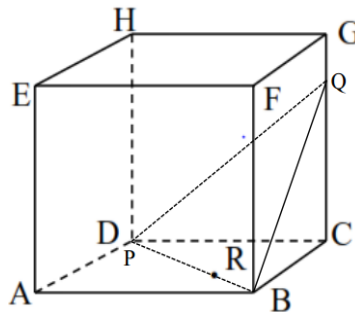
Answer 2: If  $PD > QC$



$PD = QC$ . Connect TS via R  
 Connect TP, PQ, and QS.  
 Then the field  $\alpha =$  level of PQST because  
 $PQ \parallel TS$  and  $TP \parallel SQ$ .

Other correct answers can be obtained if: (1) Q and C coincide. (2) BRD is not in line. (3) Point P and point D coincide, so also point Q and point C coincide.

Answer 3: If point P and point D coincide and BRD is in line.



Point P and point D coincide; and BRD in line.  
 Connect PQ. Connect BQ.  
 Then the plane  $\alpha =$  PQB triangle plane.

By assigning divergent and varied tasks or problems, students are expected to be trained and accustomed to communicating their own opinions and ideas, both in writing and orally / verbally when students are asked to present their findings. The assignment of divergent tasks / problems that are solved using analogous reasoning is expected to: (1) be a revealer of the growth of quality GAR; (2) gives students the opportunity to choose how to communicate their findings so that the idea can be captured by other parties (Da Veiga, Lovadina, & Vacca, 2017).

## II. METHODS

This research was conducted using qualitative research approach focused on student's divergence problem analysis based on geometric analogy reasoning. There are six first year-Mathematic students from Department of Mathematic Education, Faculty of Science and Technology, Universitas Islam Negeri Walisongo, Semarang was involved in this research as

the subject. For the first observation, all student was asked to finish geometric problem, then the answer was confirmed with in-depth interview. The qualitative explanation was dug up using in-depth interview, all the data then compared with affective, learning activity, portfolio and self-assessment of student capability in geometric analogy reasoning problem solving. The interview session was done using double blinded method using guided questionnaire and developed based on the student's answer.

All collected data then tabulated and coded to utilize the massive information followed by reduced unnecessary or junk data. The clean data then was interpreted, verified, and concluded before displayed.

### III. RESULTS AND DISCUSSION

Divergence problems can be used as a tool to train the quality of geometric analogy reasoning, because with divergence problems students can develop their ideas in forming GAR naturally. Based on the analysis result of student's GAR types. The type that many students have is the visual type of GAR. The amount reaches 47%. While the GAR type of structural or conceptual analogy was around 32%. For the problem-solving analogy of GAR is around 21%. This is quite reasonable because students tend to be able to work on type 1 because the similarity in working on the questions is almost the same as the example given by the lecturer. While in the 3rd type, namely the analogy in how to solve it, not many students have it. Students need to have adequate intelligence and often practice working on varied problems.

Based on observations made shows that the divergence problems approach can be used as a tool to train the growth of quality GAR. That is because divergence problems give students the opportunity to find several correct answers and different problem-solving strategies. In addition, the findings obtained from the results of student interviews show that there are three types of GAR, namely a visual analogy, a structural or conceptual analogy, and an analogy in how to solve it. Both the visual analogy, conceptual and way of solving shows that students can identify the similarity of problems between the target and the problems found at the source, while identifying differences.

After finished the task, every the student was grouped into three different group then interviewed, as the result of the interview :

- Q : is representing the interviewer.  
 S1 : is representing the students who performed mathematical communication using visual type  
 S2 : is representing the student who performed mathematical communication using analogy structure or conceptual to answer geometric

cases.

- S3 : is representing the student who performed communication in problem-solving with analogy type of how to completion the geometric case

#### First group

- Q : "Do you see the similarity of the questions you just finish with the question were discussed in previous lecture? Give an explanation of the similarity."  
 S1 : "Yes. This problem has the same thing that is building space which is a problem that is in the form of a cube."  
 Q : "Are they all the same? or is there something different?"  
 S1 : "Not all the same, Although building space on the problem that I face with what has been discussed has the same shape, which is a cube, but there are different things."  
 Q : "What's the difference?"  
 S1 : "Position of a known point on the side of the cube."  
 Q : "Then how do you look for a solution?"  
 S1 : "In the same way, **I first draw a line by connecting two points on the ribs but still in the same plane.**"  
 Q : "Why do you use this method?"  
 S1 : "because it is easier for me to determine the next step after I have obtained the picture from connecting the two dots"

#### Second Group

- Q : "Do you see the similarity of the questions you just finish with the question were discussed in previous lecture? Give an explanation of the similarity."  
 S2 : "Yes, they are, both of the question has similarity. The concept used to complete the slice of building space, with the help of the affinity axis."  
 Q : "Are they all the same? or is there something different?"  
 S2 : "There is something different."  
 Q : "What's the difference?"  
 S2 : "The affinity axis sought can be in the upper plane."  
 Q : "How do you find a solution?"  
 S2 : "**I finished by applying the same concept, which is about the affinity axis in the upper plane.**"  
 Q : "Why do you use this method?"  
 S2 : "Because the affinity axis in the lower plane cannot be found, the help cut lines are too far away so it is difficult to find. After I tried

applying the affinity axis to the top plane, it turned out that the incision plane could be obtained.”

### Third group

- Q : “Do you see the similarity of the questions you just finish with the question were discussed in previous lecture? Give an explanation of the similarity.”
- S3 : “Yes, same. the completion steps are the same.”
- Q : “How do you know that the problem has the same solution?”
- S3 : “Because what is asked is the same.”
- Q : “Are they all the same? or is there something different?”
- S3 : “No. The shape of the building is different.”
- Q : “What's the difference?”
- S3 : “Build the space that was discussed in the form of a cube, while in this problem build a pyramid-shaped space.”
- Q : “Then how do you look for a solution?”
- S3 : “Before presenting the solution in the form of a drawing, **I first describe the steps of the solution following the completion of the problem that has been discussed, namely: the first step to make the secant lines, the second step to find the affinity axis, and the third step to find the intersection of the lines with each building area room.**”
- Q : “Why do you use this method?”
- S3 : “Because with the steps in the method of resolution that I plan to follow in solving the problems that have already been discussed, the results will be more directed and I can predict the final outcome.”

Problem solving done by students is also followed by the ability to communicate solutions to problems well through an explanation of the relationship between the two elements presented in the form of images. Specifically, students who have a more prominent visual analogy ability have the ability to identify similarities based on the shape or visualization of the existing space in the problem of the target or source (Debrenti, 2015). Mathematical communication conducted by students with visual analogy skills shows the translation of problem solving is done in the form of space sketches and field drawings. Visual representations are more easily communicated for simple building objects.

Whereas students with conceptual analogies show the ability to evaluate and clarify errors. The mathematical communication formed by students in the geometric problem based on their own GAR type. The structure or conceptual type was performed by constructing evident using mathematic symbols, mathematical models, and characteristics of similar concepts. The development of students' knowledge experience about concepts related to problem solving,

plays a role in the form of representation of the concept being communicated where students are more focused on efforts to find similar problem-solving techniques that have been faced before. In this type, the form of mathematical communication is carried out by means of solving the same method, but the steps used were different from the example problems that have been given. Difficulties in communicating this type are experienced by students who rarely deal with problems in various forms. By this repetition, student was helped to construct a holistic concept and may able to apply in other cases (Burns, Ysseldyke, Nelson, & Kanive, 2014)

Based on the results of this study, the GAR is one form of reasoning in geometry that needs to be grown. It mainly aims to improve the quality and ability of mathematics education students. This increase will certainly have a positive impact on the development of mathematical analogy reasoning abilities for students to be taught. In other words, mathematics education students as mathematics teacher candidates can provide explanations about problem solving based on solutions characterized by GAR, and teachers also have the ability to communicate problem solving to students according to GAR type.

## IV. CONCLUSIONS

Problems can be used as a tool to train the growth of qualified Geometric Analogy Reasoning, because it requires students to find some correct answers, different strategies, or ways of solving problems. The types of GAR students found in this study were of three types, namely visual, structural or conceptual analogy, and analogy in how to solve them. (3) Based on the type of GAR, there are three different ways of communicating the solution of Divergence Problems.

Based on the facts found in this study, it can be suggested that mathematics students must have the ability to make problems with the type of Divergence Problems because as an educator, are required to be able to improve student reasoning. In addition, the ability to communicate a GAR-based Divergence Problem solution needs to be developed because it is related to the ability to understand students who will be educated in the future.

## REFERENCES

- [1] Arbain, N., & Shukor, N. A. (2015). The Effects of GeoGebra on Students Achievement. *Procedia - Social and Behavioral Sciences*, 172(2007), 208–214. <https://doi.org/10.1016/j.sbspro.2015.01.356>
- [2] Ayllon, M., Gomez, I., & Ballesta-Claver, J. (2016). Mathematical thinking and creativity through mathematical problem posing and solving. *Propósitos y Representaciones*, 4(1), 169–218. <https://doi.org/http://dx.doi.org/10.20511/pyr2016.v4n1.89>
- [3] Botana, F., Hohenwarter, M., Janičić, P., Kovács, Z., Petrović, I., Recio, T., & Weitzhofer, S. (2015). Automated Theorem Proving in GeoGebra: Current Achievements. *Journal of Automated Reasoning*, 55(1), 39–59.

- <https://doi.org/10.1007/s10817-015-9326-4>
- [4] Burns, M. K., Ysseldyke, J., Nelson, P. M., & Kanive, R. (2014). Number of repetitions required to retain single-digit multiplication math facts for elementary students school Psychology Quarterly Number of Repetitions Required to Retain Single-Digit. *School Psychology Quarterly*, 30(3), 398–405. <https://doi.org/10.1037/spq0000097>
- [5] Da Veiga, L. B., Lovadina, C., & Vacca, G. (2017). Divergence free virtual elements for the stokes problem on polygonal meshes. *ESAIM: Mathematical Modelling and Numerical Analysis*, 51(2), 509–535. <https://doi.org/10.1051/m2an/2016032>
- [6] Debrenti, E. (2015). Visual Representations in Mathematics Teaching: An Experiment with Student. *Acta Didactica Napocensia*, 8(1), 1–15.
- [7] Dhayanti, D., Johar, R., & Zubainur, C. M. (2018). Improving Students' Critical and Creative Thinking through Realistic Mathematics Education using Geometer's Sketchpad. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 3(1), 25. <https://doi.org/10.23917/jramathedu.v3i1.5618>
- [8] Fava, M. (2017). Drawing Analogies to Deepen Learning. *International Journal of Art and Design Education*, 36(3), 315–324. <https://doi.org/10.1111/jade.12162>
- [9] Khairree, K. (2015). Creative Thinking in Mathematics with Tangrams and The Geometer ' s Sketchpad. *Proceedings of the 20th Asian Technology Conference in Mathematics*, 153–161.
- [10] Lauer mann, F., & König, J. (2016). Teachers' professional competence and wellbeing: Understanding the links between general pedagogical knowledge, self-efficacy and burnout. *Learning and Instruction*, 45, 9–19. <https://doi.org/10.1016/j.learninstruc.2016.06.006>
- [11] Loc, N. P., & Uyen, B. P. (2015). A Study of Mathematics Education Students' Difficulties in Applying Analogy to Teaching Mathematics: A Case of the " TWA " Model. *American International Journal of Research in Humanities, Arts and Social Sciences*, 9(3), 276–280.
- [12] Lovett, A., & Forbus, K. (2017). Modeling visual problem solving as analogical reasoning. *Psychological Review*, 124(1), 60–90. <https://doi.org/10.1037/rev0000039>
- [13] Magdas, I. (2015). Analogical Reasoning in Geometry Education. *Acta Didactica Napocensia*, 8(1), 57–65.
- [14] Nogueira, J., & Veiga, F. (2014). Relationships as a basis of engagement? Self-efficacy and school engagement of pupils in school. *Envolvimento Dos Alunos Na Escola: Perspetivas Internacionais Da Psicologia e Educação/Students' Engagement in School: International Perspectives of Psychology and Education*, 373–385.
- [15] Psycharis, S., & Kallia, M. (2017). The effects of computer programming on high school students' reasoning skills and mathematical self-efficacy and problem solving. *Instructional Science*, 45(5), 583–602. <https://doi.org/10.1007/s11251-017-9421-5>
- [16] Putra, F. G., Widyawati, S., Asyhari, A., & Putra, R. W. Y. (2018). The Implementation of Advance Organizer Model on Mathematical Communication Skills in terms of Learning Motivation. *Tadris: Jurnal Keguruan Dan Ilmu Tarbiyah*, 3(1), 41. <https://doi.org/10.24042/tadris.v3i1.2208>
- [17] Rohrer, D., Dedrick, R. F., & Stershic, S. (2015). Interleaved practice improves mathematics learning. *Journal of Educational Psychology*, 107(3), 900–908. <https://doi.org/10.1037/edu0000001>
- [18] Santos, L., & Semana, S. (2015). Developing mathematics written communication through expository writing supported by assessment strategies. *Educational Studies in Mathematics*, 88(1), 65–87. <https://doi.org/10.1007/s10649-014-9557-z>
- [19] Tieng, P. G., & Eu, L. K. (2014). Improving Students' Van Hiele Level of Geometric Thinking Using Geometer's Sketchpad. *Malaysia Online Journal of Educational Technology*, 2(3), 20–31.
- [20] Tunteler, E., Pronk, C. M. E., & Resing, W. C. M. (2008). Inter- and intra-individual variability in the process of change in the use of analogical strategies to solve geometric tasks in children: A microgenetic analysis. *Learning and Individual Differences*, 18(1), 44–60. <https://doi.org/10.1016/j.lindif.2007.07.007>
- [21] Yang, D. C., Tseng, Y. K., & Wang, T. L. (2017). A comparison of geometry problems in middle-grade mathematics textbooks from Taiwan, Singapore, Finland, and the United States. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(7), 2841–2857. <https://doi.org/10.12973/eurasia.2017.00721a>