International Journal of Computational Intelligence Systems
Vol. 13(1), 2020, pp. 1072–1091
DOI: https://doi.org/10.2991/ijcis.d.200717.001; ISSN: 1875-6891; eISSN: 1875-6883
https://www.atlantis-press.com/journals/ijcis/

Research Article

Specific Types of *q*-Rung Picture Fuzzy Yager Aggregation Operators for Decision-Making

Peide Liu[1,*], Gulfam Shahzadi[2], Muhammad Akram[2]

[1]School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan, Shandong, China
[2]Department of Mathematics, University of the Punjab, New Campus, Lahore, Pakistan

ARTICLE INFO

Article History

Received 08 Jun 2020
Accepted 16 Jul 2020

Keywords

q-rung picture fuzzy numbers
Yager operators
Arithmetic
Geometric
Multi-attribute decision-making problems

ABSTRACT

q-rung picture fuzzy sets can handle complex fuzzy and impression information by changing a parameter q based on the different hesitation degree, and Yager operator is a useful aggregation technology that can control the uncertainty of valuating data from some experts and thus get intensive information in the process of decision-making. Thus, in this paper, we develop specific types of operators, namely, q-rung picture fuzzy Yager weighted average, q-rung picture fuzzy Yager ordered weighted average, q-rung picture fuzzy Yager hybrid weighted average, q-rung picture fuzzy Yager weighted geometric, q-rung picture fuzzy Yager ordered weighted geometric and q-rung picture fuzzy Yager hybrid weighted geometric operators. We propose q-rung picture fuzzy Yager aggregation operators to handle multiple attribute decision-making problems in a modernize way. Moreover, we discuss the effect of parameter on the decision-making results. To demonstrate the superiority and advantage of our proposed method, a comparison with existing methods is presented.

1. INTRODUCTION

Decision-making (DM) plays a vital role in the practical life activities of human beings as it refers to a process that lays out all the options according to the assessment data of the decision makers (DMrs) and then selects the excellent one, mostly happening in our everyday lives. In the early era of social development, DMrs utilized the real numbers as a rule to offer their assessment information. As the multi-attribute decision-making (MADM) problems are becoming complex, the experts cannot give exact real numbers to assess the alternatives. The ambiguities and imprecision of human judgements highlighted the deficiency of the crisp set theory. Therefore, Zadeh [50] laid the foundations of the fuzzy set (FS) theory for uncertain knowledge that permits the experts to describe their satisfaction level (membership degree [MD]) regarding performance of a member within the unit interval. To solve DM problems, many operators in fuzzy environment were introduced. Song *et al.* [38] studied few operators in fuzzy environment. Atanassov [9] introduced intuitionistic fuzzy set (IFS) which has both MD μ and nonmembership degree (NMD) ν with condition $\mu + \nu \leq 1$. Xu [45] studied IF aggregation operators (AOs). Zhao *et al.* [51] developed generalized AOs for IFS. Tan *et al.* [39,40] developed generalized geometric AOs in IF environment. For aggregating the various alternatives, with the help of different AOs, many researchers gave more attention to IFS problems [1,11–13,28,37,46]. Cuong [10] introduced picture fuzzy set (PFS) as a generalization of IFS. PFS handles the situations when the expert's judgment is of a type like yes, abstinence, no and rejection. PFS is denoted by triplet (μ, η, ν), positive MD, neutral MD and negative MD, respectively with condition $\mu + \eta + \nu \leq 1$. Garg [16] developed picture fuzzy AOs. Wei [43] studied PFAOs and discussed some of its applications in DM. The Einstein AOs under picture fuzzy (PF) environment were handled by Khan *et al.* [22]. Jana *et al.* [20] gave the theory of PF Dombi AOs. Many new operations on interval-valued PFS and interval-valued PF soft set were defined by Khalil *et al.* [21]. Lin *et al.* [23] discussed the MULTIMOORA-based MADM problem under PF environment.

Yager [48] introduced Pythagorean fuzzy set (PyFS). The main characteristic of this model is that it replaces the constraint of IFS with the condition $0 \leq \mu^2 + \nu^2 \leq 1$. Yager [49] proposed q-rung orthopair fuzzy set (q-ROFS) in which sum of q^{th} power of MD and NMD is less than or equal to 1. Liu and Wang [29] discussed q-rung orthopair fuzzy (q-ROF) weighted AOs. The idea of Bonferroni mean weighted AOs were extended to q-ROF information by Liu and Liu [27]. Liu *et al.* [25] developed multi-attribute group decision-making (MAGDM) technique using q-ROF power Maclaurin AOs. Dombi AOs for q-ROFS were defined by Jana *et al.* [19]. The neutrality AOs of q-ROFS were studied by Garg and Chen [14]. Garg *et al.* [15] proposed power AOs and VIKOR methods for complex q-ROFS. As an extension of PFS, Gundogdu and Kahraman [17] introduced spherical fuzzy set (SFS). SFS replaces the condition $\mu+\eta+\nu \leq 1$ of PFS with $\mu^2+\eta^2+\nu^2 \leq 1$.

*Corresponding author. Email: peide.liu@gmail.com

Ashraf *et al.* [7] gave the notion of spherical fuzzy AOs. Li *et al.* [31] studied the conception of q-rung picture fuzzy set (q-RPFS) with constraint $\mu^q + \eta^q + \nu^q \leq 1$ in 2018. In DM problems, q-RPFS is more efficient than PFS and SFS. He *et al.* [18] explained the framework of q-RPF Dombi Hamy mean operators. For other terminologies not discussed in the paper, the readers are referred to [2–6,8,24,26,30,32–36,41,42,44,47].

The motivations of this article are summarized as follows:

1. q-RPFS is more flexible than PFS and SFS to study DM problems.
2. The assessment of the best alternative in a q-RPF environment is a very difficult MADM problem and has several imprecise factors. In the present MADM techniques, assessment data is simply portrayed by picture fuzzy and spherical fuzzy numbers which may prompt data mutilation. Therefore, we need a more general model to elaborate the potential of alternatives.
3. Taking into account that Yager AOs are a straight forward, however ground-breaking, approach for solving DM issues, this article, in general, aims to define Yager AOs in the q-RPF context to tackle difficult problems of choice.
4. Yager AOs make the decision results more precise and exact when applied to real-life MADM problems based on the q-RPF environment as compared to existing operators.
5. Yager AOs are very simplest and short approach for the evaluation of a single choice in the list of various choices.
6. The drawbacks and limitations of existing operators are run over by proposed operators as these operators are more general that work excellently not only for q-RPF information but also for picture fuzzy and spherical fuzzy data.

The contributions of this research are specified as follows:

1. The theory of Yager AOs is extended to q-rung picture fuzzy numbers (q-RPFNs) and some basic results related to them are discussed.
2. An algorithm is proposed to deal complex practical problems with q-RPF data. The proposed algorithm is supported by two MADM problems, one is the selection of suitable emerging technology enterprise and second is selection of the suitable company for investment.
3. The effect of various values of parameter on DM results is discussed.
4. The importance of these operators is depicted through comparison analysis.

The structure of remaining paper is as follows: Section 2 provides basic definitions. In Section 3, Yager operations for q-RPFNs are developed. In Section 4, we study the q-rung picture fuzzy Yager weighted arithmetic (q-RPFYWA) operator, q-rung picture fuzzy Yager ordered weighted arithmetic (q-RPFYOWA) operator, q-rung picture fuzzy Yager hybrid weighted arithmetic (q-RPFYHWA) operator, q-rung picture fuzzy Yager weighted geometric (q-RPFYWG) operator, q-rung picture fuzzy Yager ordered weighted geometric (q-RPFYOWG) operator, q-rung picture fuzzy Yager hybrid weighted geometric (q-RPFYHWG) operator and some results of these operators. In Section 5, an algorithm is provided for MADM problems. In Section 6, we study two MADM problems under these operators, the effect of different parameter's values on DM results and comparison analysis of proposed model with other existing model to show the validity of model. Section 7 provides the conclusion about proposed theory.

2. PRELIMINARIES

Definition 2.1. [9] An IFS I on nonempty set $\mathcal{V}$ is defined as

$$I = \{\langle x, \mu_I(x), \nu_I(x)\rangle\},$$

where $\mu_I : \mathcal{V} \to [0,1]$ and $\nu_I : \mathcal{V} \to [0,1]$ specify MD and NMD of an element $x \in \mathcal{V}$, respectively. $\varpi_I(x) = 1 - \mu_I(x) - \nu_I(x)$ is indeterminacy degree (InD) of an element $x \in \mathcal{V}$.

Definition 2.2. [48] A PyFS P_y on nonempty set $\mathcal{V}$ is defined as

$$P_y = \{\langle x, \mu_{P_y}(x), \nu_{P_y}(x)\rangle\},$$

where $\mu_{P_y} : \mathcal{V} \to [0,1]$ and $\nu_{P_y} : \mathcal{V} \to [0,1]$ specify MD and NMD of an element, respectively. $\varpi_{P_y}(x) = \sqrt{1 - (\mu_{P_y}(x))^2 - (\nu_{P_y}(x))^2}$ is InD.

Definition 2.3. [49] A q-ROFS $\mathcal{F}'$ on nonempty set $\mathcal{V}$ is defined as

$$\mathcal{F}' = \{\langle x, \mu_{\mathcal{F}'}(x), \nu_{\mathcal{F}'}(x)\rangle\},$$

where $\mu_{\mathcal{F}'} : \mathcal{V} \to [0,1]$ and $\nu_{\mathcal{F}'} : \mathcal{V} \to [0,1]$ indicate the MD and NMD of an element $x \in \mathcal{V}$, respectively. $\varpi_{\mathcal{F}'}(x) = \sqrt[q]{1 - (\mu^q_{\mathcal{F}'}(x) + \nu^q_{\mathcal{F}'}(x))}$ is InD.

Definition 2.4. [10] A PFS P on nonempty set $\mathcal{V}$ is represented by

$$P = \{\langle x, \mu_P(x), \eta_P(x), \nu_P(x)\rangle\},$$

where $\mu_P(x) : \mathcal{V} \to [0,1]$, $\eta_P(x) : \mathcal{V} \to [0,1]$ and $\nu_P(x) : \mathcal{V} \to [0,1]$ are positive, neutral and negative MD, respectively, of an element $x \in \mathcal{V}$, respectively. $\pi_P(x) = 1 - (\mu_P(x) + \eta_P(x) + \nu_P(x))$ is refusal MD.

Definition 2.5. [17] A SFS S on nonempty set $\mathcal{V}$ is represented by

$$S = \{\langle x, \mu_S(x), \eta_S(x), \nu_S(x)\rangle\},$$

where $\mu_S(x) : \mathcal{V} \to [0,1]$, $\eta_S(x) : \mathcal{V} \to [0,1]$ and $\nu_S(x) : \mathcal{V} \to [0,1]$ are positive, neutral and negative MD, respectively, of an element $x \in \mathcal{V}$, respectively. $\pi_S(x) = \sqrt[2]{1 - ((\mu_S(x))^2 + (\eta_S(x))^2 + (\nu_S(x))^2)}$ is refusal MD.

Definition 2.6. [31] A q-rung PFS $\mathcal{F}$ on nonempty set $\mathcal{V}$ is represented by

$$\mathcal{F} = \{\langle x, \mu_{\mathcal{F}}(x), \eta_{\mathcal{F}}(x), \nu_{\mathcal{F}}(x)\rangle\},$$

where $\mu_{\mathcal{F}}(x) : \mathcal{V} \to [0,1]$, $\eta_{\mathcal{F}}(x) : \mathcal{V} \to [0,1]$ and $\nu_{\mathcal{F}}(x) : \mathcal{V} \to [0,1]$ are positive, neutral and negative MD, respectively, of an element $x \in \mathcal{V}$, respectively. $\pi_{\mathcal{F}}(x) = \sqrt[q]{1 - ((\mu_{\mathcal{F}}(x))^q + (\eta_{\mathcal{F}}(x))^q + (\nu_{\mathcal{F}}(x))^q)}$ is refusal MD. For easiness, $\mathcal{F} = \{\langle x, \mu_{\mathcal{F}}(x), \eta_{\mathcal{F}}(x), \nu_{\mathcal{F}}(x)\rangle\}$, called q-RPFN is represented by $\mathcal{F} = \langle \mu_{\mathcal{F}}, \eta_{\mathcal{F}}, \nu_{\mathcal{F}}\rangle$.

3. YAGER OPERATIONS FOR q-RPFNs

Definition 3.1. Let $\mathcal{F}_1 = \langle \mu_1, \eta_1, \nu_1\rangle$ and $\mathcal{F}_2 = \langle \mu_2, \eta_2, \nu_2\rangle$ be two q-RPFNs, $\vartheta > 0$ and $\zeta > 0$. Yager t-norm and t-conorm operations of q-RPFNs are

1. $\mathcal{F}_1 \oplus \mathcal{F}_2 = \left\langle \sqrt[q]{\min\left(1, (\mu_1^{q\vartheta} + \mu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\eta_1^q)^{\vartheta} + (1-\eta_2^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\nu_1^q)^{\vartheta} + (1-\nu_2^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$

2. $\mathcal{F}_1 \otimes \mathcal{F}_2 = \left\langle \sqrt[q]{1 - \min\left(1, \left((1-\mu_1^q)^{\vartheta} + (1-\mu_2^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, (\eta_1^{q\vartheta} + \eta_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, (\nu_1^{q\vartheta} + \nu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)} \right\rangle.$

3. $\zeta\mathcal{F}_1 = \left\langle \sqrt[q]{\min\left(1, (\zeta\mu_1^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\eta_1^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\nu_1^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)} \right\rangle,$

4. $\mathcal{F}_1^{\zeta} = \left\langle \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\mu_1^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, (\zeta\eta_1^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, (\zeta\nu_1^{q\vartheta})^{\frac{1}{\vartheta}}\right)} \right\rangle.$

Example 3.1. Let $\mathcal{F}_1 = \langle 0.9, 0.3, 0.4\rangle$, $\mathcal{F}_2 = \langle 0.5, 0.4, 0.6\rangle$ be two q-RPFNs, then by Definition 3.1 for $q = 3, \vartheta = 4, \zeta = 5$:

1. $\mathcal{F}_1 \oplus \mathcal{F}_2 = \left\langle \sqrt[3]{\min\left(1, (0.9^{12} + 0.5^{12})^{\frac{1}{4}}\right)}, \sqrt[3]{1 - \min\left(1, ((1-0.3^3)^4 + (1-0.4^3)^4)^{\frac{1}{4}}\right)}, \right\rangle$
   $\sqrt[3]{1 - \min\left(1, ((1-0.4^3)^4 + (1-0.6^3)^4)^{\frac{1}{4}}\right)} = \langle 0.90, 0, 0\rangle.$

2. $\mathcal{F}_1 \otimes \mathcal{F}_2 = \left\langle \sqrt[3]{1 - \min\left(1, ((1-0.9^3)^4 + (1-0.5^3)^4)^{\frac{1}{4}}\right)}, \sqrt[3]{\min\left(1, (0.3^{12} + 0.4^{12})^{\frac{1}{4}}\right)}, \sqrt[3]{\min\left(1, (0.4^{12} + 0.6^{12})^{\frac{1}{4}}\right)} \right\rangle$
   $= \langle 0.50, 0.40, 0.60\rangle.$

3. $5\mathcal{F}_1 = \left\langle \sqrt[3]{\min\left(1, (5(0.9)^{12})^{\frac{1}{4}}\right)}, \sqrt[3]{1 - \min\left(1, (5(1-0.3^3)^4)^{\frac{1}{4}}\right)}, \sqrt[3]{1 - \min\left(1, (5(1-0.4^3)^4)^{\frac{1}{4}}\right)} \right\rangle$
   $= \langle 1, 0, 0\rangle.$

4. $\mathcal{F}_1^5 = \left\langle \sqrt[3]{1 - \min\left(1, (5(1-0.9^3)^4)^{\frac{1}{4}}\right)}, \sqrt[3]{\min\left(1, (5(0.3)^{12})^{\frac{1}{4}}\right)}, \sqrt[3]{\min\left(1, (5(0.4)^{12})^{\frac{1}{4}}\right)} \right\rangle$
   $= \langle 0.84, 0.34, 0.46\rangle.$

Theorem 3.1. *Let $\mathcal{F} = \langle \mu, \eta, \nu \rangle, \mathcal{F}_1 = \langle \mu_1, \eta_1, \nu_1 \rangle, \mathcal{F}_2 = \langle \mu_2, \eta_2, \nu_2 \rangle$ be three q-RPFNs, then*

1. $\mathcal{F}_1 \oplus \mathcal{F}_2 = \mathcal{F}_2 \oplus \mathcal{F}_1$,
2. $\mathcal{F}_1 \otimes \mathcal{F}_2 = \mathcal{F}_1 \otimes \mathcal{F}_2$,
3. $\zeta(\mathcal{F}_1 \oplus \mathcal{F}_2) = \zeta\mathcal{F}_1 \oplus \zeta\mathcal{F}_2$,
4. $(\zeta_1 + \zeta_2)\mathcal{F} = \zeta_1\mathcal{F} \oplus \zeta_2\mathcal{F}$,
5. $(\mathcal{F}_1 \otimes \mathcal{F}_2)^\zeta = \mathcal{F}_1^\zeta \otimes \mathcal{F}_2^\zeta, \zeta > 0$,
6. $\mathcal{F}^{\zeta_1} \otimes \mathcal{F}^{\zeta_2} = \mathcal{F}^{(\zeta_1+\zeta_2)},\ \zeta_1, \zeta_2 > 0$.

Proof. For three q-RPFNs $\mathcal{F}, \mathcal{F}_1, \mathcal{F}_2$ and $\zeta, \zeta_1, \zeta_2 > 0$, by Definition 3.1,

1. $$\mathcal{F}_1 \oplus \mathcal{F}_2 = \left\langle \sqrt[q]{\min\left(1, (\mu_1^{q\vartheta} + \mu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\eta_1^q)^\vartheta + (1-\eta_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\nu_1^q)^\vartheta + (1-\nu_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$
$$= \left\langle \sqrt[q]{\min\left(1, (\mu_2^{q\vartheta} + \mu_1^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\eta_2^q)^\vartheta + (1-\eta_1^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\nu_2^q)^\vartheta + (1-\nu_1^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$
$$= \mathcal{F}_2 \oplus \mathcal{F}_1.$$

2. $$\mathcal{F}_1 \otimes \mathcal{F}_2 = \left\langle \sqrt[q]{1 - \min\left(1, \left((1-\mu_1^q)^\vartheta + (1-\mu_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, (\eta_1^{q\vartheta} + \eta_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, (\nu_1^{q\vartheta} + \nu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)} \right\rangle$$
$$= \left\langle \sqrt[q]{1 - \min\left(1, \left((1-\mu_2^q)^\vartheta + (1-\mu_1^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min(1, \left(\eta_2^{q\vartheta} + \eta_1^{q\vartheta}\right)^{\frac{1}{\vartheta}})}, \sqrt[q]{\min\left(1, (\nu_2^{q\vartheta} + \nu_1^{q\vartheta})^{\frac{1}{\vartheta}}\right)} \right\rangle = \mathcal{F}_2 \otimes \mathcal{F}_1.$$

3. $$\zeta(\mathcal{F}_1 \oplus \mathcal{F}_2) = \zeta \left\langle \sqrt[q]{\min\left(1, (\mu_1^{q\vartheta} + \mu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\eta_1^q)^\vartheta + (1-\eta_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((1-\nu_1^q)^\vartheta + (1-\nu_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$
$$= \left\langle \sqrt[q]{\min\left(1, (\zeta\mu_1^{q\vartheta} + \zeta\mu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\eta_1^q)^\vartheta + \zeta(1-\eta_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \right.$$
$$\left. \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\nu_1^q)^\vartheta + \zeta(1-\nu_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle,$$
$$\zeta\mathcal{F}_1 \oplus \zeta\mathcal{F}_2 = \left\langle \sqrt[q]{\min\left(1, (\zeta\mu_1^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\eta_1^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\nu_1^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle \oplus$$
$$\left\langle \sqrt[q]{\min\left(1, (\zeta\mu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\eta_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\nu_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$
$$= \left\langle \sqrt[q]{\min\left(1, (\zeta\mu_1^{q\vartheta} + \zeta\mu_2^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\eta_1^q)^\vartheta + \zeta(1-\eta_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \right\rangle$$
$$\left. \sqrt[q]{1 - \min\left(1, \left(\zeta(1-\nu_1^q)^\vartheta + \zeta(1-\nu_2^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle = \zeta(\mathcal{F}_1 \oplus \mathcal{F}_2).$$

4. $$\zeta_1\mathcal{F} \oplus \zeta_2\mathcal{F} = \left\langle \sqrt[q]{\min\left(1, (\zeta_1\mu^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta_1(1-\eta^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta_1(1-\nu^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle \oplus$$
$$\left\langle \sqrt[q]{\min\left(1, (\zeta_2\mu^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta_2(1-\eta^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\zeta_2(1-\nu^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$
$$= \left\langle \sqrt[q]{\min\left(1, ((\zeta_1+\zeta_2)\mu^{q\vartheta})^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((\zeta_1+\zeta_2)(1-\eta^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left((\zeta_1+\zeta_2)(1-\nu^q)^\vartheta\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$
$$= (\zeta_1 + \zeta_2)\mathcal{F}.$$

Similarly, others can be verified.

Definition 3.2. Consider a q-RPFN $\mathcal{F} = \langle \mu_{\mathcal{F}}, \eta_{\mathcal{F}}, \nu_{\mathcal{F}} \rangle$. The score $\mathcal{S}(\mathcal{F})$ and accuracy functions $\mathrm{A}(\mathcal{F})$ of $\mathcal{F}$ are

$$\mathcal{S}(\mathcal{F}) = \mu_{\mathcal{F}}^q - \eta_{\mathcal{F}}^q - \nu_{\mathcal{F}}^q, \text{where } \mathcal{S}(\mathcal{F}) \in [-1, 1],$$

$$\mathrm{A}(\mathcal{F}) = \mu_{\mathcal{F}}^q + \eta_{\mathcal{F}}^q + \nu_{\mathcal{F}}^q, \text{where } \mathrm{A}(\mathcal{F}) \in [0, 1].$$

Definition 3.3. Consider two q-RPFNs $\mathcal{F}_1 = \langle \mu_{\mathcal{F}_1}, \eta_{\mathcal{F}_1}, \nu_{\mathcal{F}_1} \rangle$ and $\mathcal{F}_2 = \langle \mu_{\mathcal{F}_2}, \eta_{\mathcal{F}_2}, \nu_{\mathcal{F}_2} \rangle$. Then

1. If $\mathcal{S}(\mathcal{F}_1) < \mathcal{S}(\mathcal{F}_2)$, then $\mathcal{F}_1 < \mathcal{F}_2$,
2. If $\mathcal{S}(\mathcal{F}_1) > \mathcal{S}(\mathcal{F}_2)$, then $\mathcal{F}_1 > \mathcal{F}_2$,
3. If $\mathcal{S}(\mathcal{F}_1) = \mathcal{S}(\mathcal{F}_2)$, then
   (a) If $\mathrm{A}(\mathcal{F}_1) < \mathrm{A}(\mathcal{F}_2)$, then $\mathcal{F}_1 < \mathcal{F}_2$,
   (b) If $\mathrm{A}(\mathcal{F}_1) > \mathrm{A}(\mathcal{F}_2)$, then $\mathcal{F}_1 > \mathcal{F}_2$,
   (c) If $\mathrm{A}(\mathcal{F}_1) = \mathrm{A}(\mathcal{F}_2)$, then $\mathcal{F}_1 \sim \mathcal{F}_2$.

4. *q*-RUNG PICTURE FUZZY YAGER AOs

4.1. *q*-rung Picture Fuzzy Yager Hybrid-Weighted Arithmetic Operators

Here, we define Yager weighted arithmetic operators under q-RPF environment.

Definition 4.1. Let $\mathcal{F}_i = \langle \mu_i, \eta_i, \nu_i \rangle (i = 1, 2, \cdots, \mathfrak{s})$ be a number of q-RPFNs. The q-RPFYWA operator is a function $\mathcal{Q}^{\mathfrak{s}} \to \mathcal{Q}$ s.t.

$$q - RPFYWA_{\chi}(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{\mathfrak{s}}) = \bigoplus_{i=1}^{\mathfrak{s}} (\chi_i \mathcal{F}_i),$$

where $\chi = (\chi_1, \chi_2, \cdots, \chi_{\mathfrak{s}})^T$ is the weight vector (WV) of $\mathcal{F}_i$ with $\chi_i > 0$ and $\sum_{i=1}^{\mathfrak{s}} \chi_i = 1$.

Theorem 4.1. *Let $\mathcal{F}_i = (\mu_i, \eta_i, \nu_i)$ be a number of q-RPFNs, then aggregated value of them by the q-RPFYWA operator is a q-RPFN and*

$$\begin{aligned} q - RPFYWA_{\chi}(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{\mathfrak{s}}) &= \bigoplus_{i=1}^{\mathfrak{s}} (\chi_i \mathcal{F}_i) \\ &= \left\langle \sqrt[q]{\min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i \mu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i (1 - \eta_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \right. \\ &\quad \left. \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i (1 - \nu_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle. \end{aligned} \tag{1}$$

Proof. The mathematical induction is used to prove the theorem.

(i) when $\mathfrak{s} = 2$,

As

$$\chi_1 \mathcal{F}_1 = \left\langle \sqrt[q]{\min(1, (\chi_1 \mu_1^{q\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1 - \min(1, \left(\chi_1 (1 - \eta_1^q)^{\vartheta}\right)^{\frac{1}{\vartheta}})}, \sqrt[q]{1 - \min(1, \left(\chi_1 (1 - \nu_1^q)^{\vartheta}\right)^{\frac{1}{\vartheta}})} \right\rangle,$$

$$\chi_2 \mathcal{F}_2 = \left\langle \sqrt[q]{\min(1, (\chi_2 \mu_2^{q\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1 - \min(1, \left(\chi_2 (1 - \eta_2^q)^{\vartheta}\right)^{\frac{1}{\vartheta}})}, \sqrt[q]{1 - \min(1, \left(\chi_2 (1 - \nu_2^q)^{\vartheta}\right)^{\frac{1}{\vartheta}})} \right\rangle.$$

Therefore,

$$\chi_1\mathcal{F}_1 \oplus \chi_2\mathcal{F}_2 = \left\langle \sqrt[q]{\min(1,(\chi_1\mu_1^{q\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1-\min(1,(\chi_1(1-\eta_1^q)^{\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1-\min(1,(\chi_1(1-\nu_1^q)^{\vartheta})^{\frac{1}{\vartheta}})} \right\rangle \oplus$$

$$\left\langle \sqrt[q]{\min(1,(\chi_2\mu_2^{q\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1-\min(1,(\chi_2(1-\eta_2^q)^{\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1-\min(1,(\chi_2(1-\nu_2^q)^{\vartheta})^{\frac{1}{\vartheta}})} \right\rangle$$

$$= \left\langle \sqrt[q]{\min(1,(\chi_1\mu_1^{q\vartheta}+\chi_2\mu_2^{q\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1-\min(1,(\chi_1(1-\eta_1^q)^{\vartheta}+\chi_2(1-\eta_2^q))^{\frac{1}{\vartheta}})},\right.$$

$$\left.\sqrt[q]{1-\min(1,(\chi_1(1-\nu_1^q)^{\vartheta}+\chi_2(1-\nu_2^q))^{\frac{1}{\vartheta}})}\right\rangle$$

$$= \left\langle \sqrt[q]{\min\left(1,\left(\sum_{i=1}^{2}(\chi_i\mu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{2}(\chi_i(1-\eta_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{2}(\chi_i(1-\nu_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

Hence, Equation (1) is true for $\mathfrak{s} = 2$.

(ii) Let Equation (1) holds for $\mathfrak{s} = k$,

$$q-RPFYWA_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_k) = \bigoplus_{i=1}^{k}(\chi_i\mathcal{F}_i)$$

$$= \left\langle \sqrt[q]{\min\left(1,\left(\sum_{i=1}^{k}(\chi_i\mu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{k}(\chi_i(1-\eta_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\right.$$

$$\left.\sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{k}(\chi_i(1-\nu_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

Now for $\mathfrak{s} = k+1$.

$q-RPFYWA_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{k+1})$

$$= \left\langle \sqrt[q]{\min\left(1,\left(\sum_{i=1}^{k}\left(\chi_i\mu_i^{q\vartheta}\right)\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{k}\left(\chi_i\left(1-\eta_i^q\right)^{\vartheta}\right)\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{k}\left(\chi_i\left(1-\nu_i^q\right)^{\vartheta}\right)\right)^{\frac{1}{\vartheta}}\right)} \right\rangle \oplus$$

$$\left\langle \sqrt[q]{\min\left(1,\left(\chi_{k+1}\mu_{k+1}^{q\vartheta}\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\chi_{k+1}\left(1-\eta_{k+1}^q\right)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\chi_{k+1}\left(1-\nu_{k+1}^q\right)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$

$$= \left\langle \sqrt[q]{\min\left(1,\left(\sum_{i=1}^{k+1}\left(\chi_i\mu_i^{q\vartheta}\right)\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{k+1}\left(\chi_i\left(1-\eta_i^q\right)^{\vartheta}\right)\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{k+1}\left(\chi_i\left(1-\nu_i^q\right)^{\vartheta}\right)\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

Hence, Equation (1) is true for $\mathfrak{s} = k+1$. Thus, Equation (1) is true, $\forall\, \mathfrak{s}$.

Example 4.1. Let $\mathcal{F}_1 = \langle 0.6, 0.2, 0.9\rangle, \mathcal{F}_2 = \langle 0.5, 0.4, 0.5\rangle, \mathcal{F}_3 = \langle 0.7, 0.4, 0.7\rangle$ and $\mathcal{F}_4 = \langle 0.9, 0.1, 0.6\rangle$ be q-RPFNs with a WV $\chi = (0.1, 0.4, 0.2, 0.3)^T$ and $\vartheta = 2$. By applying Theorem 4.1, the aggregated value of q-RPFNs for $q = 3$ is

$$\begin{aligned}
q-RPFYWA_\chi(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4) &= \underset{\mathfrak{i}=1}{\overset{4}{\oplus}}(\chi_\mathfrak{i}\mathcal{F}_\mathfrak{i}) \\
&= \left\langle \sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{4}(\chi_\mathfrak{i}\mu_\mathfrak{i}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1, \left(\sum_{\mathfrak{i}=1}^{4}(\chi_\mathfrak{i}(1-\eta_\mathfrak{i}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \right. \\
&\qquad \left. \sqrt[q]{1-\min\left(1, \left(\sum_{\mathfrak{i}=1}^{4}(\chi_\mathfrak{i}(1-\nu_\mathfrak{i}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle \\
&= \left\langle \sqrt[3]{\min(1, (0.1(0.6)^6 + 0.4(0.5)^6 + 0.2(0.7)^6 + 0.3(0.9)^6)^{\frac{1}{2}}}), \right. \\
&\qquad \sqrt[3]{1-\min\left(1, (0.1(1-0.2^3)^2 + 0.4(1-0.4^3)^2 + 0.2(1-0.4^3)^2 + 0.3(1-0.1^3)^2)^{\frac{1}{2}}\right)}, \\
&\qquad \left. \sqrt[3]{1-\min\left(1, (0.1(1-0.9^3)^2 + 0.4(1-0.5^3)^2 + 0.2(1-0.7^3)^2 + 0.3(1-0.6^3)^2)^{\frac{1}{2}}\right)} \right\rangle \\
&= \langle 0.60, 0.34, 0.62\rangle.
\end{aligned}$$

Theorem 4.2. *(Idempotency). If all q-RPFNs are identical, i.e.,* $\mathcal{F}_\mathfrak{i} = \mathcal{F}$ *then*

$$q-RPFYWA(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_\mathfrak{s}) = \mathcal{F}.$$

Proof. As $\mathcal{F}_\mathfrak{i} = \langle \mu_\mathfrak{i}, \eta_\mathfrak{i}, \nu_\mathfrak{i}\rangle = \mathcal{F}(\mathfrak{i} = 1, 2, \cdots, \mathfrak{s})$. Then by Equation (1),

$$\begin{aligned}
q-RPFYWA_\chi(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_\mathfrak{s}) &= \underset{\mathfrak{i}=1}{\overset{\mathfrak{s}}{\oplus}}(\chi_\mathfrak{i}\mathcal{F}_\mathfrak{i}) \\
&= \left\langle \sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_\mathfrak{i}\mu_\mathfrak{i}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_\mathfrak{i}(1-\eta_\mathfrak{i}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \right. \\
&\qquad \left. \sqrt[q]{1-\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_\mathfrak{i}(1-\nu_\mathfrak{i}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle \\
&= \left\langle \sqrt[q]{\min(1, (\mu^{q\vartheta})^{\frac{1}{\vartheta}})}, \sqrt[q]{1-\min(1, ((1-\eta^q)^\vartheta)^{\frac{1}{\vartheta}})}, \sqrt[q]{1-\min(1, ((1-\nu^q)^\vartheta)^{\frac{1}{\vartheta}})} \right\rangle \\
&= \left\langle \sqrt[q]{\min(1, \mu^q)}, \sqrt[q]{1-\min(1, (1-\eta^q))}, \sqrt[q]{1-\min(1, (1-\nu^q))} \right\rangle = \langle \mu, \eta, \nu\rangle = \mathcal{F}.
\end{aligned}$$

Theorem 4.3. *(Boundedness). Let* $\mathcal{F}_\mathfrak{i} = (\mu_\mathfrak{i}, \eta_\mathfrak{i}, \nu_\mathfrak{i})$ *be a collection of q-RPFNs. Let* $\mathcal{F}^- = \min(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_\mathfrak{s})$ *and* $\mathcal{F}^+ = max(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_\mathfrak{s})$*. Then*

$$\mathcal{F}^- \le q-RPFYWA(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_\mathfrak{s}) \le \mathcal{F}^+.$$

Proof. Suppose $\mathcal{F}^- = \min(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_\mathfrak{s}) = (\mu^-, \eta^-, \nu^-)$ and $\mathcal{F}^+ = \max(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_\mathfrak{s}) = (\mu^+, \eta^+, \nu^+)$, where $\mu^- = \min(\mu_\mathfrak{i}), \eta^- = \max(\eta_\mathfrak{i}), \nu^- = \max(\nu_\mathfrak{i}), \mu^+ = \max(\mu_\mathfrak{i}), \nu^+ = \min(\nu_\mathfrak{i}), \nu^+ = \min(\nu_\mathfrak{i})$. Thus

$$\sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_\mathfrak{i}\mu^{-q\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \le \sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_\mathfrak{i}\mu_\mathfrak{i}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \le \sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_\mathfrak{i}\mu^{+q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}.$$

Similarly,

$$\sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i(1-\eta^{+^q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \leq \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i(1-\eta_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \leq \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i(1-\eta^{-^q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}.$$

Similarly,

$$\sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i(1-\nu^{+^q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \leq \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i(1-\nu_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \leq \sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i(1-\nu^{-^q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}.$$

Therefore, $\mathcal{F}^- \leq q-RPFYWA(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}}) \leq \mathcal{F}^+$.

Theorem 4.4. *(Monotonicity). Let $\mathcal{F}'_i = \{\mathcal{F}'_1,\mathcal{F}'_2,\cdots,\mathcal{F}'_s\}$ and $\mathcal{F}_i = \{\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}}\}$ be two collections of q-RPFNs. If $\mu'_i \leq \mu_i$, $\eta'_i \geq \eta_i$ and $\nu'_i \geq \nu_i$, $\forall$ i. Then*

$$q-RPFYWA(\mathcal{F}'_1,\mathcal{F}'_2,\cdots,\mathcal{F}'_{\mathfrak{s}}) \leq q-RPFYWA(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}}).$$

Proof. Let $q-RPFYWA(\mathcal{F}'_1,\mathcal{F}'_2,\cdots,\mathcal{F}'_{\mathfrak{s}}) = (\mathcal{G}',\mathcal{U}',\mathcal{K}')$ and $q-RPFYWA(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}}) = (\mathcal{G},\mathcal{U},\mathcal{K})$. First, we show that $\mathcal{G}' \leq \mathcal{G}$. As $\mu'_i \leq \mu_i$, $\mu'^q_i \leq \mu^q_i$. Moreover,

$$\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\mu'^{q\vartheta}_i)\right)^{\frac{1}{\vartheta}} \leq \left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\mu^{q\vartheta}_i)\right)^{\frac{1}{\vartheta}}$$

$$\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\mu'^{q\vartheta}_i)\right)^{\frac{1}{\vartheta}}\right) \leq \min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\mu^{q\vartheta}_i)\right)^{\frac{1}{\vartheta}}\right)$$

$$\sqrt[q]{\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\mu'^{q\vartheta}_i)\right)^{\frac{1}{\vartheta}}\right)} \leq \sqrt[q]{\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\mu^{q\vartheta}_i)\right)^{\frac{1}{\vartheta}}\right)}.$$

Hence, $\mathcal{G}' \leq \mathcal{G}$. Similarly, $\mathcal{U}' \geq \mathcal{U}$, $\mathcal{K}' \geq \mathcal{K}$. Thus, $(\mathcal{G}',\mathcal{U}',\mathcal{K}') \leq (\mathcal{G},\mathcal{U},\mathcal{K})$, i.e.,

$$q-RPFYWA(\mathcal{F}'_1,\mathcal{F}'_2,\cdots,\mathcal{F}'_{\mathfrak{s}}) \leq q-RPFYWA(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}}).$$

Theorem 4.5. *(Reducibility). Let $\mathcal{F}_i = (\mu_i,\eta_i,\nu_i)$ be a collection of q-RPFNs with WV$\chi = (\chi_1,\chi_2,\cdots,\chi_{\mathfrak{s}})^T = (\frac{1}{\mathfrak{s}},\frac{1}{\mathfrak{s}},\cdots,\frac{1}{\mathfrak{s}})^T$. Then, q-RPFYWA operator is*

$$q-RPFYWA_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}}) = \left\langle \sqrt[q]{\min\left(1,\frac{1}{\mathfrak{s}}\left(\sum_{i=1}^{\mathfrak{s}}(\mu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\frac{1}{\mathfrak{s}}\left(\sum_{i=1}^{\mathfrak{s}}(1-\eta_i^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1-\min\left(1,\frac{1}{\mathfrak{s}}\left(\sum_{i=1}^{\mathfrak{s}}(1-\nu_i^q)^{\vartheta}\right)^{\frac{1}{\vartheta}}\right)}\right\rangle.$$

We now define the q-RPFYOWA operators.

Definition 4.2. Let $\mathcal{F}_i = \langle\mu_i,\eta_i,\nu_i\rangle$ be a family of q-RPFNs with WV$\chi = (\chi_1,\chi_2,\cdots,\chi_{\mathfrak{s}})^T$ s.t. $\chi_i > 0$ and $\sum_{i=1}^{\mathfrak{s}}\chi_i = 1$. The q-RPFYOWA operator is a function $\mathcal{Q}^{\mathfrak{s}} \to \mathcal{Q}$ s.t.

$$q-RPFYOWA_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}}) = \bigoplus_{i=1}^{\mathfrak{s}}(\chi_i\mathcal{F}_{\varrho(i)}),$$

where $(\varrho(1),\varrho(2),\cdots,\varrho(\mathfrak{s}))$ is the permutation of $(i = 1,2,\cdots,\mathfrak{s})$ s.t. $\mathcal{F}_{\varrho(i-1)} \geq \mathcal{F}_{\varrho(i)}$, $\forall$ i.

Theorem 4.6. *Let $\mathcal{F}_i = (\mu_i, \eta_i, \nu_i)$ be a collection of q-RPFNs with WV $\chi = (\chi_1, \chi_2, \cdots, \chi_{\mathfrak{s}})^T$ s.t. $\chi_i > 0$ and $\sum_{i=1}^{\mathfrak{s}} \chi_i = 1$, then clumped value of them by q-RPFYOWA operator is a q-RPFN and*

$$q - RPFYOWA_{\chi}(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{\mathfrak{s}}) = \bigoplus_{i=1}^{\mathfrak{s}} (\chi_i \mathcal{F}_{\varrho(i)}) \tag{2}$$

$$= \left\langle \sqrt[q]{\min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i \mu_{\varrho(i)}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i (1 - \eta_{\varrho(i)}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i (1 - \nu_{\varrho(i)}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

Example 4.2. Let $\mathcal{F}_1 = \langle 0.8, 0.3, 0.7\rangle, \mathcal{F}_2 = \langle 0.6, 0.4, 0.7\rangle, \mathcal{F}_3 = \langle 0.6, 0.4, 0.8\rangle$ and $\mathcal{F}_4 = \langle 0.9, 0.1, 0.4\rangle$ be q-RPFNs with a WV $\chi = (0.2, 0.3, 0.1, 0.4)^T$ and $\vartheta = 2$. By applying Theorem 4.6, the aggregated value of q-RPFNs for $q = 3$ is

$$\mathcal{S}(\mathcal{F}_1) = 0.8^3 - 0.3^3 - 0.7^3 = 0.14,$$

$$\mathcal{S}(\mathcal{F}_2) = 0.6^3 - 0.4^3 - 0.7^3 = -0.19,$$

$$\mathcal{S}(\mathcal{F}_3) = 0.6^3 - 0.4^3 - 0.8^3 = -0.36,$$

$$\mathcal{S}(\mathcal{F}_4) = 0.9^3 - 0.1^3 - 0.4^3 = 0.66.$$

Since $\mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_1) > \mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_3)$, therefore

$$\mathcal{F}_{\varrho(1)} = \mathcal{F}_4 = \langle 0.9, 0.1, 0.4\rangle,$$

$$\mathcal{F}_{\varrho(2)} = \mathcal{F}_1 = \langle 0.8, 0.3, 0.7\rangle,$$

$$\mathcal{F}_{\varrho(3)} = \mathcal{F}_2 = \langle 0.6, 0.4, 0.7\rangle,$$

$$\mathcal{F}_{\varrho(4)} = \mathcal{F}_3 = \langle 0.6, 0.4, 0.8\rangle.$$

Thus by applying the q-RPFYOWA operator,

$$q - RPFYOWA_{\chi}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4) = \bigoplus_{i=1}^{4} (\chi_i \mathcal{F}_{\varrho(i)})$$

$$= \left\langle \sqrt[q]{\min\left(1, \left(\sum_{i=1}^{4} (\chi_i \mu_{\varrho(i)}^{2\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{4} (\chi_i (1 - \nu_{\varrho(i)}^{2})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle$$

$$= \left\langle \sqrt[3]{\min(1, (0.2(0.8)^6 + 0.3(0.6)^6 + 0.1(0.6)^6 + 0.4(0.9)^6)^{\frac{1}{2}})}, \right.$$

$$\sqrt[3]{1 - \min\left(1, (0.2(1 - 0.1^3)^2 + 0.3(1 - 0.3^3)^2 + 0.1(1 - 0.4^3)^2 + 0.4(1 - 0.4^3)^2)^{\frac{1}{2}}\right)},$$

$$\left. \sqrt[3]{1 - \min\left(1, (0.2(1 - 0.7^3)^2 + 0.3(1 - 0.7^3)^2 + 0.1(1 - 0.8^3)^2 + 0.4(1 - 0.4^3)^2)^{\frac{1}{2}}\right)} \right\rangle$$

$$= \langle 0.81, 0.34, 0.61\rangle.$$

Remark 4.1. q-RPFYOWA operators satisfy the properties 4.2, 4.3, 4.4 and 4.5.

We now define the q-RPFYHWA operators.

Definition 4.3. A $q-RPFYHWA$ is a function $\mathcal{Q}^{\mathfrak{s}} \to \mathcal{Q}$, with correlated WV $\chi = (\chi_1, \chi_2, \cdots, \chi_{\mathfrak{s}})^T$ with $\chi_{\mathfrak{i}} > 0$ and $\sum_{\mathfrak{i}=1}^{\mathfrak{s}} \chi_{\mathfrak{i}} = 1$ s.t.

$$q-RPFYHWA_{\chi}(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{\mathfrak{s}}) = \bigoplus_{\mathfrak{i}=1}^{\mathfrak{s}} (\chi_{\mathfrak{i}} \dot{\mathcal{F}}_{\varrho(\mathfrak{i})}) \tag{3}$$

$$= \left\langle \sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}} (\chi_{\mathfrak{i}} \dot{\mu}_{\varrho(\mathfrak{i})}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}} (\chi_{\mathfrak{i}} (1 - \dot{\eta}_{\varrho(\mathfrak{i})}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}} (\chi_{\mathfrak{i}} (1 - \dot{\nu}_{\varrho(\mathfrak{i})}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

where $\dot{\mathcal{F}}_{\varrho(\mathfrak{i})}$ is the $\mathfrak{i}th$ biggest weighted q-rung picture fuzzy values $\dot{\mathcal{F}}_{\mathfrak{i}} (\dot{\mathcal{F}}_{\mathfrak{i}} = \mathfrak{s}\chi_{\mathfrak{i}}\mathcal{F}_{\mathfrak{i}}, \mathfrak{i} = 1, 2, \cdots, \mathfrak{s})$ and $\mathfrak{s}$ is the balancing coefficient.

Remark 4.2. For $\chi = (\frac{1}{\mathfrak{s}}, \frac{1}{\mathfrak{s}}, \cdots, \frac{1}{\mathfrak{s}})^T$, q-RPFYWA and q-RPFYOWA operators are particular example of q-RPFYHWA operator. Thus, q-RPFYHWA operator is a generalization of them.

4.2. q-rung Picture Fuzzy Yager Hybrid-Weighted Geometric Operators

Here, we discuss Yager weighted geometric operators under q-RPF environment.

Definition 4.4. Let $\mathcal{F}_{\mathfrak{i}} = \langle \mu_{\mathfrak{i}}, \eta_{\mathfrak{i}}, \nu_{\mathfrak{i}} \rangle$ be a number of q-RPFNs. The q-RPFYWG operator is a function $\mathcal{Q}^{\mathfrak{s}} \to \mathcal{Q}$ s.t.

$$q-RPFYWG_{\chi}(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{\mathfrak{s}}) = \bigotimes_{\mathfrak{i}=1}^{\mathfrak{s}} \mathcal{F}_{\mathfrak{i}}^{\chi_{\mathfrak{i}}},$$

where $\chi = (\chi_1, \chi_2, \cdots, \chi_{\mathfrak{s}})^T$ is the WV of $\mathcal{F}_{\mathfrak{i}}$ with $\chi_{\mathfrak{i}} > 0$ and $\sum_{\mathfrak{i}=1}^{\mathfrak{s}} \chi_{\mathfrak{i}} = 1$.

Theorem 4.7. *Let $\mathcal{F}_{\mathfrak{i}} = \langle \mu_{\mathfrak{i}}, \eta_{\mathfrak{i}}, \nu_{\mathfrak{i}} \rangle$ be a number of q-RPFNs, then clumped value of them by q-RPFYWG operator is a q-RPFN and*

$$q-RPFYWG_{\chi}(\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{\mathfrak{s}}) = \bigotimes_{\mathfrak{i}=1}^{\mathfrak{s}} \mathcal{F}_{\mathfrak{i}}^{\chi_{\mathfrak{i}}} \tag{4}$$

$$= \left\langle \sqrt[q]{1 - \min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}} (\chi_{\mathfrak{i}} (1 - \mu_{\mathfrak{i}}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}} (\chi_{\mathfrak{i}} \eta_{\mathfrak{i}}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, \left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}} (\chi_{\mathfrak{i}} \nu_{\mathfrak{i}}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

Proof. It is similar to Theorem 4.1.

Example 4.3. Consider Example 4.1 and by Theorem 4.7, the clumped value for q-RPFNs is

$$q-RPFYWG_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3,\mathcal{F}_4)=\bigotimes_{i=1}^{4}(\mathcal{F}_i)^{\chi_i}$$

$$=\left\langle\sqrt[3]{1-\min\left(1,\left(\sum_{i=1}^{4}(\chi_i(1-\mu_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\sqrt[3]{\min(1,\left(\sum_{i=1}^{4}(\chi_i\nu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\right.$$

$$\left.\sqrt[3]{\min\left(1,\left(\sum_{i=1}^{4}(\chi_i\nu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}\right\rangle$$

$$=\left\langle\sqrt[3]{1-\min(1,(0.2(1-0.6^3)^2+0.3(1-0.5^3)^2+0.1(1-0.7^3)^2+0.4(1-0.9^3)^2)^{\frac{1}{2}}},\right.$$

$$\sqrt[3]{\min(1,(0.2(0.2)^6+0.3(0.4)^6+0.1(0.4)^6+0.4(0.1)^6)^{\frac{1}{2}})},$$

$$\left.\sqrt[3]{\min(1,(0.2(0.9)^6+0.3(0.5)^6+0.1(0.7)^6+0.4(0.6)^6)^{\frac{1}{2}})}\right\rangle=\langle 0.70,0.34,0.72\rangle.$$

Remark 4.3. q-RPFYWG operators satisfy the properties 4.2, 4.3, 4.4 and 4.5.

We now define q-RPFYOWG operators.

Definition 4.5. Let $\mathcal{F}_i=\langle\mu_i,\eta_i,\nu_i\rangle$ be a collection of q-RPFNs with WV $\chi=(\chi_1,\chi_2,\cdots,\chi_{\mathfrak{s}})^T$ s.t. $\chi_i>0$ and $\sum_{i=1}^{\mathfrak{s}}\chi_i=1$. The q-RPFYOWG operator is a function $\mathcal{Q}^{\mathfrak{s}}\to\mathcal{Q}$ s.t.

$$q-RPFYOWG_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}})=\bigotimes_{i=1}^{\mathfrak{s}}(\mathcal{F}_{\varrho(i)})^{\chi_i},$$

where $(\varrho(1),\varrho(2),\cdots,\varrho(\mathfrak{s}))$ is the permutation of $(i=1,2,\cdots,\mathfrak{s})$ s.t. $\mathcal{F}_{\varrho(i-1)}\geq\mathcal{F}_{\varrho(i)},\forall\, i$.

Theorem 4.8. *Let $\mathcal{F}_i=\langle\mu_i,\eta_i,\nu_i\rangle$ be a number of q-RPFNs with WV $\chi=(\chi_1,\chi_2,\cdots,\chi_{\mathfrak{s}})^T$ s.t. $\chi_i>0$ and $\sum_{i=1}^{\mathfrak{s}}\chi_i=1$ then clumped value of them by q-RPFYOWG operator is a q-RPFN and*

$$q-RPFYOWG_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}})=\bigotimes_{i=1}^{\mathfrak{s}}(\mathcal{F}_{\varrho(i)})^{\chi_i} \tag{5}$$

$$=\left\langle\sqrt[q]{1-\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i(1-\mu_{\varrho(i)}^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\sqrt[q]{\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\eta_{\varrho(i)}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\right.$$

$$\left.\sqrt[q]{\min\left(1,\left(\sum_{i=1}^{\mathfrak{s}}(\chi_i\nu_{\varrho(i)}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}\right\rangle.$$

Proof. It is similar to Theorem 4.1.

Example 4.4. Consider Example 4.2 and by Theorem 4.8, the clumped value for q-RPFNs is

$$q-RPFYOWG_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3,\mathcal{F}_4)=\bigotimes_{\mathfrak{i}=1}^{4}(\mathcal{F}_{\varrho(\mathfrak{i})})^{\chi_{\mathfrak{i}}}$$

$$=\left\langle \sqrt[3]{1-\min\left(1,\left(\sum_{\mathfrak{i}=1}^{4}(\chi_{\mathfrak{i}}(1-\mu_{\varrho(\mathfrak{i})}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\sqrt[3]{\min\left(1,\left(\sum_{\mathfrak{i}=1}^{4}(\chi_{\mathfrak{i}}\nu_{\varrho(\mathfrak{i})}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\sqrt[3]{\min\left(1,\left(\sum_{\mathfrak{i}=1}^{4}(\chi_{\mathfrak{i}}\nu_{\varrho(\mathfrak{i})}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}\right\rangle$$

$$=\left\langle\sqrt[3]{1-\min(1,(0.2(1-0.9^3)^2+0.3(1-0.8^3)^2+0.1(1-0.6^3)^2+0.4(1-0.6^3)^2)^{\frac{1}{2}}},\sqrt[3]{\min(1,(0.2(0.1)^6+0.3(0.3)^6+0.1(0.4)^6+0.4(0.4)^6)^{\frac{1}{2}})},\sqrt[3]{\min(1,(0.2(0.4)^6+0.3(0.7)^6+0.1(0.7)^6+0.4(0.8)^6)^{\frac{1}{2}})}\right\rangle=\langle 0.72,0.36,0.73\rangle.$$

Remark 4.4. q-RPFYOWG operators satisfy the properties 4.2, 4.3, 4.4 and 4.5.

Now, we define q-RPFYHWG operators.

Definition 4.6. A q-RPFYHWG operator is a function $Q^{\mathfrak{s}}\to Q$, with correlated WV $\chi=(\chi_1,\chi_2,\cdots,\chi_{\mathfrak{s}})^T$ with $\chi_{\mathfrak{i}}>0$ and $\sum_{\mathfrak{i}=1}^{\mathfrak{s}}\chi_{\mathfrak{i}}=1$ s.t.

$$q-RPFYHWG_{\chi}(\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_{\mathfrak{s}})=\bigotimes_{\mathfrak{i}=1}^{\mathfrak{s}}(\dot{\mathcal{F}}_{\varrho(\mathfrak{i})})^{\chi_{\mathfrak{i}}} \tag{6}$$

$$=\left\langle \sqrt[q]{1-\min\left(1,\left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_{\mathfrak{i}}(1-\dot{\mu}_{\varrho(\mathfrak{i})}^{q})^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\sqrt[q]{\min\left(1,\left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_{\mathfrak{i}}\dot{\eta}_{\varrho(\mathfrak{i})}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)},\sqrt[q]{\min\left(1,\left(\sum_{\mathfrak{i}=1}^{\mathfrak{s}}(\chi_{\mathfrak{i}}\dot{\nu}_{\varrho(\mathfrak{i})}^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}\right\rangle,$$

where $\dot{\mathcal{F}}_{\varrho(\mathfrak{i})}$ is the $\mathfrak{i}th$ biggest weighted q-rung picture fuzzy values $\dot{\mathcal{F}}_{\mathfrak{i}}(\dot{\mathcal{F}}_{\mathfrak{i}}=\mathcal{F}_{\mathfrak{i}}^{\mathfrak{s}\chi_{\mathfrak{i}}},\mathfrak{i}=1,2,\cdots,\mathfrak{s})$.

5. MATHEMATICAL APPROACH FOR MADM UNDER q-RPF ENVIRONMENT

Here, we discuss the MADM problems with q-RPF information by using q-RPF Yager AOs proposed in the preceding sections. The following notations are used to show the MADM problem for the selection of alternative with q-RPF information. Let $\mathcal{L}=\{\mathcal{L}_1,\mathcal{L}_2,\cdots,\mathcal{L}_m\}$ be a set of alternatives and $\chi=\{\chi_1,\chi_2,\cdots,\chi_{\mathfrak{s}}\}$ is the WV of the attributes $\mathcal{K}=\{\mathcal{K}_1,\mathcal{K}_2,\cdots,\mathcal{K}_{\mathfrak{s}}\}$, where $\chi_{\mathfrak{i}}>0$ and $\sum_{\mathfrak{i}=1}^{\mathfrak{s}}\chi_{\mathfrak{i}}=1$. Suppose that $\tilde{\mathcal{N}}=(\mu_{l\mathfrak{i}},\eta_{l\mathfrak{i}},\nu_{l\mathfrak{i}})_{m\times\mathfrak{s}}$ is the q-RPF decision matrix (DMx), where $\mu_{l\mathfrak{i}}$ represents the positive MD that the alternative $\mathcal{L}_l$ satisfies the attribute $\mathcal{K}_{\mathfrak{i}}$ given by the decision maker (DMr), $\eta_{l\mathfrak{i}}$ represents the neutral MD that the alternative $\mathcal{L}_l$ does not satisfy the attribute $\mathcal{K}_{\mathfrak{i}}$, and $\nu_{l\mathfrak{i}}$ represents the negative MD that the alternative $\mathcal{L}_l$ does not satisfy the attribute $\mathcal{K}_{\mathfrak{i}}$ given by the DMr, where $0\le\mu_{l\mathfrak{i}}^q+\eta_{l\mathfrak{i}}^q+\nu_{l\mathfrak{i}}^q\le 1$.

For solving a MADM problem, the Algorithm 1 is given as:

Algorithm 1: Steps to deal MADM problem by q-RPFYWA (or q-RPFYWG) operator

1. **Input**:
   $\mathcal{L}$: Set of alternatives,
   $\mathcal{K}$: Set of attributes,
   χ : WV for attributes.
2. Using the q-RPFYWA (or q-RPFYWG) operator to evaluate the information in q-RPFDMx, find preference values $\mathfrak{g}_l, l = 1, 2, ..., m$ of the alternatives $\mathcal{L}_l$.

$$\mathfrak{g}_l = q - RPFYWA(\mathcal{L}_{l1}, \mathcal{L}_{l2}, \cdots, \mathcal{L}_{l\mathfrak{s}})$$

$$= \left\langle \sqrt[q]{\min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i \mu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i (1 - \eta_i^q)^{\vartheta})\right)\right)^{\frac{1}{\vartheta}}}, \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i (1 - \nu_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

OR

$$\mathfrak{g}_l = q - RPFYWG(\mathcal{L}_{l1}, \mathcal{L}_{l2}, \cdots, \mathcal{L}_{l\mathfrak{s}})$$

$$= \left\langle \sqrt[q]{1 - \min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i (1 - \mu_i^q)^{\vartheta})\right)^{\frac{1}{\vartheta}}\right)}, \sqrt[q]{\min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i \eta_i^{q\vartheta})\right)\right)^{\frac{1}{\vartheta}}}, \sqrt[q]{\min\left(1, \left(\sum_{i=1}^{\mathfrak{s}} (\chi_i \nu_i^{q\vartheta})\right)^{\frac{1}{\vartheta}}\right)} \right\rangle.$$

3. Compute the score values.
4. Use the score values $S(\mathfrak{g}_l)$, to rank the alternatives $\mathcal{L}_l, l = 1, 2, ..., m$. For equal score, use the accuracy function for ranking of alternatives.

Output: The alternative with greatest score will be the decision.

6. NUMERICAL EXAMPLES

6.1. Selection of Emerging Technology Enterprise

In this section, we present a numerical result to build up the reasonable assessment of technology commercialization with q-RPF data in such a way to get the desired result of our proposed approach in this article. There is a committee which takes four possible emerging technology enterprises $\mathcal{L}_l (l = 1, 2, \cdots, 4)$. The attributes for the selection are as follows:

$\mathcal{K}_1$: Technical advancement

$\mathcal{K}_2$: Potential market and market risk

$\mathcal{K}_3$: Industrialization framework, human resources and financial investments

1. The q-RPFDMx is shown in Table 1.
2. The weights assigned by the DMr are

$$\chi_1 = 0.3, \chi_2 = 0.4, \chi_3 = 0.3 \text{ and } \sum_{i=1}^{3} \chi_i = 1.$$

We proceed to select the suitable alternative by q-RPFYWA operator. The steps are given as:

Table 1 q-rung picture fuzzy decision matrix (q-RPFDMx).

$\tilde{\mathcal{N}}$	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$
$\mathcal{L}_1$	(0.7, 0.06, 0.2)	(0.6, 0.01, 0.3)	(0.5, 0.04, 0.4)
$\mathcal{L}_2$	(0.6, 0.09, 0.3)	(0.4, 0.02, 0.4)	(0.5, 0.03, 0.3)
$\mathcal{L}_3$	(0.7, 0.01, 0.2)	(0.3, 0.01, 0.5)	(0.3, 0.05, 0.5)
$\mathcal{L}_4$	(0.6, 0.08, 0.2)	(0.4, 0.05, 0.5)	(0.5, 0.05, 0.4)

Step 1. The performance values $\mathfrak{g}_{\mathfrak{i}}$ of the alternatives by q-RPFYWA operator for $q = 3$ are

$$\mathfrak{g}_1 = (0.63, 0.04, 0.32),$$
$$\mathfrak{g}_2 = (0.54, 0.06, 0.35),$$
$$\mathfrak{g}_3 = (0.61, 0.03, 0.44),$$
$$\mathfrak{g}_4 = (0.54, 0.09, 0.41).$$

Step 2. The scores $S(\mathfrak{g}_{\mathfrak{i}})$ of all q-RPFNs are

$$S(\mathfrak{g}_1) = 0.22,$$
$$S(\mathfrak{g}_2) = 0.11,$$
$$S(\mathfrak{g}_3) = 0.14,$$
$$S(\mathfrak{g}_4) = 0.09.$$

Step 3. Ranking of alternatives according to scores $S(\mathfrak{g}_{\mathfrak{i}}), 1 \leq \mathfrak{i} \leq 4$ is

$$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4.$$

Step 4. $\mathcal{L}_1$ is best alternative. If the q-RPFYWG operator is used for selection, the best alternative can be chosen in a similar way. Now the steps are as follows:

Step 1. The performance values $\mathfrak{g}_{\mathfrak{i}}$ of the alternatives by q-RPFYWG operator for $q = 3$ are

$$\mathfrak{g}_1 = (0.60, 0.05, 0.35),$$
$$\mathfrak{g}_2 = (0.50, 0.08, 0.37),$$
$$\mathfrak{g}_3 = (0.46, 0.04, 0.48),$$
$$\mathfrak{g}_4 = (0.50, 0.07, 0.46).$$

Step 2. The scores $S(\mathfrak{g}_{\mathfrak{i}})$ of all q-RPFNs are

$$S(\mathfrak{g}_1) = 0.22,$$
$$S(\mathfrak{g}_2) = 0.11,$$
$$S(\mathfrak{g}_3) = 0.14,$$
$$S(\mathfrak{g}_4) = 0.09.$$

Step 3. Ranking of alternatives according to scores $S(\mathfrak{g}_{\mathfrak{i}}), 1 \leq \mathfrak{i} \leq 4$ is

$$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4.$$

Step 4. $\mathcal{L}_1$ is best alternative.

6.2. Selection of the Suitable Company for Investment

Let suppose another MADM problem in which a DMr wants to select a company for the investment of money. Let $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_4$ be the possible companies for the investment where $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$ and $\mathcal{L}_4$ are the food, mobile, car and fabric companies, respectively. The DMr selects three components to assess companies which are given as follows:

$$\mathcal{K}_1 \ : \ \text{Profit-loss ratio}$$
$$\mathcal{K}_2 \ : \ \text{Company managements}$$
$$\mathcal{K}_3 \ : \ \text{Competitive power}$$

1. The q-RPFDMx is shown in Table 2.
2. The weights assigned by the DMr are

$$\chi_1 = 0.4, \chi_2 = 0.3, \chi_3 = 0.3 \text{ and } \sum_{\mathfrak{i}=1}^{3} \chi_{\mathfrak{i}} = 1.$$

Table 2 q-rung picture fuzzy decision matrix (q-RPFDMx).

$\tilde{\mathcal{N}}$	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$
$\mathcal{L}_1$	(0.9, 0.01, 0.5)	(0.4, 0.02, 0.4)	(0.7, 0.03, 0.2)
$\mathcal{L}_2$	(0.3, 0.05, 0.6)	(0.8, 0.4, 0.5)	(0.6, 0.04, 0.3)
$\mathcal{L}_3$	(0.4, 0.09, 0.5)	(0.3, 0.041, 0.5)	(0.7, 0.4, 0.6)
$\mathcal{L}_4$	(0.5, 0.06, 0.4)	(0.7, 0.01, 0.2)	(0.8, 0.07, 0.1)

We proceed to select the suitable alternative by q-RPFYWA operator. The steps are given as:

Step 1. The performance values $\mathfrak{g}_i$ of the alternatives by q-RPFYWA operator for $q = 3$ are

$$\mathfrak{g}_1 = (0.82, 0.02, 0.40),$$
$$\mathfrak{g}_2 = (0.70, 0.26, 0.50),$$
$$\mathfrak{g}_3 = (0.61, 0.27, 0.53),$$
$$\mathfrak{g}_4 = (0.72, 0.06, 0.35).$$

Step 2. The scores $S(\mathfrak{g}_i)$ of all q-RPFNs are

$$S(\mathfrak{g}_1) = 0.10,$$
$$S(\mathfrak{g}_2) = 0.20,$$
$$S(\mathfrak{g}_3) = 0.06,$$
$$S(\mathfrak{g}_4) = 0.35.$$

Step 3. Ranking of alternatives according to scores $S(\mathfrak{g}_i), 1 \leq i \leq 4$ is

$$\mathcal{L}_4 > \mathcal{L}_2 > \mathcal{L}_1 > \mathcal{L}_3.$$

Step 4. $\mathcal{L}_4$ is best alternative.

If the q-RPFYWG operator is used for selection, the best alternative can be chosen in a similar way. Now the steps are as follows:

Step 1. The performance values $\mathfrak{g}_i$ of the alternatives by q-RPFYWA operator for $q = 3$ are

$$\mathfrak{g}_1 = (0.67, 0.03, 0.44),$$
$$\mathfrak{g}_2 = (0.57, 0.35, 0.55),$$
$$\mathfrak{g}_3 = (0.49, 0.35, 0.55),$$
$$\mathfrak{g}_4 = (0.64, 0.06, 0.36).$$

Step 2. The scores $S(\mathfrak{g}_i)$ of all q-RPFNs are

$$S(\mathfrak{g}_1) = 0.21,$$
$$S(\mathfrak{g}_2) = -0.02,$$
$$S(\mathfrak{g}_3) = -0.09,$$
$$S(\mathfrak{g}_4) = 0.22.$$

Step 3. Ranking of alternatives according to scores $S(\mathfrak{g}_i), 1 \leq i \leq 4$ is

$$\mathcal{L}_4 > \mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_3.$$

Step 4. $\mathcal{L}_4$ is best alternative. The framework for the selection of investment company is shown in Figure 1.

6.3. Effect of Parameter ϑ on Decision-Making Results

To analyze the effect of parameter ϑ on results, we take distinct values of ϑ to handle the Application 6.1. The results for distinct values of ϑ for q-RPFYWA operator are shown in Table 3 and Figure 2, and for q-RPFYWG operator are shown in Table 4 and Figure 3.

From above tables and figures, it can be seen that by taking various values of ϑ, the best alternative is same from both operators and best choice is $\mathcal{L}_1$. However, the overall ranking is slightly different. It is clear from Table 3 and Figure 2 that by using q-RPFYWA operator for $\vartheta = 1, 2$, the ranking of alternatives is different and for $\vartheta = 3, 4, \cdots, 10$ the ranking is same. But for all values, $\mathcal{L}_1$ is the best. The score values

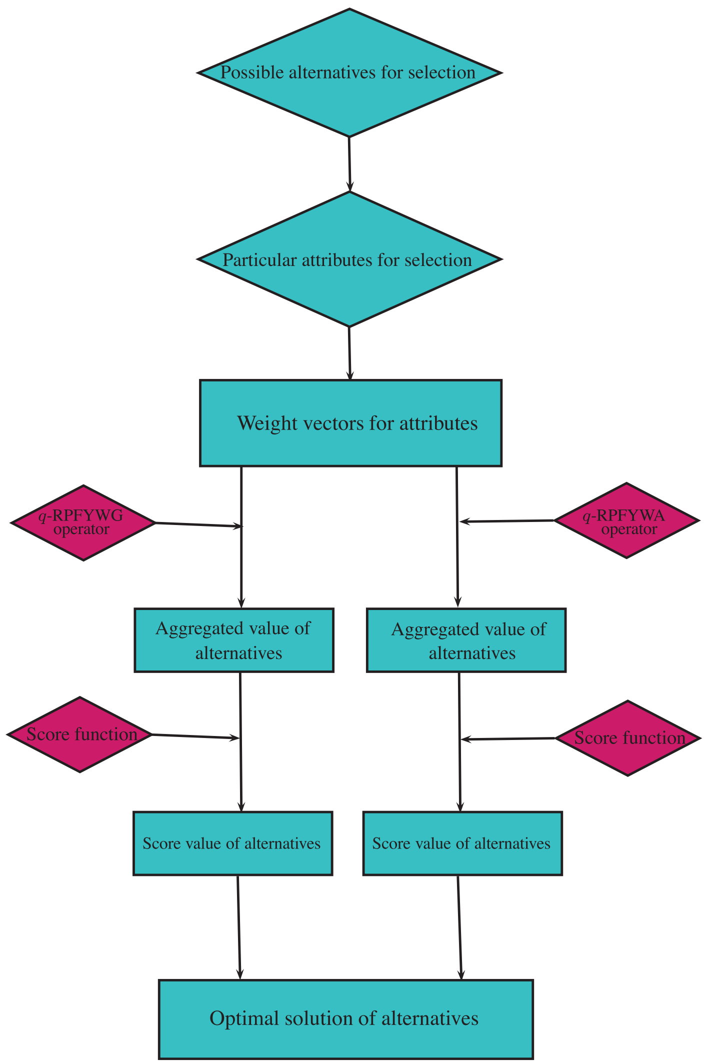

Figure 1 Framework for the selection of investment company.

Table 3 Ranking order (RO) using q-rung picture fuzzy Yager weighted arithmetic (q-RPFYWA) operator.

ϑ	$\mathcal{S}(\mathfrak{g}_1)$	$\mathcal{S}(\mathfrak{g}_2)$	$\mathcal{S}(\mathfrak{g}_3)$	$\mathcal{S}(\mathfrak{g}_4)$	RO
1	0.19	0.08	0.03	0.05	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
2	0.21	0.10	0.09	0.07	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_3 > \mathcal{L}_4$
3	0.22	0.11	0.14	0.09	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
4	0.23	0.12	0.16	0.10	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
5	0.24	0.13	0.19	0.11	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
6	0.26	0.14	0.20	0.11	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
7	0.26	0.15	0.21	0.12	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
8	0.27	0.15	0.22	0.12	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
9	0.27	0.15	0.23	0.13	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
10	0.27	0.16	0.23	0.14	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$

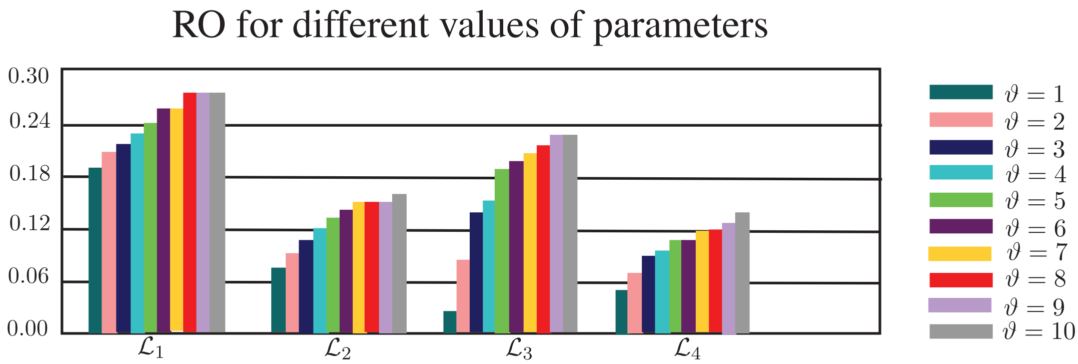

Figure 2 RO for different values of parameters using q-rung picture fuzzy Yager weighted arithmetic (-RPFYWA).

Table 4 RO using q-rung picture fuzzy Yager weighted arithmetic (q-RPFYWG) operator.

ϑ	$\mathcal{S}(\mathfrak{g}_1)$	$\mathcal{S}(\mathfrak{g}_2)$	$\mathcal{S}(\mathfrak{g}_3)$	$\mathcal{S}(\mathfrak{g}_4)$	RO
1	0.19	0.08	0.03	0.05	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
2	0.19	0.08	0.01	0.04	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
3	0.17	0.07	−0.01	0.03	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
4	0.17	0.07	−0.03	0.01	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
5	0.15	0.06	−0.03	0.01	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
6	0.15	0.06	−0.04	0.01	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
7	0.15	0.06	−0.04	0.01	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
8	0.14	0.05	−0.04	−0.001	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
9	0.14	0.05	−0.05	−0.001	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
10	0.14	0.05	−0.06	−0.001	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$

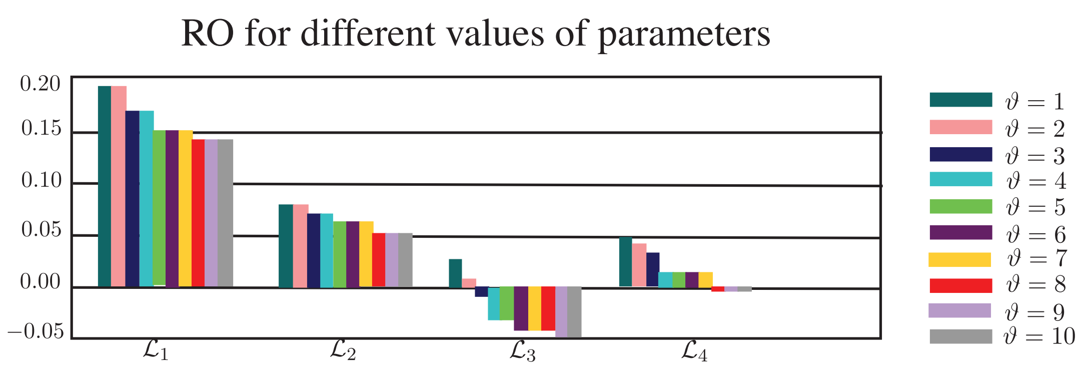

Figure 3 RO for different values of parameters using q-rung picture fuzzy Yager weighted arithmetic (-RPFYWG).

of alternatives increase if value of ϑ increased. From Table 4 and Figure 3, it is clear that by using q-RPFYWG operator for $\vartheta = 1, 2, \cdots, 10$, the ranking of alternatives is same and best one is $\mathcal{L}_1$.

6.4. Comparison Analysis and Discussion

To compute performance and validity of our proposed operators, here we aggregate the same data using different operators, namely, picture fuzzy Einstein weighted average (PFEWA) [22], spherical fuzzy weighted average (SFWA) [7], spherical fuzzy weighted geometric (SFWG) [7] and picture fuzzy Dombi average (PFDWA) [20] operators. The computed results by applying these operators are summarized in Table 5 and shown in Figure 4.

It is clear from Table 5 and Figure 4 that best alternative obtained by using PFEWA, SFWA, SFWG and PFDWA operators remains same as obtained from proposed operators. This implies that our proposed methods are authentic and can be applied in DM problems. The main logic behind our proposed approach is that PFS and SFS handle the situations like $\mu + \eta + \nu \leq 1$ and $\mu^2 + \eta^2 + \nu^2 \leq 1$, respectively but fail in situations where $\mu^q + \eta^q + \nu^q \leq 1$, where $q \geq 3$. That's why we need q-RPFS. The results from proposed theory are more accurate and closest to original results. However, in q-RPF Yager AOs, we can also discuss the effect of parameters.

Table 5 Comparison analysis with PFEWA, SFWA, SFWG and PFDWA operators (suppose $q = 3$).

Methods	$\mathcal{S}(\mathfrak{g}_1)$	$\mathcal{S}(\mathfrak{g}_2)$	$\mathcal{S}(\mathfrak{g}_3)$	$\mathcal{S}(\mathfrak{g}_4)$	Ranking Order
q-RPFYWA	0.22	0.11	0.14	0.09	$\mathcal{L}_1 > \mathcal{L}_3 > \mathcal{L}_2 > \mathcal{L}_4$
q-RPFYWG	0.17	0.07	−0.01	0.03	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
PFEWA	0.20	0.09	0.03	0.01	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_3 > \mathcal{L}_4$
SFWA	0.20	0.09	0.06	0.01	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_3 > \mathcal{L}_4$
SFWG	0.18	0.07	−0.03	0.04	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_4 > \mathcal{L}_3$
PFDWA	0.22	0.10	0.05	0.04	$\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_3 > \mathcal{L}_4$

q-RPFYWA, q-rung picture fuzzy Yager weighted arithmetic; PFEWA, picture fuzzy Einstein weighted average; SFWA, spherical fuzzy weighted average; SFWG, spherical fuzzy weighted geometric; PFDWA, picture fuzzy Dombi average.

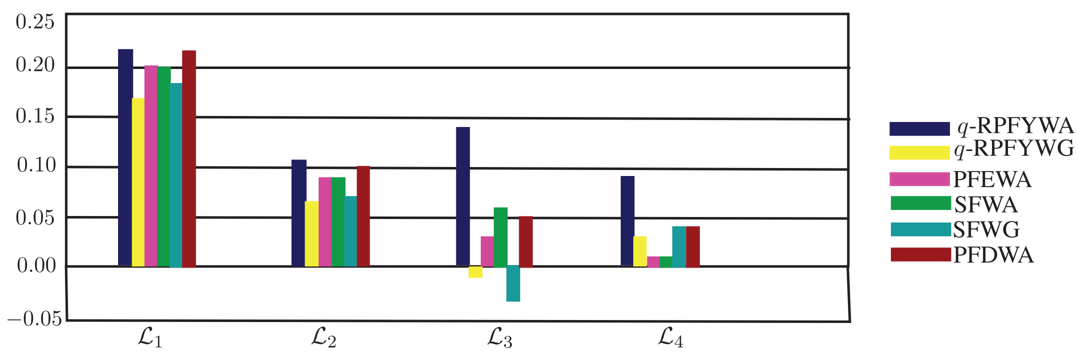

Figure 4 Comparison with existing operators.

7. CONCLUSIONS AND FUTURE DIRECTIONS

AOs are mathematical functions and imperative tools of unifying the many inputs into single valuable output. The q-RPFS, as a new generalization of PFS and SFS, allows to handle the situations with more generality than PFS and SFS, because it still works in the cases where $\mu+\eta+\nu \leq 1$ and $\mu^2+\eta^2+\nu^2 \leq 1$ but it satisfies $\mu^q+\eta^q+\nu^q \leq 1$, where $q \geq 3$. Here, we have discussed MADM problems using q-RPF information. We have studied arithmetic and geometric operations to develop some q-RPF Yager AOs from the motivation of Yager operators as q-RPFYWA, q-RPFYOWA, q-RPFYHWA, q-RPFYWG, q-RPFYOWG and q-RPFYHWG operators. Different properties of these operators including idempotency, boundedness, monotonicity and reducibility are considered. Then, we have applied these operators to expand a few techniques to discuss MADM issues. Finally, practical example for emerging technology enterprise system selection is provided to develop a strategy. The proposed method is compared with the existing methods to show its advantages and appropriateness. Another example for the selection of a suitable company for investment under q-RPF data shows its importance. Moreover, we have discussed the DM results by taking distinct values of parameter. We will extend our work on the following DM problems:

- An in-depth study of the Yager AOs for q-rung picture fuzzy information such as induced q-rung picture fuzzy Yager AOs, q-rung picture fuzzy Yager interactive AOs will be a hot topic in the future.
- A MADM problem in medical diagnosis under q-rung picture fuzzy soft data using Yager AOs will be discussed.
- A MADM problem for the selection of a smartphone under q-rung picture fuzzy soft data using Yager AOs will be discussed.
- A robust DM model under m-polar fuzzy soft Yager AOs will be discussed.

CONFLICT OF INTEREST

The authors declare no conflicts of interest.

AUTHORS' CONTRIBUTIONS

Conceptualization, Muhammad Akram; Methodology, Peide Liu; Investigation, Gulfam Shahzadi. Writing—original Draft Preparation, Gulfam Shahzadi and Muhammad Akram; and Writing—review and Editing, Peide Liu.

ACKNOWLEDGMENTS

This paper is supported by the National Natural Science Foundation of China (No. 71771140), Project of cultural masters and "the four kinds of a batch" talents, the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), Major bidding projects of National Social Science Fund of China (19ZDA080).

REFERENCES

[1] M. Akram, W.A. Dudek, J.M. Dar, Pythagorean Dombi fuzzy aggregation operators with application in multi-criteria decision-making, Int. J. Intell. Syst. 34 (2019), 3000–3019.
[2] M. Akram, W.A. Dudek, F. Ilyas, Group decision-making based on Pythagorean fuzzy TOPSIS method, Int. J. Intell. Syst. 34 (2019), 1455–1475.
[3] M. Akram, H. Garg, F. Ilyas, Multi-criteria group decision-making based on ELECTRE I method in Pythagorean fuzzy information, Soft Comput. 24 (2020), 3425–3453.
[4] M. Akram, H. Garg, K. Zahid, Extensions of ELECTRE-I and TOPSIS methods for group decision-making under complex Pythagorean fuzzy environment, Iran. J. Fuzzy Syst. 17 (2020), 147–164.
[5] M. Akram, G. Shahzadi, A hybrid decision-making model under q-rung orthopair fuzzy Yager aggregation operators, Granular Comput. (2020), 1–5.
[6] M. Akram, N. Yaqoob, G. Ali, W. Chammam, Extensions of Dombi aggregation operators for decision making under m-polar fuzzy information, J. Math. 2020 (2020), 4739567.
[7] S. Ashraf, S. Abdullah, Spherical aggregation operators and their application in multi-attribute group decision-making, Int. J. Intell. Syst. 34 (2019), 493–523.
[8] S. Ashraf, S. Abdullah, T. Mahmood, Spherical fuzzy Dombi aggregation operators and their application in group decision-making problems, J. Amb. Intell. Hum. Comput. 11 (2019), 1–19.
[9] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986), 87–96.
[10] B.C. Cuong, Picture fuzzy sets, J. Comput. Sci. Cybern. 30 (2014), 409.
[11] G. Deschrijver, C. Cornelis, E.E. Kerre, On the representation of intuitionistic fuzzy t-norms and t-conorms, IEEE Trans. Fuzzy Syst. 12 (2004), 45–61.
[12] G. Deschrijver, E.E. Kerre, A generalization of operators on intuitionistic fuzzy sets using triangular norms and conorms, Notes Intuitionistic Fuzzy Sets. 8 (2002), 19–27.
[13] H. Garg, A novel trigonometric operation-based q-rung orthopair fuzzy aggregation operator and its fundamental properties, Neural Comput. Appl. (2020).
[14] H. Garg, S.M. Chen, Multi-attribute group decision-making based on neutrality aggregation operators of q-rung orthopair fuzzy sets, Inf. Sci. 517 (2020), 427–447.
[15] H. Garg, J. Gwak, T. Mahmood, Z. Ali, Power aggregation operators and VIKOR methods for complex q-rung orthopair fuzzy sets and their applications, Mathematics. 8 (2020), 538.
[16] H. Garg, Some picture fuzzy aggregation operators and their applications to multi-criteria decision-making, Arab. J. Sci. Eng. 42 (2017), 5275–5290.
[17] F.K. Gundogdu, C. Kahraman, Spherical fuzzy sets and spherical fuzzy TOPSIS method, J. Intell. Fuzzy Syst. 36 (2019), 337–352.
[18] J. He, X. Wang, R. Zhang, L. Li, Some q-rung picture fuzzy Dombi Hamy Mean operators with their application to project assessment, Mathematics. 7 (2019), 468.
[19] C. Jana, G. Muhiuddin, M. Pal, Some Dombi aggregation of q-rung orthopair fuzzy numbers in multiple-attribute decision-making, Int. J. Intell. Syst. 34 (2019), 3220–3240.
[20] C. Jana, T. Senapati, M. Pal, R.R. Yager, Picture fuzzy Dombi aggregation operators: application to MADM process, Appl. Soft Comput. 74 (2019), 99–109.
[21] A.M. Khalil, S.G. Li, H. Garg, H. Li, S. Ma, New operations on interval-valued picture fuzzy set, interval-valued picture fuzzy soft set and their applications, IEEE Access. 7 (2019), 51236–51253.
[22] S. Khan, S. Abdullah, S. Ashraf, Picture fuzzy aggregation information based on Einstein operations and their application in decision-making, Math. Sci. 13 (2019), 213–229.
[23] M. Lin, C. Huang, Z. Xu, MULTIMOORA based MCDM model for site selection of car sharing station under picture fuzzy environment, Sustain. Cities Soc. 53 (2020), 101873.
[24] M. Lin, W. Xu, Z. Lin, R. Chen, Determine OWA operator weights using kernel density estimation, Econ. Res. Ekon. Istraiz. 33 (2020), 1441–1464.
[25] P. Liu, S.M. Chen, P. Wang, Multiple-attribute group decision-making based on q-rung orthopair fuzzy power maclaurin symmetric mean operators, IEEE Trans. Syst. Man Cybern. Syst. (2018), 1–16.
[26] P. Liu, S.M. Chen, Y. Wang, Multi-attribute group decision-making based on intuitionistic fuzzy partitioned Maclaurin symmetric mean operators, Inf. Sci. 512 (2020), 830–854.
[27] P. Liu, J. Liu, Some q-rung orthopair fuzzy Bonferroni mean operators and their application to multi-attribute group decision-making, Int. J. Intell. Syst. 33 (2018), 315–347.

[28] P. Liu, J. Liu, S.M. Chen, Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision-making, J. Oper. Res. Soc. 69 (2018), 1–24.
[29] P. Liu, P. Wang, Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision-making, Int. J. Intell. Syst. 33 (2018), 259–280.
[30] P. Liu, M. Akram, A. Sattar, Extensions of prioritized weighted aggregation operators for decision-making under complex q-rung orthopair fuzzy information, J. Intell. Fuzzy Syst. (2020), 1–25.
[31] L. Li, R. Zhang, J. Wang, X. Shang, K. Bai, A novel approach to multi-attribute group decision-making with q-rung picture linguistic information, Symmetry. 10 (2018), 172.
[32] X. Ma, M. Akram, K. Zahid, J.C.R. Alcantud, Group decision-making framework using complex Pythagorean fuzzy information, Neural Comput. Appl. (2020). 1–21.
[33] X. Peng, J. Dai, H. Garg, Exponential operation and aggregation operator for q-rung orthopair fuzzy set and their decision-making method with a new score function, Int. J. Intell. Syst. 33 (2018), 2255–2282.
[34] X. Peng, Y. Yang, Some results for Pythagorean fuzzy sets, Int. J. Intell. Syst. 30 (2015), 1133–1160.
[35] X. Peng, Y. Yang, Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators, Int. J. Intell. Syst. 31 (2016), 444–487.
[36] X. Peng, H. Yuan, Fundamental properties of Pythagorean fuzzy aggregation operators, Fundam. Inform. 147 (2016), 415–446.
[37] G. Shahzadi, M. Akram, A.N. Al-Kenani, Decision-making approach under Pythagorean fuzzy Yager weighted operators, Mathematics. 8 (2020), 70.
[38] Q. Song, A. Kandel, M. Schneider, Parameterized fuzzy operators in fuzzy decision-making, Int. J. Intell. Syst. 18 (2003), 971–987.
[39] C. Tan, Generalized intuitionistic fuzzy geometric aggregation operator and its application to multi-criteria group decision-making, Soft Comput. 15 (2011), 867–876.
[40] C. Tan, W. Yi, X. Chen, Generalized intuitionistic fuzzy geometric aggregation operators and their application to multi-criteria decision-making, J. Oper. Res. Soc. 66 (2015), 1919–1938.
[41] L. Wang, H. Garg, N. Li, Interval-Valued q-rung orthopair 2-tuple linguistic aggregation operators and their applications to decision making process, IEEE Access. 7 (2019), 131962–131977.
[42] L. Wang, N. Li, Pythagorean fuzzy interaction power Bonferroni mean aggregation operators in multiple attribute decision-making, Int. J. Intell. Syst. 35 (2020), 150–183.
[43] G. Wei, Picture fuzzy aggregation operators and their application to multiple attribute decision-making, J. Intell. Fuzzy Syst. 33 (2017), 713–724.
[44] Y. Xing, R. Zhang, Z. Zhou, J. Wang, Some q-rung orthopair fuzzy point weighted aggregation operators for multi-attribute decision-making, Soft Comput. 23 (2019), 11627–11649.
[45] Z. Xu, Intuitionistic fuzzy aggregation operators, IEEE Trans. Fuzzy Syst. 15 (2007), 1179–1187.
[46] Z. Xu, X. Cai, Intuitionistic Fuzzy Information Aggregation, Springer, Berlin, Germany, 2012, p. 1102.
[47] R.R. Yager, Aggregation operators and fuzzy systems modeling, Fuzzy Sets Syst. 67 (1994), 129-145.
[48] R.R. Yager, Pythagorean fuzzy subsets, in 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), IEEE, Edmonton, Canada, 2013, pp. 57–61.
[49] R.R. Yager, Generalized orthopair fuzzy sets, IEEE Trans. Fuzzy Syst. 25 (2016), 1222–1230.
[50] L.A. Zadeh, Fuzzy sets, Inf. Control. 8 (1965), 338–353.
[51] H. Zhao, Z. Xu, M. Ni, S. Liu, Generalized aggregation operators for intuitionistic fuzzy sets, Int. J. Intell. Syst. 25 (2010), 1–30.