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Optimal Investment Model for New Technology Project

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Abstract — **One of the most important directions in the study of the behavior of dynamic models in economy is the analysis of effective trajectories, i.e. the analysis of their common properties, not depending on the trajectory, but determined only by the dynamic model. This approach makes it possible to make long-term forecasts and analyze long-term growth rates and structural changes in the economy, to form a perspective view of the general features of behavior in the economy sector. The theoretical basis for such an analysis is the asymptotic (well-known at infinity horizon) properties of the finite sections of the effective trajectories found in solving the stationary problem that are widely known in economic dynamics. The subject of the study is a project model that allows full analysis of the possibility of investing in innovative technologies in enterprises. The aim of the work is to substantiate the possibility of using a probabilistic approach to build a single-product model of transients using a new technology. In the work, the problem of substantiating investment investments in the project of transition from the used (old) technology to the new one with a higher growth rate for the case of a one-dimensional model is solved. It was assumed that the transition was spasmodic at the time the accumulated investment reached a certain threshold value, which is a random variable. As a result of the study, the following significant points were formulated and determined: the optimal investment strategy was formulated as a function of the current "state" of the system, it was substantiated that there was a probability of three types of trajectory behavior, determined by the relationships between growth rates, and the presence of a "risk area" was established, in the event of a hit in which the system is very likely to come to complete destructuring. The paper provides an overview of literary sources in the subject area, as well as an economic interpretation of the results.**

Key words — optimality, single-product model, project, strategy.

I. Introduction

Recently, variants of models of economic dynamics have begun to be intensively studied, in which the potential for a “spasmodic” change in technology (for example, as a result of scientific and technological progress, structural changes, etc.) is laid down. A general description scheme for such models was proposed in [1]. The main features of this scheme are the presence of one or more projects to create a new (or new) technology that extends the capabilities of the system and makes it more efficient. The moment of project implementation (and, consequently, the emergence of new technology) is considered as a random variable, with respect to the distribution of which certain proposals are built. In a number of works [2, 3], such a distribution is considered exogenously given; at the same time, it is possible to build a stationary mode of the trunk type, as well as to carry out the corresponding model calculations. However, the most interesting, in our opinion, are models in which the implementation of projects requires additional costs due to the diversion of some resources from the "production" sphere and the creation of certain "innovative" funds, and the time of the project depends on the level of these funds. For such problems, the existence of a system of stimulating prices was proved in [4], and in [5] it was shown that optimal strategies are close to the optimal strategy of some stationary problem. This property makes it possible to use the corresponding stationary task for a qualitative analysis of both trajectories and investment strategies for the implementation of a new technology project. From an applied point of view, despite the fundamental possibility of solving such problems, the problems of efficiently constructing a solution and analyzing it are of particular importance. This circumstance forces us to consider large-aggregate models, and, in particular, single-product ones. In them we deal only with the cost expression of the state of the economic system and, solving the problem of the distribution of resources between production and the "innovative" sector, we digress from the traditional problem of the distribution of capital investments between individual sectors of the economy. This approach allows giving some estimate of the size of investments in the innovation sector and then separate considering the task of allocating funds within the economy. The single-product model of the transition to a new technology is described below, which is a one-dimensional linear version of a more general model from [6]. The effective procedure for numerically finding a solution to the stationary problem has been constructed for it. This procedure is implemented on a computer, it makes it possible to calculate the volume of investment in the development of a new technology depending on the state of the economic system, as well as make a number of qualitative conclusions.

II. One-product model of transition to a new technology

Let the system function within the framework of some (traditional) technology in the dark. This means that the gross product of the system, which in the one-dimensional model is characterized by a non-negative number x, increases per unit time in times. At some point, it becomes known that there is a fundamentally feasible project of new technology with a higher pace. However, its implementation requires additional costs for the creation of a fund for the implementation of a new technology project (FRP), the volume of which should reach a certain "threshold" value. The exact value of this threshold is not known in advance, and we will consider it a random variable (CB).

Thus, from the moment the period of accumulation of the FRF begins, during which the state of the system is specified by a pair where it has the previous meaning, and the number is the current volume of the FRF. The control parameter in state X is the amount of investment for the upcoming unit time interval. We believe that a non-negative increasing function of investment efficiency is given, which characterizes the increment in the volume of the FRF for current investments c. The main case is considered in the work, when the increment in the volume of FRP coincides with the volume of investments, but nonlinear dependence of efficiency on investments is theoretically acceptable.

Dynamics (in discrete time) in the accumulation period is described by the transition rule

$$X \to X' = (x', z')$$

$$\text{а) } x \to x' = a_0(x - c), \quad 0 \le c \le x, \tag{1}$$

$$\text{б) } z \to z' = z + h(c),$$

which defines a state space mapping

$$\Omega := \{X = (x, z) \mid x \ge 0,\ z \in R\} \tag{2}$$

in yourself.

For everybody $X_0 \in \Omega$ an investment program, or simply a program, we will call a finite or infinite sequence $\chi = \chi(X_0) := \{(x_t, c_t), t = 0,1,...\}$, bound by rule (1, a). The program χ generates a sequence in accordance with (1, б); the accumulation $\{z_t\}$ period ends at a random moment of stopping τ, when the current volume of FRP will be sufficient for the implementation of the project

$$\tau := \min\{t \mid z_t \ge \xi\}. \tag{3}$$

We believe that at the moment τ there is an instant transition (jump) to a new technology, and from this moment the system develops according to the law

$$y_t = x_\tau \alpha_1^{t-\tau}, t \ge \tau. \tag{4}$$

A probabilistic description of the stopping moment is based on an exogenously given a priori distribution function (PD) π of a threshold random variable $P(\tau \le t) = P(z_t \ge \xi) = \pi(z_t)$.

Remark 1. It follows that the project investment model is uniform in time; if at the moment t_0 the system is in a state $X = (x, z) \in \Omega,$ and the FRF level z is still insufficient, then the future stop moment is a conditional CB τ_z with distribution

$$\begin{aligned} F_z(t) &:= P(\tau \le t \mid \tau > t_0) = P(z_t \ge \xi \mid \xi > z) = \\ &= P(\xi \in (z, z(t)) \mid \xi > z = \frac{\pi(z_t) - \pi(z)}{1 - \pi(z)}, t > t_0, \end{aligned} \tag{5}$$

which depends only on z and c_t, $t \ge t_0$, but not on t_0 the background of the process z_t, $t < t_0$. In particular, if in state X a decision is made that increases the volume of the FRF from z to $z' > z$, then the probability that this will be sufficient to reach the threshold and transition the system over time to a new technology will be

$$\beta = \beta(z, z') := \frac{\pi(z') - \pi(z)}{1 - \pi(z)}. \tag{6}$$

Thus, for each state $x_0 = (x_0, z_0) \subset \Omega$, the investment program $\chi(x_0)$ generates a random stop moment τ_{z_0} of the FR of the form (5) and, thereby, a random trajectory $\{b_t, t = 0,1,...\}$ of the gross product of the system:

$$b_t := \begin{cases} x_t, t < \tau, \\ y_t, t \ge \tau, \end{cases}$$

where x_t is the *x*-component of the program $\chi(x_0)$, and are defined in (4).

To formulate the optimization problem, it is necessary to select a criterion for evaluating the investment program. We restrict ourselves to a criterion of the terminal type, which depends only on the final product b_T, where t is the planning horizon. Note that there are two different approaches to the formulation of the terminal problem related to the global attitude of the system to new technology [7, 8].

In the first of these, the transition to a new technology is a strategic task of the economic system, given out of model considerations. Therefore, if the project is not implemented in the planning period $(\tau > T)$, we assign a zero rating to the final state, otherwise the final product will be the natural state estimate. This approach, which can be called strategic, corresponds to the following task:

$$\alpha_1^{-T} E(b_T I_{(\tau \le T)}) \to \max_{\chi(X_0)} =: \Phi_T(X_0), \tag{7}$$

where is I_A the indicator of event *A* (unit for events from the set A and zero otherwise), and the maximum is taken for all programs $\chi(x_0)$ of length T emanating from the point $X_0 \in \Omega$.

In the second approach – pragmatic – the goal of the system is simply to maximize the expected output at the end of the planned interval, regardless of the technology within which the system will be located. The corresponding task takes the form

$$\alpha_1^{-T} Eb_T \to \max_{\chi(X_0)} =: \Psi_T(X_0), \quad (8)$$

where, as above, the maximum is taken for all programs $\chi(x_0)$ of length T emanating from the point $X_0 \in \Omega$.

We note that always $\Phi_T(X) \le \Psi_T(X)$, and the normalizing factor α_1^{-T} in (7) and (8) does not affect the decision made, but allows going to the limit stationary model for $T \to \infty$.

The main thing that is of interest to us in this problem is the asymptotic behavior of the functionals $\Phi_T(X), \Psi_T(X): \Omega \to R$ and their corresponding investment $C_T, C^T: \Omega \to R$ strategies that correspond to the planning horizon *T*. The value of the function $C_T(C^T)$ at the point $X_0 \in \Omega$ is the investment amount $\overline{c_0}$ prescribed by the corresponding maximizing program $\chi^T(X_0) = \{(\overline{x_t}, \overline{c_T}), t = 0,1,...,T\}$. We will see below that the asymptotic behavior does not depend on strategic or pragmatic approaches to the statement of the problem, although for a finite horizon the difference can be significant.

III. STATIONARY MODEL. PRELIMINARY ANALYSIS

We denote by *W* the class of continuous functions Φ defined on Ω and satisfying the condition $0 \le \Phi(x,z) \le x \; \forall (x,z) \in \Omega.$. We introduce the natural ordering in *W* $\Phi_1 \le \Phi_2 \leftrightarrow \Phi_1(X) \le \Phi_2(X) \forall X \in \Omega$ and metric

$$\rho(\Phi_1, \Phi_2) := \sup_{X=(x,z)} \frac{|\Phi_1(X) - \Phi_2(X)|}{x}.$$

By definition, $\Phi \le \Phi \; \forall \Phi \in W$, where $\Phi(x,z) \equiv x$

Moreover it is easy to show that the class *W* is complete.

Consider the sequence of functionals (7) and (8) with increasing values of T. When $T = 0$, by definition $\Phi_0(X) = 0, \quad \Psi_0(X) = x$. For $T \ge 1$, taking into account Remark 1, the recurrence relations

$$\Phi_T = \Gamma\Phi_{T-1}, \Psi_T = \Gamma\Psi_{T-1}, T = 1,2,.... \quad (9)$$

in which the Bellman operator Γ acts by the formula

$$\Gamma\Phi(X) := \frac{1}{\alpha_1} \max_{0 \le c \le x} [(1-\beta)\Phi(X') + \beta x'] \quad (10)$$

where $X' = (x', z')$ and β are determined according to (1), (6); in this case $C_T(X)$ – the value of *c*, maximizing the right-hand side of (10) with $\Phi = \Phi_{T-1}$, and $C^T(X)$ the value of c maximizing the right-hand side of (10) with $\Phi = \Psi_{T-1}$.

The following statement is true.

Statement. a) The operator Γ is a contracting map of *W* into itself, and, therefore, there exists a unique function $G \in W$ satisfying the Bellman equation:

$$G = \Gamma G, \quad (11)$$

and the strategy $C(\bullet)$;

b) the functions $G(X)$ and $C(X)$ – the functional and optimal strategy of the problem

$$E\alpha_1^{-\tau} x_\tau \to \max_{\chi(x)}; \quad (12)$$

c) for any $z \in R$ function $G(x,z)$ increases and is convex in *x*; moreover,

$$G \le \lambda\Phi, \lambda := \frac{\alpha_0}{\alpha_1} < 1; \quad (13)$$

d) sequences of functions Φ_T and Ψ_T belong to the class *W* and converge (for $T \to \infty$) to *G*, respectively, monotonically increasing (Φ_T) or monotonically decreasing (Ψ_T).

Optimization problem (12) characterizes the "asymptotic" planning horizon ($T = \infty$). We will call it the stationary task, and the corresponding investment strategy $C(X)$ – the stationary optimal strategy. Note that the stationary problem will be the same for different statements of the terminal problems (7) and (8); moreover, the corresponding finite-step functionals can be considered as its lower and upper approximations: $\Phi_T \le G \le \Psi_T$ for any *T*.

Statement. Let $c_T(X) \in C^T(X)$ be a sequence of optimal moves for a state $X \in \Omega$ with an increasing planning horizon. Then:

a) $d(c_T(X), C(X)) \to 0$ for $T \to \infty$, where $d(a, A)$ is the distance from the point *a* to the set A;

b) if the strategy $C(\bullet)$ is unique, then for any compact $K \subset \Omega \; c_T(X) \to C(X)$ at $T \to \infty$ uniformly $X \in K$.

Next, we study the stationary problem for a uniform a priori distribution of the threshold ξ . Such a distribution corresponds to a situation where only the range of possible values ξ is known and there is no reason to prefer one of them to the other. By choosing a reference point and a scale unit, we can reduce an arbitrary uniform distribution of a random variable ξ to a standard one on the interval [0, 1]. Therefore, we accept

$$\pi(z) = \begin{cases} 0, & z < 0, \\ z, & z \in [0,1], \\ 1, z \ge 1. \end{cases}$$

Obviously, in the state space (2) the region $z \ge 1$ is of no interest, since in it from any state a transition to a new technology will immediately take place and the concept of "accumulation period" loses its meaning. Formally, the solution of equation (11) is the functions

$$G = \lambda\Phi, C = 0. \tag{14}$$

Therefore, the stationary optimal problem requires investigation only when $z < 1$, in this case, the "normal" region is substantial. In states with a "subnormal" volume of FRP $z < 0$, preliminary investments are required to enter the normal region, after which there is a chance for a transition to a new technology.

We examine each area individually.

1. The normal area. Here, equation (11) takes the form

$$G(X) = \alpha_1^{-1} \max_{0 \le c \le \min(1-z,x)} \left[\left(1 - \frac{c}{1-z}\right) G(x', z+c) + \frac{c}{1-z} x' \right], \quad X \in \Omega, \tag{15}$$

where $x' = \alpha_0 (x - c)$, while his decision is representable in the form

$$G(x,z) = (1-z) g(p), C(x,z) = (1-z) S(p), \; p := \frac{x}{1-z}, \tag{16}$$

where the function g of the non-negative scalar argument p satisfies the equation

$$g(p) = \alpha_1^{-1} \max_{0 \le s \le \min(1,p)} K(g(q), s, q), \; p \ge 0. \tag{17}$$

$$q = L(p,s) := \alpha_0 \frac{p-s}{1-s} \ge 0 \tag{18}$$

($q = p'$ – new condition)

$K(\upsilon, s, q) := (1-s)^2 \upsilon + s(1-s) q$ – the kernel of the Bellman operator, $S(p)$ is the optimal move on the right-hand side of (17). Note that, according to (16) $g(p) = G(p, 0)$, $S(p) = C(p, 0)$; in addition, the function *g* is monotonic, convex, and $g(p) \le \lambda p$. Variable *p* and the functions $g(p)$, $S(p)$ can be considered as "relative" analogues of the variable *x* and the functions $G(X)$, $C(X)$ respectively. Indeed, according to (16), the transition from "absolute" to relative values is obtained if we take the "deficit FRP" $1 - z$, i.e. the amount of investment required in state *X* to guarantee the achievement of new technology. From this point of view $s = S(p)$, the value can be considered as the proportion of the deficit paid off in the state *p*. We now state the main properties of the solution.

1. In the entire (two-dimensional) normal region $G(X)$, the function is convex and monotonic in $X = (x, z)$.

2. The optimal course $S(p)$ does not decrease along the *p*.

3. The transition mapping of equation (17) $p \to q = Q(p) := L(p, S(p))$ also increases with respect to *p*; therefore, the sequence of states $p_t = Q(p_{t-1})$ (*p* is the trajectory of the system) is monotonic – it either increases or decreases with time.

4. For $p \ge p^0 := \frac{2-\lambda}{1-\lambda} (p^0 > 2)$ the function *g* is linear

$$g(p) = \lambda(p-1) \tag{19}$$

and the optimal move is a direct transition to a new technology, i.e. $S(p) = 1$, $Q(p) \to \infty$ in this case $S(p) < 1$, at $p < p^0$, i.e. $p^0 = Q^{-1}(\infty)$.

5. In the region $p < p^0$, the function g continues from right to left as piecewise quadratic

$$g(p) = g_k(p) := A_k p^2 + B_k p + C_k, \; p \in \Delta_k, \; k = 1, 2, ... \tag{20}$$

where intervals Δ_k and functions g_k are constructed recursively using the relation

$$\Delta_0 := \{p \mid p \ge p^0\}, \quad \Delta_k := \{p \mid Q(p) \in \Delta_{k-1}\}, k = 1, 2.... \tag{21}$$

Since it $Q(p)$ increases in p and $Q^{-1}(\infty) = p^0$, these intervals are adjacent to each other, i.e. are formed by a decreasing sequence of left ends $\{a_k, k = 1, 2, ...\}$ according to the rule $\Delta_k = [a_k, a_{k-1}]$, where $a_k = Q^{-1}(a_{k-1})$, at the same time $a_0 = p^0$. Note that the recurrence relations for the coefficients of the right-hand side of (19) are written explicitly, and therefore their values for all *k* can be calculated $Q(p)$ $p \in \Delta_k$ and sequence $\{a_k\}$ also computable. The recurrence process (20) "extends" the solution from right to left. With increasing *k*, the considered values $p \in \Delta_k$ decrease; values $S(p)$, are also reduced and three cases are possible.

Case 1. For some $k = j$ on the interval Δ_j, the function $S(p)$ reaches its smallest value – zero, i.e. $S(p) = 0$ at some point $p \in \Delta_j$. Then at $p < p$ $S(p) \equiv 0$ ("waiting" mode – refusal to invest the project), and the process (19) turns into the next

$$Q(p) = \alpha_0 p, \; g(p) = \alpha_1^{-1} g(\alpha_0 p), \quad p \le p \tag{21}$$

Thus, the "quadratic staircase" contains a finite number of steps *j* and reaches the point p to the left of which the waiting mode operates. In two other cases, there are no waiting states: the quadratic ladder is infinite, and $S(a_k) > 0 \; \forall k$.

Case 2. An infinite quadratic staircase covers the entire semi-axis $\{p > 0\}$, т.е. $a_k \to 0$.

Case 3. An infinite sequence converges to a nonzero point, which is a fixed point of the transition map $Q(p)$. Using the methods proposed in [8], it can be found analytically:

$$p^* = 1 - \frac{1}{\theta}(\alpha_0 - 1), \quad \theta := \sqrt{1 - \lambda\alpha_0}, \tag{22}$$

where in

$$S(p) = \frac{\theta}{1+\theta}p, \; g(p) = g^*(p) := \frac{\lambda}{1+\theta}\frac{p^2}{2}, \; p \le p^*. \tag{23}$$

A necessary and sufficient condition for the existence of such a "stationary" point is a condition $a_0 + \lambda < 2$ that implies inequality $p^* > 0$.

Remark 2. By constructing a "ladder" everywhere on it $p < Q(p)$.

In fig. Figure 1 shows the regions in the space of model parameters corresponding to the three indicated cases. In fig. Figure 2 shows the graphs of the functions S and Q (only for; $p \le p^0$ since the $p > p^0$ transition to a new technology is optimal).

We comment on these three cases.

Case 1 corresponds to a situation where the pace a_0 of traditional technology is high enough, and the difference in pace a_1 and a_0 is not very significant. Then $Q(p) > p \quad \forall p$ each trajectory p_t increases with time, passing with probability one to a new technology in a finite number of steps. Moreover, when $p^0 > \hat{p}$ investing begins immediately, and when $p^0 < \hat{p}$ – only after a certain period of waiting p_t , when the current state reaches a value $\hat{p}$.

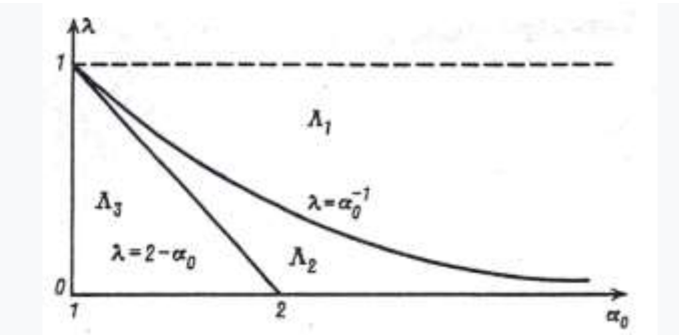

Fig. 1. Partitioning the parameter space

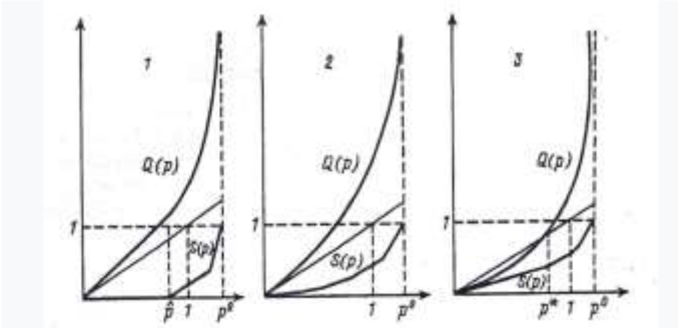

Fig. 2. Optimal stroke and transition function in the "normal" region

Case 2 is intermediate; it differs from the first only in that the investment of the project begins immediately with any $p_0 > 0$.

Case 3 – the difference in pace a_1 and a_0 manifests itself significantly. Here the situation is fundamentally different: when $p_0 > p^*$ it is similar to case 2, and when $p_0 < Q(p_o) < p_0$ and $p_t \to 0$ when $t \to \infty$. The fixed point p^* is unstable and the p_t trajectories "run away" from it.

2. Continuation of the decision in the "subnormal" area. If the volume $X \in \Omega$ of the FRF is negative in the state, then preliminary investments are necessary to enter the positive area, and only after that a non-zero probability of transition to a new technology appears. With the investment efficiency function under consideration $h(c) = c$, the total increment of funds in a few steps $(\Delta z_i = c_i)$ is equivalent to one increment in the total investment $(\Delta z = \sum \Delta z_i = \sum c_i)$, so it makes no sense to pre-invest in small doses, but it is more profitable to do this in a single act, postponing it as much as possible to increase the gross product x in the framework of the original traditional technology. Therefore, for each $z < 0$, there exists a certain minimum value such $x = m(z)$ that $C(X) = 0$ for $x < m(z)$ and $C(X) + z > 0$ for $x \ge m(z)$. It is clear that $m(z) \ge -z$ since $C(X) \le x$. The curve $y = \{X \in \Omega \mid x = m(z)\}$ is the boundary of the zones Ω^0 and Ω^+. In the zone Ω^+, optimal investments include upfront costs in volume $(-z)$ and "stimulating" investments $\tilde{c}$ that increase the likelihood of new technology: $C(X) = -z + \tilde{c}$. Moreover, the choice of the optimal value $\tilde{c}$ in state X is equivalent to the choice of the same value in state $\tilde{X} = (x - (-z), 0) = (x + z, 0)$, i.e. $C(X) = -z + S(x + z)$ this implies

$$G(X) = G(u, 0) = g(u), \; X = (x, z) \in \Omega^+, \; u := x + z. \tag{24}$$

On the other hand, in the zone Ω^0 $C(X) = 0$, and therefore

$$G(x, z) = \alpha_1^{-1} G(\alpha_0, x, z) \tag{25}$$

At the interface γ, both relations (24) and (25) are valid, and if $(x,z)\in\gamma\subset\Omega^+$, then $(\alpha_0,x,z)\subset\Omega^+$, and, therefore, the boundary itself is determined by the equation

$$\gamma: g(x+z)=\alpha_1^{-1}g(\alpha_0 x+z), \quad (26)$$

in which the function g should be considered known – it has already been defined.

It can be shown that:

1) for each $x\geq 0$, relation (26), regarded as an equation with respect to z, has a unique solution $z=m^{-1}(x)$ in the region $z\leq 0$ moreover $|m^{-1}(x)|$ and increases in x (in case 1 $m^{-1}(x)=0$ with $x\leq\hat{p}$), thus, the curve γ is determined uniquely;

2) the relative product $p=x/(1-z)$ is limited on the entire curve γ by a value $\bar{p}:=((a_1-1)/(a_1-a_0))>1$ and when $z<z^0:=1-a_1/(a_0-1)$ the curve γ lies on the beam $\{x=\bar{p}(1-z),\quad z\leq 1\}$ (Fig. 3). Thus, relations (24) – (26) completely describe the solution in the considered "subnormal" region.

Remark 3. It is easy to see that the arguments of Sec. 2 directly carry over to the case of an arbitrary distribution $\pi(z)$ with support on the positive semiaxis $\{z\geq 0\}$. Indeed, the whole logic of reasoning regarding preliminary investments is based on the following property of the investment efficiency function $h(c):\sum h(c_i)\leq h\sum(c_i)$. This relation holds for convex functions. Therefore, for such efficiency functions and any distribution function $\pi(z)$ with a carrier R_+, the overall picture in the region $z<0$ has the same form as for the main case studied here. Note that the convexity of the function h reflects the effect of "concentration of effort." A different picture is observed for concave functions. So, for power functions $h(c)=c^{\gamma},\quad 0<\gamma<1,$ it is advisable to make small investments. In this case, the zone of zero investment will be completely absent.

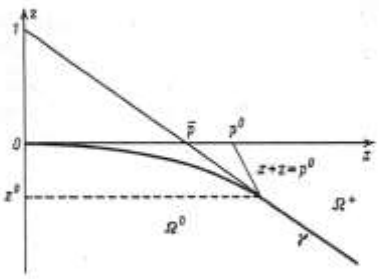

Fig. 3. Boundary Curve γ

IV. Final picture. Field of trajectories

Figures 4 and 5 show the field of optimal trajectories (x_t, z_t) of the stationary model in the state space Ω. Hitting a level means $z=1$ moving to a new technology. Three types of behavior are possible depending on the position of the parametric point (α_0,λ) ($\Lambda_1-\Lambda_3$ the area in Fig. 1). Comment on them.

In all cases, there is an area Ω_w of waiting states when investment is delayed: $C(X)=0, X\in\Omega_w$. In cases 2 and 3, the region Ω_w coincides with the zone Ω_w of subnormal states with a right boundary γ. In the first case Ω_w, it includes, in addition to Ω^0, a part of the region of normal states in Fig. 4 (case 1), located to the left of the segment l connecting the points $(0, 1)$ and $(\hat{p},0)$, so that the right boundary γ is supplemented by this segment. Whatever the initial state X_0 with a non-zero product x_0, the investment of the project will necessarily begin at once $X_0\notin\Omega_w$, and at $X_0\in\Omega_w$ – after a certain waiting period during which the phase point moves horizontally in the region Ω_w $(z_t=z_0,\ x_t=a_0^t x_0)$ until it crosses the border γ. Having left the waiting area Ω_w the trajectory never returns there again; in other words, once started, project investment is never interrupted. If the initial level of FRF $z_0\leq z^0$, then, leaving Ω_w, you must immediately make investments that guarantee the transition to new technology in one step [9].

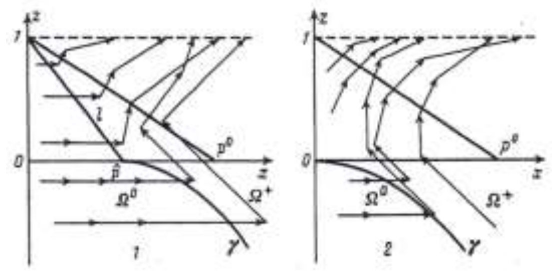

Fig. 4. Field of trajectories for cases 1 and 2

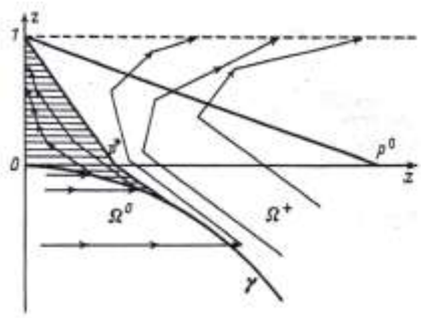

Fig. 5. Trajectory field for case 3

Cases 1 and 2 are characterized by the fact that in a finite number of steps the system moves to a new technology, reaching a level, if necessary $z=1$. The same picture will be in case 3, if only, leaving the zone Ω_w, the phase point does not fall into the risk area, shaded in Fig. 5. The presence of a risk area is a special feature of the case 3. Let us consider it in more detail.

If $X_0 \in \Omega_r$ the volume of the FRF z_t is either positive from the very beginning ($z_0 > 0$), or it will be so, starting with $t = 1$. For simplicity we put that $z_0 > 0$. When the $z > 0$ optimal behavior is described by formulas (23), whence it follows that in any state $X \in \Omega$ the investments are $C(X)$ proportional to *x*, therefore

$$x_t = \eta^t x_0,\ z_t = z_0 + \frac{\theta(1+\eta^{t+1})}{(1+\theta)(1-\eta)} x_0, \text{ где } \eta := \frac{\alpha_0}{1+\theta} < 1.$$

Thus, in the risk zone, the gross product x_t tends to zero, and the level of FRP $z_t \to z_\infty = z_0 + x_0 / p^*$. Note that

$$\frac{1}{1-z_0}\sum_{t=0}^{\infty} C(X_t) = \frac{z_\infty - z_0}{1-z_0} = \frac{p_0}{p^*} = \pi_{z_0}(z_\infty),\ p_0 < p^*.$$

This means that the ratio p_0 / p^* can be interpreted as the asymptotic fraction of the initial deficit $1 - z_0$ that the system is able to cover, falling into the risk area and acting in an optimal way, and at the same time as the probability of transition to a new technology; the value p^* itself is the minimum value of the relative product *p*, at which the transition to a new technology is guaranteed. With small ones p_0, such "risky behavior", which threatens complete ruin, turns out to be more profitable than a "reasonable" program of guaranteed transition to a new technology. In fact, able $X_0 = (p_0, 0)$, there is a guaranteed program $\chi(X_0)$ in the state: wait for the moment t_0 when the value p_{t_0} becomes more than two, then give the product unit to the FRP, and with the remaining unit go to the new technology. Let us estimate the functional value of the stationary problem Φ with such a program; we have for small p_0

$$p_{t_0} = p_0\alpha_0^{t_0} \approx 2,\ t \approx \frac{\log(2/p_0)}{\log\alpha_0},$$
$$\Phi = 1 \cdot \alpha_1^{-t_0} = e^{-t_0 \log\alpha_1} \approx e^{-v\log(2/p_0)} = (p_0/2)^v, \quad (27)$$
$$v := \log\alpha_1 / \log\alpha_0$$

In the region Λ_3 $v > 2$, therefore, for small p_0 functional (27) is less than the quadratic functional (23). In the region Λ_1, on the contrary, $v < 2$ and there, for small ones p_0, a waiting mode does take place, but not as rough as proposed in (27).

Now, we will discuss the problems that arise in the transition from discrete time to continuous, and ways to overcome them. We start with some rethinking and a slightly different interpretation of the model parameters α_0 and α_1. According to the statement of the problem, they are the growth factors of the system per unit time. Emphasizing the dependence of these parameters on Δt, it is convenient to represent them in the form

$$\alpha_0 = e^{p_0\Delta t},\ \alpha_1 = e^{p1\Delta t}, \quad (28)$$

where p_0, p_1 are some absolute indicators, with a dimension of a year. We will interpret the time Δt unit adopted in the model, in accordance with the transition rule, as the duration of the development of investments made in the FRF. As part of the economic interpretation, p_0~ 0.02-0.05 (2-5% per year), p_1~ 0.04-0.08 ($p_1 > p_0$) can be taken; then dimensionless parameter $v = p_1 / p_0$ will vary widely: $1 < v < 4$, and cases 1-3 can occur.

Small lag development. Is it possible in the above analysis of the solution to go to the limit at $\Delta t \to 0$, which would correspond to the "instant" development of investments in continuous time? When $\Delta t \to 0$ we have $\alpha_0, \alpha_1, \lambda \to 1$ the parametric point (α_0, λ) rushes to (1, 1), where all three areas are joined (Fig. 1). This means that it is impossible to formally substitute values $\alpha_0 = \alpha_1 = 1$ into the model: a solution, if it exists, will be either non-unique or degenerate.

Indeed, the functionals Φ_T can be directly substituted $\alpha_0 = \alpha_1 = 1$. The calculation shows that the corresponding sequence of functionals g_T and optimal strategies S_T has the form

$$g_T(p) = \begin{cases} \dfrac{p^2}{2p_T}, & p \le p_T, \quad p_T = \dfrac{T+1}{T}, \quad T = 1, 2, \ldots \\ p - \dfrac{1}{2}p_T, & p > p_T \end{cases} \quad (29)$$

here *T* is interpreted as the number of a member of the sequence, i.e. iteration number; calendar planning period $T\Delta t$. When $T = \infty$ we obtain a stationary functional g_∞ corresponding to the value $g_\infty = 1$, and a degenerate strategy $S_\infty(p) \equiv 0$ that satisfy the conditions (16). But this solution is not the only one; equation (16) has an infinite number of solutions, but in all solutions the strategy is degenerate: $S(p) \equiv 0$. The solution, in particular, is the family of functionals with parameter v

$$g(v, p) := \begin{cases} Mp^v, & p \le \bar{p}, \\ p - \dfrac{1}{2}, & p > \bar{p}, \end{cases}$$

$$\bar{p} = \bar{p}(v) := \frac{v}{2(v-1)},\ M = M(v) := \frac{1}{v\bar{p}^{v-1}}, 1 < v \le 2,$$

(at the point $\bar{p}$ the conjugation has first-order smoothness). If we try to go to the limit $\Delta t \to 0$ with parameters (28) with fixed values p_0, p_1, then with $v < 2$ we get a solution of the form (30), and with $v \ge 2$ – of the form (29) from. $p_T = p_\infty = 1$. In various situations, this expression has a different meaning. In the case $v < 2$, the strategy $S(p) = 0$ corresponds $p \le \bar{p}$ to the waiting mode, and at $p > \bar{p}$ to the investment mode of the FRF in infinitely small portions; the case $v > 2$ corresponds to the same regime on the entire axis

$p > 0$. Note that investing in infinitely small portions can not be interpreted as the intensity of investment in continuous time, since this requires that $S(p) = O(\Delta t)$.

Fixed lag development. The unit of time Δt plays a double role in the model – this is both the lag of investment development and the step of the discrete grid of decision-making moments. Therefore, we can tend to zero the step of the time grid, preserving the development lag and taking it as a fixed unit of time: $\Delta t = 1$ [5]. Consider a discrete-continuous version of the model in which the system dynamics differs from the discrete transition rule (1) in only one respect: if no investments are made at time t0, the system is not required to wait a whole unit of time, but can plan to start investing at any time $t > t_0$. If at some point non-zero investments are made, then new investments can be made, as before, only after a unit of time. The stationary Bellman equation in this model is somewhat different from (11):

$$G(X) = \max[e^{-p_1 dt} G(X_{dt})], \Gamma G(X)], \ X = (x,z) \in \Omega, \quad (31)$$

where dt is the time differential. The first member on the right side corresponds to the opportunity to "wait", the second – "start investing". If the first opportunity is realized, then

$$G(x,z) = (1 - p_1 dt) G(x(1 + p_0 dt), z) + o(dt) = G(x,z) + \\ + \left\{ p_0 x \frac{\partial G}{\partial x} - p_1 G \right\} dt + o(dt),$$

those $\{...\} = 0$, and therefore (31) is reduced to the form $G(X) = \max\left[v^{-1} x \frac{\partial G}{\partial x}, \Gamma G(X) \right]$, where $v = p_1 / p_0$, as before.

V. Conclusions

The constructed model allows stating that in the normal region we have obtained a one-dimensional equation for relative variables (15). Note that the investment, having begun, is no longer interrupted in the future, therefore, in the new model the strategy will not change, and in cases 2 and 3 the solution is fully preserved. In case 1, the functional will change only in the region $p \le \hat{p}$. The trajectory field also retains its structure, only the horizontal sections of the trajectories in the waiting area Ω_w will no longer cross the border γ, but will "rest" on it and jump further into the normal region. The results of studies of the properties of the constructed model show its viability and its practical use will lead to a significant reduction in time and material costs when implementing investment projects.

References

[1] O.V. Konina, "Scientific and methodological foundations of an innovative company management", Lect Notes Networks Syst., no. 91, pp. 495–502, 2020.

[2] N. Natocheeva, A. Borodin, N. Rud, G. Kutsuri, M. Zholamanova, A. Namitulina, "Development of tools for realizing the potential of financial stability of enterprises", Entrepreneurship and Sustainability Issues, vol. 7, no. 2, pp. 1654–1665, 2019.

[3] OV. Konina, "Scientific and methodological foundations of an innovative company management", Lect Notes Networks Syst., no. 91, pp. 495–502, 2020.

[4] I.I. Galimova, O.M. Pasynkova, L.N. Tchaikovskaya "Assessment of investment attractiveness of the Voronezh region", Bull. of the Voronezh State Univer. of Engineer. Technol., vol. 81, no. 2, pp. 285–289, 2019. Retrieved from: https://doi.org/10.20914/2310-1202-2019-2-285-289

[5] M.L. Lapshina, A.A. Meshcheryakova, T.V. Zaitseva, E. A. Zaitsev, "The rationale for using the properties of inequalities in the formation of price indices", Sci. J., vol. 7, no. 3, 2019. Retrieved from: http://moit.vivt.ru/ doi.org: 10.26102/2310-6018/2019.26.3.008

[6] A.N. Ayvazly, "Scientific research on improving commercial banks and applying innovations", Bull. of the Voronezh State Univer. of Engineer. Technol., vol. 81, no. 2, pp. 280–284, 2019. Retrieved from: https://doi.org/10.20914/2310-1202-2019-2-280-284

[7] H. Setiawan, B. Erdogan, "Key factors for successful corporate entrepreneurship: A study of indonesian contractors", Int. J. of Construct. Manag., vol. 20, no. 3, pp. 252–268, 2020.

[8] L.V. Kislitsyna, S.A. Kapustina, "Financial equilibrium as a special state of capital within the holding: the possibility of collateral", Bull. of the Voronezh State Univer. of Engineer. Technol., vol. 81, no. 1, pp. 351–356, 2019. Retrieved from: https://doi.org/10.20914/2310-1202-2019-1-351-356

[9] V.N. Vasilenko, L.N. Frolova, A.A. Derkanosova, N.A. Mikhailova, A.A. Schepkina, A.M. Davydov, "Mathematical support of the process of extrusion of anomalously viscous media by experimental design methods", Bull. of VGUIT, vol. 80, no. 3, pp. 37–42, 2018. Retrieved from: https://doi.org/10.20914/2310-1202-2018-3-37-42

[10] N. Natocheeva, A. Borodin, N. Rud, G. Kutsuri, M. Zholamanova, A. Namitulina, "Development of tools for realizing the potential of financial stability of enterprises", Entrepreneurship and Sustainability Issues, vol. 7, no. 2, pp. 1654–1665, 2019.