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Research Article

A Deng-Entropy-Based Evidential Reasoning Approach for Multi-expert Multi-criterion Decision-Making with Uncertainty

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ABSTRACT

The evidential reasoning (ER) approach has been widely applied to aggregate evaluation information in multi-expert multi-criterion decision-making (MEMCDM) problems with uncertainties. However, the comprehensive results derived by the ER approach remain uncertain. In this study, we propose a Deng-entropy-based ER approach for MEMCDM problems to reduce the uncertainty. Firstly, we reassign the remaining belief of the uncertain evaluation information to the focal elements of the given evaluations. Afterward, we introduce the Deng entropy to respectively calculate the objective weights of criteria and those of experts, so as to reduce the subjective uncertainty in MEMCDM. Then, the ER approach is applied twice to generate the comprehensive evaluations of alternatives. A method is introduced to rank alternatives corresponding to their comprehensive evaluations, forming a Deng-entropy-based ER approach for MEMCDM problems with uncertainty. An illustrative example of screening the people at high risk of lung cancer is provided, and comparative analyses are given to show the rationality and superiority of the proposed method.

1. INTRODUCTION

Multi-expert multi-criterion decision-making (MEMCDM) is a process where several experts are invited to evaluate a set of alternatives under multiple criteria. As MEMCDM problems usually are complex, it is difficult for each expert to give crisp evaluation values based on limited information and knowledge. To deal with the uncertainty in MCDM, the evidential reasoning (ER) approach [1] was proposed by introducing the concept of belief structure. This method can model the uncertainty in evaluations by assigning the remaining belief to its unique identification framework, and then aggregate such uncertain evaluations by calculating their orthogonal sum. Due to its effectiveness in modeling and handling uncertain information, the ER approach has been introduced to solve MEMCDM problems with uncertainty in many fields, such as research and development of production modeling [2], emergency response assessment [3] and risk assessment [4].

Although the ER approach is a good method to deal with uncertain evaluation, the aggregation results of this method still have a high degree of uncertainty. Theoretically, there are two kinds of uncertainty in the evidence theory. One is the ambiguity uncertainty caused by the weight distribution of evidence; the other is the probability uncertainty caused by the undistributed belief of evidence, which we usually call the probability uncertainty ignorance. The number of elements contained in focal elements again divides the ignorance into three categories: (1) if a focal element contains all elements of the discernment framework, it is called the global ignorance; (2) if a focal element contains more than one element but does not reach the total number of elements, it is called the local ignorance; (3) if all focal elements are single element focal elements, then the basic probability assignment function degenerates into the traditional probability function which is neither the global ignorance nor the local ignorance. In the original Dempster–Shafer (DS) theory [5,6], the remaining belief was assigned to the whole identification framework, which includes both the global ignorance and local ignorance. Yager [7]'s method allocated the remaining belief to the whole discernment framework, which includes the global ignorance. Smet [8]'s method allocated the remaining belief to an empty set. Although this method avoided the global and local ignorance, it did not substantially solve the problem of belief allocation, and according to many calculations, Smet [8]'s allocation method increased rather than decreased the uncertainty of the evidence. Later, Smet [9]'s pignistic probability translation method did allocate the ignorance belief to single element focal elements, but it essentially allocated the existing ignorance proportionally, and the remaining belief was still allocated to the global ignorance and then proportional to all single element focal elements that did not even appear. In the original definition of probabilistic linguistic term sets [10], the unknown probability was supposed to be assigned to the given linguistic terms proportionately, which made the original evaluation information lose its uncertainty. In this regard, Fang *et al.* [11] assigned the remaining probability of a probabilistic linguistic term set to the envelope of the linguistic terms, which still

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magnified the uncertainty of evaluations to some extent like the ER approach. In this sense, how to assign the remaining belief and reduce the uncertainty in the ER approach is still challengeable. In this regard, we propose a method in this study to assign the remaining belief to given evaluation rather than envelope of linguistic terms.

In addition, in the ER approach, the weights of criteria and experts are usually supposed to be provided subjectively. To make up for this insufficiency, there have been a few attempts on how to generate weights objectively [9,12–14,15,16]. For instance, Zhou *et al.* [12] built relevant optimization models to obtain the weights of criteria based on the given ranges of weights. Diego *et al.* [13] introduced the principal components analyses to learn the weights of criteria based on big data. From the perspective of the evidence conflict, scholars proposed to measure the similarity of evidence by distance measures, such as the Jousselme distance measure [14], Pignistic probability distance measure [9] and Hellinger distance measure [15], and then obtain the weights of criteria. However, such methods based on distance measures were applicable to high-conflict evidence, and if there is no obvious conflict between the bodies of evidence, the weights of criteria obtained by such methods may be very close. To avoid such limitations and measure weights efficiently, the concept of entropy were introduced in some studies [16–19]. However, the original entropy cannot deal with the remaining belief in the ER approach. In this regard, Deng [20] proposed the Deng entropy to calculate the weights with the remaining belief in evaluations. Using subjective weights or objective weights alone may obtain biased data. To reduce deviations, we try to combine subjective and objective weight generation methods to obtain the corresponding comprehensive weights in this study. Firstly, we use the Deng entropy to generate the weights objectively in the ER approach. Then, combining the generated objective weights with the subjective weights given by the decision-maker, we can generate weights comprehensively to reduce the uncertainty in the ER approach. The Deng entropy can not only help to measure the uncertainty of basic probability assignment, but also theoretically verify that the proposed method to allocate remaining belief reduces the uncertainty of evidence. See Example 1 in Section 2.2. for a detailed explanation.

After reducing the uncertainty by assigning the remaining belief and generating the weights of criteria and experts comprehensively, there is a need to aggregate all evaluations with respect to each alternative. The ER approach [1] can describe various uncertainties in MCDM problems by establishing a unified confidence framework. For an individual decision matrix, the uncertain evaluations on all criteria can be aggregated to generate the comprehensive evaluation of each alternative by the ER approach. Then, on the premise that experts are independent of each other, we can apply the ER approach again to aggregate the evaluations of all experts to obtain the collective evaluation of each alternative. In the ER approach, due to several evaluation grades and the incomplete information in evaluations, it is not easy to make a direct pairwise comparison between two pieces of evaluations. As an outranking method, the PROMETHEE method [21] ranks alternatives by integrating the positive outranking flow and negative outranking flow based on the preference relations between alternatives. It is a robust method to rank alternatives. In this study, we propose a PROMETHEE-based ranking method adapted to the ER approach for ranking alternatives, which considers both the utility values of evaluation grades and the pairwise comparisons between alternatives.

According to the above analyses, in this study, we propose a Deng-entropy-based ER approach for MEMCDM problems with uncertainty, and verify its validity by a case study concerning screening people at high risk of lung cancer. The main innovations of this study can be summarized as follows:

1. By assigning the remaining belief to the set of focal elements, the uncertainty in the ER approach can be reduced.
2. To generate the weights of criteria and those of experts accurately, we introduce the Deng entropy to calculate the objective weights in the ER approach, and consider the subjective weights given by the expert simultaneously.
3. We improve the PROMETHEE method to rank alternatives by constructing a probability preference matrix for each evaluation grade, forming a Deng-entropy-based ER approach for MEMCDM problems with uncertainty.

The rest of this paper is arranged as follows: Section 2 reviews the ER approach, Deng entropy and PROMETHEE method. Section 3 introduces the Deng-entropy-based ER approach for MEMCDM with uncertainty. To verify the effectiveness of the proposed method, an illustrative example of screening the people at high risk of lung cancer is provided and the corresponding comparison analyses are given in Section 4. Section 5 concludes our study.

2. PRELIMINARIES

This section briefly introduces the ER approach, Deng entropy and PROMETHEE method.

2.1. ER Approach

In the process of multiple criteria analyses, how to deal with uncertain information is a vital problem. The Bayes approach based on probability theory focused on the processing of quantitative information and needed a large amount of historical data to determine the prior distribution and other parameters [22]. To deal with this limitation, Dempster [5] and Shafer [6] proposed the DS theory by introducing a belief function to promote the traditional Bayes reasoning approach, which makes it possible to deal with uncertain information without knowing the prior probability. Considering the effectiveness of the DS theory in dealing with uncertain information, Yang and Xu [1] proposed the ER approach by introducing weights of criteria to adjust the conflicts between evidences for MCDM, which can not only aggregate such incomplete information effectively, but also can avoid counterintuitive results. Due to the superiority of the ER approach in dealing with uncertain evaluations, Zhang and Deng [23] and Dong *et al.* [24] used the evidence theory to analysis fault diagnosis problems in uncertain environment. Akhoundi *et al.* [25] used the ER approach to sustainability evaluation of wastewater reuse alternatives. Ng and Law [26] used ER to analysis sentiment words in social networks to investigate consumer preferences. Tian *et al.* [27] combined the probabilistic linguistic term set and ER methods and considered the psychological preferences of decision-makers to solve multi-criterion decision-making problems.

Next, we briefly introduce the ER approach. For an MCDM problem with I alternatives x_i $(i = 1, 2, \cdots, I)$ being evaluated on J criteria c_j $\left(j = 1, 2, \cdots, J\right)$ whose weight vector is $W = \left(w_1, w_2, \cdots, w_J\right)^T$ satisfying $0 \leq w_j \leq 1$and $\sum_{j=1}^{j} \omega_j = 1$, the alternatives are evaluated under the frame of discernment $H = \{H_n | n = 1, 2, \cdots, N\}$ which is collectively exhaustive and mutually exclusive. The utility of H_n, $u\left(H_n\right)$, satisfies $0 = u\left(H_1\right) \leq u\left(H_2\right) \leq \cdots \leq u\left(H_N\right) = 1$. Based on H, experts can evaluate alternative x_i on criterion c_j and give a distributed evaluation as $e_j\left(x_i\right) = \left\{\left(H_n, \beta_{n,j}\left(x_i\right)\right), n = 1, 2, \cdots, N\right\}$ where $0 \leq \beta_{n,j}\left(x_i\right) \leq 1$ and $\sum_{n=1}^{N} \beta_{n,j}\left(x_i\right) \leq 1$.

Firstly, considering the weights of criteria, all the belief degrees in the evaluations can be transformed into basic probability assignments, which are in the following forms [1]:

$$m_{n,j}\left(x_i\right) = w_j \beta_{n,j}\left(x_i\right) \tag{1}$$

$$m_{H,j}\left(x_i\right) = \tilde{m}_{H,j}\left(x_i\right) + \overline{m}_{H,j}\left(x_i\right) \tag{2}$$

$$\tilde{m}_{H,j}\left(x_i\right) = w_j \left(1 - \sum_{n=1}^{N} \beta_{n,j}\left(x_i\right)\right) \tag{3}$$

$$\overline{m}_{H,j}\left(x_i\right) = 1 - w_j \tag{4}$$

where $m_{n,j}\left(x_i\right)$ refers to the probability mass assigned to grade H_n on criterion c_j. $m_{H,j}\left(x_i\right)$ refers to the remaining probability mass unassigned to any individual grade.

The basic probability mass of each alternative can be calculated by the following formulas:

$$\begin{aligned} &m_{n,I(j+1)}\left(x_i\right) \\ &= K_{I(j+1)}\left(x_i\right)\left[m_{n,I(j)}\left(x_i\right) m_{n,j+1}\left(x_i\right) + m_{H,I(j)}\left(x_i\right) m_{n,j+1}\left(x_i\right)\right. \\ &\quad \left. + m_{n,I(j)}\left(x_i\right) m_{H,j+1}\left(x_i\right)\right] \end{aligned} \tag{5}$$

$$m_{H,I(j)}\left(x_i\right) = \tilde{m}_{H,I(j)}\left(x_i\right) + \overline{m}_{H,I(j)}\left(x_i\right) \tag{6}$$

$$\begin{aligned} &\tilde{m}_{H,I(j+1)}\left(x_i\right) \\ &= K_{I(j+1)}\left(x_i\right)\left[\tilde{m}_{H,I(j)}\left(x_i\right) \tilde{m}_{H,j+1}\left(x_i\right) + \overline{m}_{H,I(j)}\left(x_i\right) \tilde{m}_{H,j+1}\left(x_i\right)\right. \\ &\quad \left. + \tilde{m}_{H,I(j)}\left(x_i\right) \overline{m}_{H,j+1}\left(x_i\right)\right] \end{aligned} \tag{7}$$

$$\overline{m}_{H,I(j+1)}\left(x_i\right) = K_{I(j+1)}\left(x_i\right)\left(\overline{m}_{H,I(j)}\left(x_i\right) \overline{m}_{H,j+1}\left(x_i\right)\right) \tag{8}$$

where $m_{n,I(1)}\left(x_i\right) = m_{n,1}\left(x_i\right)$, $m_{H,I(1)}\left(x_i\right) = m_{H,1}\left(x_i\right)$ and $K_{I(j+1)}\left(x_i\right) = \left[1 - \sum_{l=1}^{N} \sum_{\substack{n=i \\ n \neq l}}^{N} m_{l,I(j)}\left(x_i\right) m_{n,j+1}\left(x_i\right)\right]^{-1}$ which is a normalization factor ensuring that $\sum_{n=1}^{N} m_{n,I(j+1)}\left(x_i\right) + m_{H,I(j)}\left(x_i\right) = 1$.

Then, the belief degrees of alternative x_i on H_n and H can be gained by

$$\beta_n\left(x_i\right) = m_{n,I(L)}\left(x_i\right) / \left(1 - \overline{m}_{H,I(L)}\left(x_i\right)\right) \tag{9}$$

$$\beta_H\left(x_i\right) = \tilde{m}_{H,I(L)}\left(x_i\right) / \left(1 - \overline{m}_{H,I(L)}\left(x_i\right)\right) \tag{10}$$

Finally, by assigning the remaining belief to H_N and H_1, respectively, we can get the upper and lower bounds of alternative x_i, and then rank the alternatives [28].

$$u\left(x_i\right)_{\max} = \sum_{n=1}^{N} u\left(H_n\right) \beta_n\left(x_i\right) + u\left(H_N\right) \beta_H\left(x_i\right) \tag{11}$$

$$u\left(x_i\right)_{\min} = \sum_{n=1}^{N} u\left(H_n\right) \beta_n\left(x_i\right) + u\left(H_1\right) \beta_H\left(x_i\right) \tag{12}$$

$$u\left(x_i\right)_{avg} = \left(u\left(x_i\right)_{\max} + u\left(x_i\right)_{\min}\right) / 2 \tag{13}$$

As a method to deal with uncertain MCDM problems, the ER approach assigns the remaining belief to the whole identification framework, so that the uncertainty can be maintained in the evaluation model and the information fusion process. In this way, the collective distributed evaluation of each alternative can be obtained and thus an appropriate decision result can be deduced. Due to the effectiveness of the ER approach in dealing with uncertain information, it may be a good attempt to apply this method to model and aggregate uncertain information for MEMCDM problems.

2.2. The Deng Entropy

For MEMCDM, before aggregating the uncertain evaluations by the ER approach, it is necessary to determine the weights of criteria and those of experts. In the original ER approach [1], the weights of criteria were supposed to be given subjectively by experts, which may be not convincing. Studies have been made to generate the weights of criteria objectively [12,13,16–19]. But such methods were based on the historical data that was not easy to collect. Shannon entropy [29] was proposed to measure the uncertainty degree of information, which provides us a good direction to generate the weights of criteria and those of experts by calculating the opposite side of uncertainty, i.e., the certainty of information under each criterion or with respect to each expert. However, the Shannon entropy cannot deal with the uncertain information in the ER approach. In this regard, Deng [20] proposed the Deng entropy based on the Shannon entropy by introducing the basic probabilistic assignment. Deng entropy can measure the entropy of information with uncertain probability.

Definition 1. [20] Given that H is a frame of discernment, let $\tilde{H}$ be a set of evaluation grades, satisfying $\tilde{H} \subseteq H$. m is a mass function defined on the frame of discernment H (if $m(\tilde{H}) > 0$, $\tilde{H}$ is called the focal element of m), and $|\tilde{H}|$ is the cardinality of $\tilde{H}$. Then, the entropy of m can be obtained by

$$E_d(m) = -\sum_{\tilde{H} \subseteq H} m(\tilde{H}) \log_2 \frac{m(\tilde{H})}{2^{|\tilde{H}|} - 1} \tag{14}$$

As can be seen, the Deng entropy is an improvement of the Shannon entropy. The difference is that the belief for each focal element in the Deng entropy is divided into $2^{|\tilde{H}|}-1$ states. Note that when there is no uncertainty, i.e., $|\tilde{H}|$=1, the Deng entropy reduces to the Shannon entropy.

Example 1. Considering an evaluation $S(y) = \{s_1(0.4), s_2(0.5)\}$, the remaining belief is 0.1. In this paper, the remaining belief is assigned to $\{s_1, s_2\}$, and then we get the evaluation as $S(y) = \{s_1(0.4), s_2(0.5), \{s_1, s_2\}(0.1)\}$. The Deng entropy of $S(y)$ is calculated by Eq. (14) as $E_d(S(y)) = -m(s_1)\log_2 m(s_1) - m(s_2)\log_2 m(s_2) - m(\{s_1, s_2\})\log_2 \frac{m(\{s_1,s_2\})}{2^2-1} = 1.52$.

As can be seen from the calculation of the above example, the magnitude of the Deng entropy is proportional to the number of elements in the focal element. The smaller the number of elements in the focal element is, the lower the entropy value is. Comparing the previous approaches which assign the remaining belief to either the whole set of the identification framework or to the envelopment of linguistic terms, we can find that the proposed approach can effectively reduce the number of elements in the set to which the remaining belief assigned. By introducing the Deng entropy, it is possible to determine the uncertainty degree of uncertain information, and then get objective weights. Combining such objective weights with the subjective weights given by experts, we can get the comprehensive weights of criteria and those of experts. In this sense, it is a good method to apply the Deng entropy to determine the objective weights of criteria and those of experts in the ER approach for MEMCDM with uncertainty.

2.3. The PROMETHEE Method

After obtaining the aggregated evaluation of each alternative by the ER approach, there is a need to choose a suitable ranking method to generate the final results. Traditional ranking methods fall into two categories: utility value-based methods and outranking methods [30]. For the first category, it needs different utility functions combining the weights of criteria to calculate the overall values of alternatives. The utility value-based methods rely heavily on the subjective evaluations of the utilities of alternatives with respect to each criterion and ignore the relations between alternatives. Outranking methods are based on the pairwise comparisons of alternatives under each criterion. The PROMETHEE method belongs to the later. It ranks alternatives by integrating the positive outranking flow and negative outranking flow based on the preference function. The PROMETHEE methods include the PROMETHEE I to VI. Among them, the PROMETHEE III ranks alternatives based on intervals; the PROMETHEE IV ranks alternatives in which the evaluations are the continuous case; the PROMETHEE V deals with MCDM including segmentation constraints, and the PROMETHEE VI involves representations of human brain [31]. Because the PROMETHEE I can only derives a partial ranking, the PROMETHEE method in this paper refers to the PROMETHEE II which derives a complete ranking of alternatives by calculating the net flow of each alternative. Due to the simplicity and practicality of the PROMETHEE II method, it has been applied widely. Liu *et al.* [32] extended the PROMETHEE II method to the 2D uncertain linguistic environment and used the cloud model to depict randomness and fuzziness. Tian *et al.* [33] proposed an image fuzziness PROMETHEE method based on the image fuzziness number and PROMETHEE II method, and combined it with the AHP to deal with the problem of tourism environmental impact assessment.

The PROMETHEE II method is described as follows:

Step 1. Suppose that e_i^j represents the evaluation of alternative x_i on criterion c_j. Each criterion c_j is associated with a matrix $D_j = \left(e_{it}^j\right)_{I\times I}$ whose element e_{it}^j is given by the difference between alternatives x_i and x_t on criterion c_j as

$$e_{it}^j = e_i^j - e_t^j,\ \text{for}\ i,t = 1,2,\cdots,I; j = 1,2,\cdots,J \tag{15}$$

Step 2. Based on six standard preference functions, we can translate e_{it}^j into a preference value within [0, 1]. Here we suppose the preference function is $f(\cdot)$ for each criterion. Then, the difference matrices $D_j = \left(e_{it}^j\right)_{I\times I}$ $(j = 1,2,\cdots,J)$ can be transformed to the preference matrices $P_j = \left(p_{it}^j\right)_{I\times I}$ $(j = 1,2,\cdots,J)$ where

$$p_{it}^j = f\left(e_{it}^j\right),\ \text{for}\ i,t = 1,2,\cdots,I; j = 1,2,\cdots,J \tag{16}$$

Step 3. Considering the weights $w_j (j = 1,2,\cdots,J)$ of criteria, we can aggregate the preferences between alternatives and obtain the matrices $Q = \left(q_{it}^j\right)_{I\times I}$ $(j = 1,2,\cdots,J)$ with

$$q_{it}^j = \sum\nolimits_{j=1}^{j} w_j \cdot p_{it}^j,\ \text{for}\ i,t = 1,2,\cdots,I; j = 1,2,\cdots,J \tag{17}$$

Step 4. For each alternative x_i, the positive outranking flow φ_i^+ and negative outranking flow φ_i^- are calculated as

$$\varphi_i^+ = \sum\nolimits_{t=1}^{I} q_{it}^j,\ \text{for}\ i = 1,2,\cdots,I \tag{18}$$

$$\varphi_i^- = \sum\nolimits_{t=1}^{I} q_{ti}^j,\ \text{for}\ i = 1,2,\cdots,I \tag{19}$$

The net flow φ_i summarizes the overall preference of each alternative over the others, which can be calculated as

$$\varphi_i = \varphi_i^+ - \varphi_i^-,\ \text{for}\ i = 1,2,\cdots,I \tag{20}$$

The positive outranking flow φ_i^+ indicates how much the alternative x_i is preferred to the other alternatives. Conversely, the negative outranking flow φ_i^- indicates how much the other alternatives are preferred to x_i. The net flow φ_i indicates the sum of preference of each alternative over the others. Based on the net outranking flow, the alternatives can be ranked in descending order of the net flow.

3. A DENG-ENTROPY-BASED ER APPROACH

For MEMCDM problems, how to deal with uncertain evaluations and get a comprehensive decision result is a key issue. In this study, we propose a Deng entropy-based ER approach for MEMCDM problems with uncertainty. Firstly, we assign the remaining belief caused by uncertain information to the set of focal elements in the evaluation. Then, we introduce the Deng entropy to obtain the objective weights of criteria and those of experts by calculating the uncertainty of evaluation information, and combine the objective weights with subjective weights to get comprehensive weights of criteria and experts, respectively. Finally, after applying the ER approach twice to obtain the comprehensive evaluations of alternatives, we propose an PROMETHEE-based method to rank alternatives, considering both the utility values of evaluation grades and the pairwise comparison relations between alternatives. Next, we will describe the proposed approach in detail.

3.1. Model the Uncertain Evaluations in an MEMCDM Problem

Let $H = \{H_n | n = 1, 2, \cdots, N\}$ be a discernment frame which is collectively exhaustive and mutually exclusive and $e(y) = \{(H_n, \beta_n(y)) | n = 1, 2, \cdots, N\}$ be an evaluation where $0 \le \beta_n(y) \le 1$ and $\sum_{n=1}^{N} \beta_n(y) \le 1$. For an evaluation $e(y)$, if $\sum_{n=1}^{N} \beta_n(y) = 1$, the evaluation is complete; if $\sum_{n=1}^{N} \beta_n(y) < 1$, the evaluation is incomplete and the remaining belief is $\overline{\beta}(y) = 1 - \sum_{n=1}^{N} \beta_n(y)$. Considering the incomplete evaluation, there is a need to model such uncertainty.

In previous studies, the ER approach [1] assigned the remaining belief generated from incomplete evaluation to the full discernment frame H. Fang *et al.* [11] assigned the remaining probability to the envelope of a linguistic term set with the minimum grade being the lower bound while the maximum grade being the upper bound. The limitation of the above assignments is that some evaluation grades the experts do not give any belief or probability. Therefore, assigning the remaining belief to the full discernment frame or the envelope of a linguistic term set may increase the uncertainty in evaluations. For example, considering an evaluation $e(y) = \{H_1(0.4), H_2(0.5)\}$ based on a discernment frame $H = \{H_1, H_2, H_3, H_4, H_5\}$, the remaining belief is 0.1. In previous literature, researchers assigned 0.1 to the universal set of H. However, except for H_1 and H_2, there are no evaluation grades with a belief degree greater than zero. Therefore, it is not reasonable to assign the remaining belief to the evaluation grades of unassigned belief on the premise that experts are independent of each other. In this sense, in this study, we assign the remaining belief to the set of focal elements in the given evaluation grades, i.e., $e = \{H_1(0.4), H_2(0.5), \{H_1, H_2\}(0.1)\}$ in the above example, which can more accurately express the uncertain information.

For an evaluation $e(y) = \{(H_n, \beta_n(y)) | n = 1, 2, \cdots, N\}$, after modeling the uncertainty by assigning the remaining belief to the set of focal elements, we can obtain

$$e(y) = \begin{cases} e(y), & \text{if } \sum_{n=1}^{N} \beta_n(y) = 1 \\ \left\{ \begin{array}{c} (H_n, \beta_n(y)), (\overline{H}, \overline{\beta}(y)), \\ n = 1, 2, \cdots, N \end{array} \right\}, & \text{if } \sum_{n=1}^{N} \beta_n(y) < 1 \end{cases}$$

where $\overline{H}$ represents the set of focal elements of the given evaluation.

3.2. Determine the Weights of Criteria and Those of Experts

In MEMCDM problems, after modeling the uncertain evaluations, how to generate the weights of criteria and those of experts reasonably is a question. In the original ER approach [1], the weights are given subjectively, which is not convincing. Existing objective weight determination methods were based on historical data [12,13,16–19] and consequently cannot effectively deal with uncertain information. Taking these two flaws into consideration, we propose a Deng-entropy-based method to determine the comprehensive weights of criteria and those of experts with uncertain evaluations, considering the subjective weights and objective weights simultaneously.

Suppose that an MEMCDM problem includes I alternatives x_i $(i = 1, 2, \cdots, I)$ evaluated on J criteria c_j $(j = 1, 2, \cdots, J)$ by K experts R_k $(k = 1, 2, \cdots, K)$, forming the initial decision matrices as $DM_k = \left[e_{ij}^k\right]_{I \times J}$ $(k = 1, 2, \cdots, K)$ on the discernment frame $H = \{H_n | n = 1, 2, \cdots, N\}$. For the expert R_k, the weight vector of criteria is subjectively denoted as $W_j^k = \left(w_1^k, w_2^k, \cdots, w_J^k\right)^T$, satisfying $0 \le w_j^k \le 1$ and $\sum_{j=1}^{j} w_j^k = 1$.

1. The Deng entropy $E_d\left(e_{ij}^k\right)$ of each expert's evaluations for all alternatives can be calculated to measure the uncertainty of evaluations. Then, the certainty a_j^k on c_j for R_k can be obtained as [32]

$$a_j^k = \left(1 - (1/I) \sum_{i=1}^{I} E_d(e_{ij}^k)\right) \Big/ \left(J - (1/I) \sum_{j=1}^{J} \sum_{i=1}^{I} E_d(e_{ij}^k)\right) \tag{21}$$

The certainty a_j^k can be seen as the objective weights of criteria. Then, we can obtain the comprehensive weights of criteria by combining the subjective weights w_j^k and objective weights a_j^k. Considering that the sum of the weights is one, the comprehensive weights adopts the form of multiplication of subjective weight and objective weight:

$$\tilde{w}_j^k = w_j^k a_j^k \Big/ \sum_{j=1}^{J} w_j^k a_j^k \tag{22}$$

2. Determine the weights of experts
After getting the Deng entropy of each evaluation by Eq. (12), we can also calculate the certainty of each expert by the following formula:

$$a^k = \left(1 - (1/(I \times J)) \sum_{j=1}^{j} \sum_{i=1}^{I} E_d(e_{ij}^k)\right) \Big/ \left(K - (1/(I \times J)) \sum_{k=1}^{k} \sum_{j=1}^{j} \sum_{i=1}^{I} E_d(e_{ij}^k)\right) \tag{23}$$

Then, let $\tilde{w}^k$ is the weights of experts with $\tilde{w}^k = a^k$.

Based on the above analyses, we can determine the weights of criteria and those of experts. In this regard, before ranking the alternatives, it is necessary to aggregate the uncertain evaluations.

3.3. Aggregate Evaluations by the ER Approach

To deal with MEMCDM problems with uncertainty, we apply the ER approach twice to aggregate the evaluations of each alternative given by different experts.

Firstly, based on the determined weights of criteria, by Eqs. (1–10), we can aggregate the evaluations of each alternative on different criteria, and get the comprehensive evaluation of x_i given by expert R_k. In this way, we can convert K decision matrices $DM_k = \left(e_{ij}^k\right)_{I\times J}$ $(k = 1, 2, \cdots, K)$ to a comprehensive decision matrix $\overline{DM} = \left(e_i^k\right)_{I\times K}$:

$$DM_k = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_I \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_J \end{array} \\ \begin{bmatrix} e_{11}^k & e_{11}^k & \cdots & e_{1J}^k \\ e_{21}^k & e_{22}^k & \cdots & e_{2J}^k \\ \vdots & \vdots & \ddots & \vdots \\ e_{I1}^k & e_{I2}^k & \cdots & e_{IJ}^k \end{bmatrix} \end{array} (k = 1, 2, \cdots, K) \to \overline{DM}$$

$$= \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_I \end{array} \begin{array}{c} \begin{array}{cccc} R_1 & R_2 & \cdots & R_K \end{array} \\ \begin{bmatrix} e_1^1 & e_1^2 & \cdots & e_1^k \\ e_2^1 & e_2^2 & \cdots & e_2^k \\ \vdots & \vdots & \ddots & \vdots \\ e_I^1 & e_I^2 & \cdots & e_I^k \end{bmatrix} \end{array}$$

Secondly, based on the determined weights of experts, by Eqs. (1–10), we can aggregate the evaluations of all the experts and get the collective evaluation of x_i as $F(x_i) = \left\{(H_n, \beta_n(x_i)), \left(\overline{H}, \overline{\beta}(x_i)\right) | n = 1, 2, \cdots, N\right\}$, for $i = 1, 2, \cdots, I$.

To get the ranking of all the alternatives, we need to rank the final evaluations $F(x_i)$, for $i = 1, 2, \cdots, I$. In this regard, we propose an PROMETHEE-based ranking method to rank them.

3.4. Rank Alternatives by a PROMETHEE-Based Ranking Method

The original ER approach [1] only considers the utility values of evaluation grades when ranking alternatives, which ignores the existence of experts' preferences. In this paper, we propose a PROMETHEE-based ranking method considering both the utility values of the evaluation grades and the pairwise comparison relations between alternatives to rank alternatives. Besides, after the final evaluation vector being obtained by the ER approach, we can determine the differences between alternatives under each evaluation grade by comparing their belief degrees. Then, preference matrices are formed by introducing a linear preference function [21]. Each evaluation grade is assigned its own utility value according to its degree of importance, and then the positive outranking flow and negative outranking flow of each alternative can be obtained by calculating the weighted sum of experts' preferences based on utility values. Next, we introduce the PROMETHEE-based ranking method in details.

Firstly, we can transform the collective evaluations of alternatives to a matrix as

$$F = \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_I \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} \left\{H_1(\beta_1), & \cdots & H_N(\beta_N, & \overline{H}\left(1 - \sum_{n=1}^N \beta_n\right)\right\} \\ \left\{H_1(\beta_1), & \cdots & H_N(\beta_N, & \overline{H}\left(1 - \sum_{n=1}^N \beta_n\right)\right\} \\ \vdots & \vdots & \vdots & \vdots \\ \left\{H_1(\beta_1), & \cdots & H_N(\beta_N, & \overline{H}\left(1 - \sum_{n=1}^N \beta_n\right)\right\} \end{bmatrix} \\ \begin{array}{cccc} H_1 & \cdots & H_N & \overline{H} \end{array} \end{array} \to D$$

$$= \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_I \end{array} \begin{bmatrix} \beta_{1,1} & \cdots & \beta_{1,N} & 1 - \sum_{n=1}^N \beta_{1,n} \\ \beta_{2,1} & \cdots & \beta_{2,N} & 1 - \sum_{n=1}^N \beta_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{I,1} & \cdots & \beta_{I,N} & 1 - \sum_{n=1}^N \beta_{2,n} \end{bmatrix}$$

Suppose that β_{it}^n represents the difference value of the evaluation that alternative x_i over x_t on the evaluation grade H_n. Each evaluation grade H_n can be associated with a difference matrix as $D_n = \left(\beta_{it}^n\right)_{I\times I}$, where

$$\beta_{it}^n = \beta_n(x_i) - \beta_n(x_t), \; i, t = 1, 2, \cdots, I; n = 1, 2, \cdots, N \tag{24}$$

By introducing a preference function, we can transform the difference matrix $D_n = \left(\beta_{it}^n\right)_{I\times I} (n = 1, 2, \cdots, N)$ into the preference matrix $P_n = \left(p_{it}^n\right)_{I\times I}$, such that

$$p_{it}^n = f\left(\beta_{it}^n\right), \; i, t = 1, 2, \cdots, I; n = 1, 2, \cdots, N \tag{25}$$

Here we suppose that $f\left(\beta_{it}^n\right)$ is a linear function in the form of $f\left(\beta_{it}^n\right) = \begin{cases} \beta_{it}^n / b, & \text{if } -b \le \beta_{it}^n \le b \\ 1, & \text{if } \beta_{it}^n < -b \text{ or } \beta_{it}^n > b \end{cases}$ and b is a threshold to distinguish the preference of an expert. When b is 0.3, the function is shown in Figure 1.

Correspondingly, for $\overline{H}$, the difference and preference matrices are $D_{\overline{H}} = \left(\beta_{it}^{\overline{H}}\right)_{I\times I}$ and $P_{\overline{H}} = \left(p_{it}^{\overline{H}}\right)_{I\times I}$ respectively. Suppose that H_{l_1} is the best grade in $\overline{H}$ and H_{l_2} is the worst grade in $\overline{H}$. Considering the utility $u(H_n)$ of the evaluation grade H_n, by Eq. (17), we can get the best expected preference matrix as $Q_B = \left(Q_{it,B}^n\right)_{I\times I}$ with $Q_{it,B}^n = \sum_{n=1}^N u(H_n) \cdot p_{it}^n + u\left(H_{l_1}\right) \cdot p_{it}^{\overline{H}}$ and the worst expected

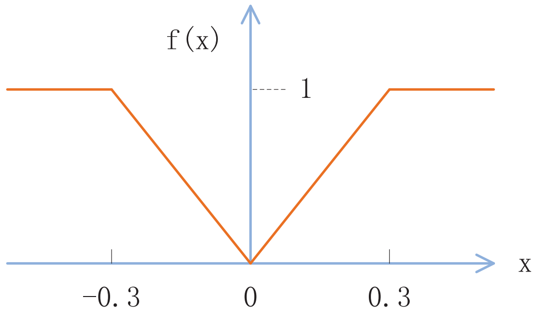

Figure 1 The preference function with the threshold being 0.3.

preference matrix as $Q_W = \left(Q_{it,W}^n\right)_{I\times I}$ with $Q_{it,W}^n = \sum_{n=1}^{N} u\left(H_n\right)\cdot p_{it}^n + u\left(H_{l_2}\right)\cdot p_{it}^{\overline{H}}$.

We can get two kinds of net flows accordingly as the best net flows φ_{iB} and the worst net flows φ_{iW} by Eqs. (18–20). Then, we can calculate the average net flow as follows:

$$\varphi_i = \frac{\varphi_{iB} + \varphi_{iW}}{2},\ \text{for } i = 1, 2, \cdots, I \tag{26}$$

Finally, the ranking of alternatives can be obtained in descending order of the average net flow.

3.5. Algorithm of the Deng-Entropy-Based ER Approach for MEMCDM with Uncertainty

Based on the above analyses, this paper proposes a Deng entropy-based ER approach for MEMCDM with uncertainty. To facilitate the application, we summarize the algorithm of this method below. The framework of this approach is illustrated in Figure 2.

Algorithm (The Deng entropy-based ER approach for MEMCDM with uncertainty)

Step 1. (Generate individual decision matrices) K experts are invited to evaluate I alternatives on J criteria and give each evaluation as $e_{ij}^k = \left\{\left(H_n, \beta_{n,j}\left(x_i\right)\right) | n = 1, 2, \cdots, N\right\}$ which can be transformed to $e_{ij}^k = \left\{\left(H_n, \beta_{n,j}\left(x_i\right)\right), \left(\overline{H}, \overline{\beta}_j\left(x_i\right)\right) | n = 1, 2, \cdots, N\right\}$ by assigning the remaining belief to the focal elements of the given evaluation. By the transformed evaluations, we can get the decision matrix of expert R_k as $DM_k = \left[e_{ij}^k\right]_{I\times J}$, for $k = 1, 2, \cdots, K$.

Step 2. (Generate the comprehensive weights of criteria and those of experts) By Eqs. (21–23), we can obtain the comprehensive weights of criteria and those of experts, which take into account the objective weights determined by the Deng entropy and the subjective weights given by experts, simultaneously.

Step 3. (Generate the collective evaluations) By aggregating the evaluations of each alternative on all criteria with the ER approach, we can get a comprehensive matrix as $\overline{DM} = \left(e_i^k\right)_{I\times K}$. Then, by Eqs. (1–10), we can aggregate the evaluations of all experts and get the collective evaluation of alternative x_i as $F(x_i) = \left\{\left(H_n, \beta_n\left(x_i\right)\right), \left(\overline{H}, \overline{\beta}\left(x_i\right)\right) | n = 1, 2, \cdots, N\right\}$.

Step 4. (Generate the preference matrices) By Eqs. (24) and (25), we can obtain the preference matrices as $P_n = \left(p_{it}^n\right)_{I\times I}$ $(n = 1, 2, \cdots, N)$ and $P_{\overline{H}} = \left(p_{it}^{\overline{H}}\right)_{I\times I}$.

Step 5. (Generate the expected preference matrices) By assigning the remaining belief to the best and worst grades of $\overline{H}$, we can get the best expected preference matrix $Q_B = \left(Q_{it,B}^n\right)_{I\times I}$ and the worst expected preference matrix $Q_W = \left(Q_{it,W}^n\right)_{I\times I}$, respectively.

Step 6. (Rank the alternatives) By Eqs. (18–20), the best net flow φ_{iB} and the worst net flow φ_{iW} can be generated. Then, by Eq. (26), we can obtain the average net flow of each alternative, and the ranking of alternatives can be generated in descending order of the average net flow.

Above all, there are four main advantages of the proposed Deng entropy-based ER approach:

1. We reassign the remaining belief to the set of focal elements of the given evaluation grades, which can effectively mitigate the uncertainty of information on the premise that experts are independent of each other.
2. We generate a comprehensive weight for each criterion and expert based on the objective weights determined by the Deng entropy and the subjective weights given by the experts, which may reduce the subjective uncertainty in the decision-making process.
3. We aggregate the evaluations of alternatives on criteria by the ER approach to get the comprehensive evaluations, and then apply the ER approach again to generate the collective evaluations of the expert group. In this way, the uncertain evaluations given by the experts can be integrated accurately.
4. We propose a PROMETHEE-based ranking method to rank alternatives, which considers both the utility values of the evaluation grades and the pairwise comparison relations between alternatives.

The proposed method provides a good attempt to deal with the MEMCDM problems with uncertainty.

4. A CASE STUDY: SCREENING THE PEOPLE AT A HIGH RISK OF LUNG CANCER

In this section, we apply the Deng entropy-based ER approach for MEMCDM with uncertainty to screen the people at a high risk of lung cancer and illustrate the effectiveness of the proposed method by comparing it with existing methods.

4.1. Screen the People at a High Risk of Lung Cancer by the Deng-Entropy-Based ER Approach

To implement the health China action, the Chinese government issued the opinions of the State Council on implementing the health

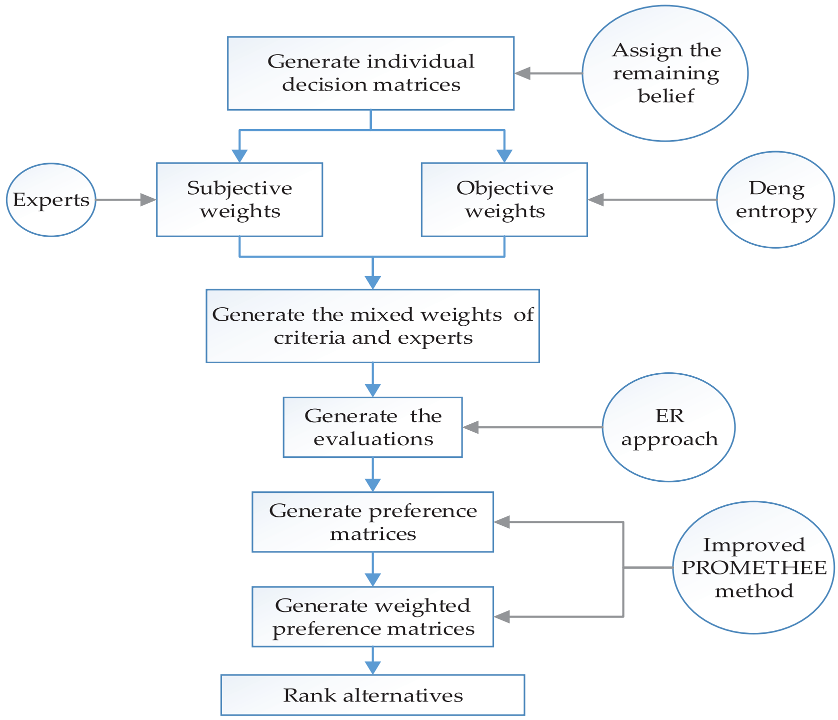

Figure 2 The flow chart of the Deng-entropy-based evidential reasoning approach.

China action in July 2019 [34], which clearly put forward the requirements of "advocating active cancer prevention, promoting early screening, early diagnosis and early treatment." According to the latest Global Cancer Statistics 2018 [35] published by the American Cancer Society, lung cancer is the first malignant tumor in the global morbidity (11.6%) and mortality (18.4%), and the most dangerous cancer in human life. In the early stage of lung cancer, most patients have no obvious symptoms [36]. Once clinical symptoms appear, the lung cancer often develops to the middle or late stage, and the possibility of cure is significantly lower than that in the early stage. It can be seen that the early diagnosis and treatment of lung cancer can be promoted through the identification of high-risk groups of lung cancer, which plays an important role in improving the cure rate and reducing the mortality rate [37–39]. Therefore, screening the people at a high risk of lung cancer is extremely important.

Before screening the high-risk population of lung cancer, relevant factors affecting lung cancer need to be determined. The etiology of lung cancer is not completely clear up to now. Currently, smoking is considered as the most important cause of lung cancer [40]. Previous studies have proved that long-term heavy smokers are 10 to 20 times more likely to have the lung cancer than nonsmokers, and the younger they start smoking, the higher the risk of lung cancer will be. Besides, the incidence of lung cancer in urban residents is higher than that in rural areas, which may be related to PM2.5 and ozone air pollution, leading to long-term residential environment. Bronchial sign may be a vital factor for screening people at a high risk of lung cancer [41]. Furthermore, lung cancer is also associated with past medical history, such as the bronchitis, chronic pneumonia and tuberculosis [42]. In addition, the blood test may be a good method to detect lung cancer [43].

Based on the above facts, we choose five relevant criteria for screening the high-risk lung cancer patients, such as c_1 (smoke state), c_2 (long-term residential environment), c_3 (bronchial sign), c_4 (history of chronic lung disease) and c_5 (blood test). Based on the impact of each criterion on smoking analyzed in [34–43], we assume that the weight vector of criteria is $(w_1, w_2, w_3, w_4, w_5)^T = (0.25, 0.15, 0.15, 0.2, 0.25)^T$.

Step 1. Generate individual decision matrices. To more accurately determine the risk degrees of lung cancer patients, a panel of three experts are invited to give their own evaluations on the above five criteria after considering the results of instrument measurement. The evaluation grades are $H = \{H_1, H_2, H_3, H_4, H_5\}$ where H_1, H_2, H_3, H_4 and H_5 represent "*normal*," "*a little bad*," "*bad*," "*very bad*" and "*extremely bad*," respectively. The individual decision matrices of the three experts are listed in Tables 1–3.

Step 2. Generate the comprehensive weights of criteria and those of experts. By Eqs. (21) and (22), we can calculate the comprehensive weights of criteria. For expert R_1, the objective weight vector of criteria is $\left(a_1^1, a_2^1, a_3^1, a_4^1, a_5^1\right)^T = (0.20, 0.11, 0.35, 0.24, 0.10)^T$ and the comprehensive weight vector of criteria is $\left(\tilde{w}_1^1, \tilde{w}_2^1, \tilde{w}_3^1, \tilde{w}_4^1, \tilde{w}_5^1\right)^T = (0.26, 0.09, 0.27, 0.25, 0.13)^T$. For expert R_2, there are $\left(a_1^2, a_2^2, a_3^2, a_4^2, a_5^2\right)^T = (0.11, 0.25, 0.29, 0.23, 0.12)^T$ and $\left(\tilde{w}_1^2, \tilde{w}_2^2, \tilde{w}_3^2, \tilde{w}_4^2, \tilde{w}_5^2\right)^T = (0.15, 0.20, 0.24, 0.25, 0.16)^T$. For expert R_3, there are $\left(a_1^3, a_2^3, a_3^3, a_4^3, a_5^3\right)^T = (0.12, 0.26, 0.28, 0.22, 0.13)^T$ and $\left(\tilde{w}_1^3, \tilde{w}_2^3, \tilde{w}_3^3, \tilde{w}_4^3, \tilde{w}_5^3\right)^T = (0.16, 0.21, 0.22, 0.24, 0.17)^T$. By Eq. (23), the comprehensive weight vector of experts can be generated as $\left(\tilde{w}^1, \tilde{w}^2, \tilde{w}^3\right)^T = \left(a^1, a^2, a^3\right)^T = (0.29, 0.36, 0.35)^T$.

Table 1 The decision-making matrix of R_1.

	c_1	c_2	c_3	c_4	c_5
x_1	$\{H_1(0.3), H_2(0.7)\}$	$\{H_2(0.5), H_3(0.5)\}$	$\{H_2(0.3), H_3(0.6), \overline{H}(0.1)\}$	$\{H_2(0.2), H_3(0.5), H_4(0.3)\}$	$\{H_1(0.4), H_2(0.5), \overline{H}(0.1)\}$
x_2	$\{H_2(0.5), H_3(0.5)\}$	$\{H_3(0.4), H_4(0.5), \overline{H}(0.1)\}$	$\{H_1(0.5), H_2(0.4), \overline{H}(0.1)\}$	$\{H_2(0.4), H_3(0.4), H_4(0.2)\}$	$\{H_3(0.2), H_4(0.6), \overline{H}(0.2)\}$
x_3	$\{H_3(0.4), H_4(0.6)\}$	$\{H_2(0.5), H_3(0.4), H_4(0.1)\}$	$\{H_4(0.5), H_5(0.5)\}$	$\{H_3(0.3), H_4(0.4), H_5(0.3)\}$	$\{H_4(0.4), H_5(0.4), \overline{H}(0.2)\}$
x_4	$\{H_2(0.5), H_3(0.3), H_4(0.2)\}$	$\{H_3(0.7), H_4(0.2), \overline{H}(0.1)\}$	$\{H_3(0.6), H_4(0.2), H_5(0.2)\}$	$\{H_2(0.2), H_3(0.8)\}$	$\{H_3(0.3), H_4(0.5), \overline{H}(0.2)\}$

Table 2 The decision-making matrix of R_2.

	c_1	c_2	c_3	c_4	c_5
x_1	$\{H_1(0.3), H_2(0.4), H_3(0.3)\}$	$\{H_1(0.2), H_2(0.6), \overline{H}(0.2)\}$	$\{H_2(0.5), H_3(0.5)\}$	$\{H_1(0.3), H_2(0.7)\}$	$\{H_2(0.5), H_3(0.3), \overline{H}(0.2)\}$
x_2	$\{H_2(0.4), H_3(0.4), H_4(0.2)\}$	$\{H_2(0.6), H_3(0.4)\}$	$\{H_1(0.1), H_2(0.8), H_3(0.1)\}$	$\{H_2(0.6), H_3(0.3), \overline{H}(0.1)\}$	$\{H_3(0.8), H_4(0.2)\}$
x_3	$\{H_3(0.6), H_4(0.4)\}$	$\{H_3(0.5), H_4(0.3), H_5(0.2)\}$	$\{H_4(0.4), H_5(0.4), \overline{H}(0.2)\}$	$\{H_2(0.2), H_3(0.8)\}$	$\{H_2(0.1), H_3(0.3), H_4(0.6)\}$
x_4	$\{H_3(0.3), H_4(0.6), \overline{H}(0.1)\}$	$\{H_2(0.5), H_3(0.2), H_4(0.2), \overline{H}(0.1)\}$	$\{H_3(0.4), H_4(0.6)\}$	$\{H_4(0.7), H_5(0.3)\}$	$\{H_3(0.5), H_4(0.2), \overline{H}(0.3)\}$

Table 3 The decision-making matrix of R_3.

	c_1	c_2	c_3	c_4	c_5
x_1	$\{H_1(0.8), H_2(0.2)\}$	$\{H_1(0.4), H_2(0.3), H_3(0.3)\}$	$\{H_2(0.5), H_3(0.3), \overline{H}(0.2)\}$	$\{H_3(0.6), H_4(0.3), \overline{H}(0.1)\}$	$\{H_3(0.5), H_4(0.5)\}$
x_2	$\{H_1(0.4), H_2(0.6)\}$	$\{H_2(0.3), H_3(0.4), H_4(0.3)\}$	$\{H_2(0.8), H_3(0.1), \overline{H}(0.1)\}$	$\{H_3(0.5), H_4(0.4), H_5(0.1)\}$	$\{H_2(0.2), H_3(0.6), H_4(0.2)\}$
x_3	$\{H_2(0.3), H_3(0.5), \overline{H}(0.2)\}$	$\{H_2(0.4), H_3(0.5), H_4(0.1)\}$	$\{H_3(0.5), H_4(0.5)\}$	$\{H_3(0.2), H_4(0.8)\}$	$\{H_4(0.5), H_5(0.3), \overline{H}(0.2)\}$
x_4	$\{H_3(0.4), H_4(0.4), \overline{H}(0.2)\}$	$\{H_2(0.5), H_3(0.3), H_4(0.2)\}$	$\{H_3(0.5), H_4(0.4), \overline{H}(0.1)\}$	$\{H_2(0.3), H_3(0.6), H_4(0.1)\}$	$\{H_4(0.7), H_5(0.3)\}$

Step 3. Generate the collective evaluations. By Eqs. (1–10), according to the previous definition, a comprehensive matrix $\overline{DM} = \left(e_i^k\right)_{I\times K}$ is obtained as presented in Table 4.

Then, by Eqs. (1–10) again, we can aggregate the evaluations of all the experts and get the collective evaluations of each alternative as

$$F(x_1) = \left\{H_1(0.15), H_2(0.44), H_3(0.30), H_4(0.07), H_5(0), \overline{H}(0.04)\right\}$$

$$F(x_2) = \left\{H_1(0.05), H_2(0.47), H_3(0.32), H_4(0.11), H_5(0.01), \overline{H}(0.04)\right\}$$

$$F(x_3) = \left\{H_1(0), H_2(0.08), H_3(0.36), H_4(0.42), H_5(0.11), \overline{H}(0.03)\right\}$$

$$F(x_4) = \left\{H_1(0), H_2(0.12), H_3(0.41), H_4(0.37), H_5(0.05), \overline{H}(0.05)\right\}$$

where $\overline{H}$ represents the universal set of focal elements in evaluations.

Step 4. Generate preference matrices. Firstly, we convert the above collective evaluations to a matrix of alternatives with respect to the evaluation grades as follows:

$$D = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccccc} H_1 & H_2 & H_3 & H_4 & H_5 & \overline{H} \end{array} \\ \begin{bmatrix} 0.15 & 0.44 & 0.30 & 0.07 & 0 & 0.04 \\ 0.05 & 0.47 & 0.32 & 0.11 & 0.01 & 0.04 \\ 0 & 0.08 & 0.36 & 0.42 & 0.11 & 0.03 \\ 0 & 0.12 & 0.41 & 0.37 & 0.05 & 0.05 \end{bmatrix} \end{array}$$

Then, by Eqs. (24) and (25), based on a preference function $f(\cdot)$ with the threshold value $b = 0.3$, we can get the preference matrices of grades H_1 and $\overline{H}$ as follows:

$$P_1 = \begin{bmatrix} 0 & 0.33 & 0.50 & 0.50 \\ -0.33 & 0 & 0.17 & 0.17 \\ -0.50 & -0.17 & 0 & 0 \\ -0.50 & -0.17 & 0 & 0 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0 & -0.10 & 1 & 1 \\ 0.10 & 0 & 1 & 1 \\ -1 & -1 & 0 & -0.13 \\ -1 & -1 & 0.13 & 0 \end{bmatrix}$$

Table 4 The comprehensive matrix of R_k.

	DM_1	DM_2	DM_3
x_1	$\{H_1(0.12), H_2(0.45), H_3(0.33), H_4(0.07), H_5(0), \overline{H}(0.03))\}$	$\{H_1(0.14), H_2(0.61), H_3(0.19), H_4(0), H_5(0), \overline{H}(0.06)\}$	$\{H_1(0.20), H_2(0.20), H_3(0.39), H_4(0.15), H_5(0), \overline{H}(0.06)\}$
x_2	$\{H_1(0.13), H_2(0.37), H_3(0.30), H_4(0.15), H_5(0), \overline{H}(0.05)\}$	$\{H_1(0.02), H_2(0.56), H_3(0.35), H_4(0.05), H_5(0), \overline{H}(0.02)\}$	$\{H_1(0.05), sH_2(0.39), H_3(0.34), H_4(0.17), H_5(0.02), \overline{H}(0.03)\}$
x_3	$\{H_1(0), H_2(0.03), H_3(0.20), H_4(0.49), H_5(0.25), \overline{H}(0.03)\}$	$\{H_1(0), H_2(0.06), H_3(0.47), H_4(0.31), H_5(0.12), \overline{H}(0.04)\}$	$\{H_1(0), H_2(0.14), H_3(0.35), H_4(0.43), H_5(0.03), \overline{H}(0.05)\}$
x_4	$\{H_1(0), H_2(0.16), H_3(0.59), H_4(0.17), H_5(0.05), \overline{H}(0.03)\}$	$\{H_1(0), H_2(0.08), H_3(0.25), H_4(0.53), H_5(0.07), \overline{H}(0.07)\}$	$\{H_1(0), H_2(0.16), H_3(0.39), H_4(0.35), H_5(0.05), \overline{H}(0.05)\}$

$$P_3 = \begin{bmatrix} 0 & -0.07 & -0.20 & -0.37 \\ 0.07 & 0 & -0.13 & -0.30 \\ 0.20 & 0.13 & 0 & -0.17 \\ 0.37 & 0.30 & 0.17 & 0 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 0 & -0.13 & -1 & -1 \\ 0.13 & 0 & -1 & -0.87 \\ 1 & 1 & 0 & 0.17 \\ 1 & 0.87 & -0.17 & 0 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0 & -0.03 & -0.37 & -0.17 \\ 0.03 & 0 & -0.33 & -0.13 \\ 0.37 & 0.33 & 0 & 0.20 \\ 0.17 & 0.13 & -0.20 & 0 \end{bmatrix},$$

$$P_{\overline{H}} = \begin{bmatrix} 0 & 0 & 0.03 & -0.03 \\ 0 & 0 & 0.03 & -0.03 \\ -0.03 & -0.03 & 0 & -0.07 \\ 0.03 & 0.03 & 0.07 & 0 \end{bmatrix}$$

Step 5. Generate the expected preference matrices. Suppose that the experts are neutral and the utility values of evaluation grades are given as $u_1 = 0$, $u_2 = 0.25$, $u_3 = 0.5$, $u_4 = 0.75$, $u_5 = 1$, and H_1 is the best grade in $\overline{H}$ and H_5 is the worst one. By assigning the preference of $\overline{H}$ to the best evaluation grade H_1 and the worst evaluation grade H_5, respectively, we can get the best expected preference matrix $Q_B = \left(Q^n_{it,B}\right)_{I\times I}$ and the worst expected preference matrix $Q_W = \left(Q^n_{it,W}\right)_{I\times I}$ as

$$Q_B = \begin{bmatrix} 0 & -0.19 & -0.97 & -0.86 \\ 0.19 & 0 & -0.90 & -0.68 \\ 0.97 & 0.90 & 0 & 0.21 \\ 0.86 & 0.68 & -0.21 & 0 \end{bmatrix},$$

$$Q_W = \begin{bmatrix} 0 & -0.19 & -0.94 & -0.89 \\ 0.19 & 0 & -0.87 & -0.71 \\ 0.94 & 0.87 & 0 & 0.14 \\ 0.89 & 0.71 & -0.14 & 0 \end{bmatrix}$$

Step 6. Rank the alternatives. Based on the generated expected preference matrices, the best net flow and the worst net flow can be calculated by Eqs. (18–20). Then, by Eq. (26), we can obtain the average net flow of each alternative as $\varphi_1 = -4.04$, $\varphi_2 = -2.78$, $\varphi_3 = 3.00$ and $\varphi_4 = 2.80$. By the PROMETHEE-based ranking method, the ranking of the risk to catch lung cancer of these four patients is $x_3 > x_4 > x_2 > x_1$, which implies that the patient x_3 has the highest risk of lung cancer.

4.2. Comparative Analyses

To show the rationality and superiority of our proposed method, we use the original ER approach [1] to solve this case at first. Moreover, from the perspective of control variables, for the second comparison, the information fusion is carried out with the original ER approach [1], and then the PROMETHEE-based ranking method is used for ranking. For the third comparison, the information fusion is carried out under the condition of considering both subjective weights and objective weights with the ER approach, and then the alternatives are ranked based on the utility-value-based method in the ER approach.

1. Rank the patients by the original ER approach
   Suppose that the experts are neutral and the utilities of evaluation grades are $u_1 = 0$, $u_2 = 0.25$, $u_3 = 0.5$, $u_4 = 0.75$, $u_5 = 1$. By Eqs. (1–10), we can fuse all the evaluations of each alternative on all criteria and obtain their utilities as $u_{avg}(x_1) = 0.33$, $u_{avg}(x_2) = 0.44$, $u_{avg}(x_3) = 0.64$, $u_{avg}(x_4) = 0.59$. Then, the ranking of the four patients can be obtained as $x_3 > x_4 > x_2 > x_1$.
2. Rank the patients by the original ER approach with the PROMETHEE-based method
   By Eqs. (1–10), we can fuse all the evaluations of each alternative on all criteria. Then, the same as Steps 3–6 in Section 4.1, we can calculate the average net flow of each alternative by the PROMETHEE-based method and obtain $\varphi_1 = -2.62$, $\varphi_2 = -0.99$, $\varphi_3 = 1.80$, $\varphi_4 = 1.14$. Therefore, the ranking of the four patients is $x_3 > x_4 > x_2 > x_1$.
3. Rank the patients by the original ER approach with comprehensive weights
   The same as Steps 1–3 in Section 4.1, we can aggregate the decision matrices with both subjective weights and objective certainty degrees by the original ER approach. With the utilities $u_1 = 0$, $u_2 = 0.25$, $u_3 = 0.5$, $u_4 = 0.75$ and $u_5 = 1$, by Eqs. (11–13), we can obtain the utilities of alternatives as: $u_{avg}(x_1) = 0.33$, $u_{avg}(x_2) = 0.39$, $u_{avg}(x_3) = 0.65$, $u_{avg}(x_4) = 0.59$. Then, the ranking of the patients is $x_3 > x_4 > x_2 > x_1$.

Based on the above analyses, we can surmise the ranking results generated by the mentioned methods in Table 5.

Table 5 The ranking results of relevant methods.

Methods	Ranking	Considering Subjective Weights	Considering Objective Weights	Considering Preference of Experts
The proposed method	$x_3 > x_4 > x_2 > x_1$	✓	✓	✓
The original ER approach	$x_3 > x_4 > x_2 > x_1$	✓	✗	✗
ER + PROMETHEE	$x_3 > x_4 > x_2 > x_1$	✓	✗	✓
ER + comprehensive weights	$x_3 > x_4 > x_2 > x_1$	✓	✓	✗

As can be seen from the ranking results of relevant methods in Table 5, all the ranking results are the same, which shows that the results generated by the proposed method are as reasonable as those derived from the existing methods. However, the original ER approach did not consider the objective weights determined by the entropy measure and the preferences of experts. The ER approach with the PROMETHEE-based ranking method only considers the preferences of experts. Though these two methods have the same ranking results, there are differences in the span between the ranking results. While applying the PROMETHEE-based ranking method in the ER approach to rank alternatives, the differences between alternatives may be magnified and can clearly show whether this alternative is superior or inferior to others, which can make the alternatives with similar performances be identified easily. In addition, by considering the subjective weights and objective weights at the same time, the subjectivity in decision-making can be reduced. In this case, the results obtained by the ER approach have little difference whether considering the objective weights or not. The main reason is that the differences between the four patients are too obvious so that the objective weights do not have enough influence to change the ranking result. However, we can still see a greater degree of differentiation between the ranking results, which means that similar goals are more easily distinguished. At the same time, it can be seen from the sensitivity analysis that when the threshold is different, the ranking results will change slightly, which also shows that our proposed method is reasonable.

To demonstrate that it is reasonable to combine subjective weights with objective weights to derive comprehensive weights, relevant results of an independent sample t-test are shown in Table 6.

As can be seen from the above table, all p values in Levene's test are greater than 0.05 (0.537, 0.777, 0.335), indicating that the variance between the two kinds of weights is homogeneous. In addition, all p values in the t-test were 1.000 (>0.05), implying that there was no significant difference between the two weights. Therefore, the combination of subjective weights and objective weights will not produce conflict results. The combination of weights is a reasonable consideration.

4.3. Sensitivity Analyses

In the above case study, we assume that the threshold of the preference function in the PROMETHEE method is 0.3. In this section, we set the threshold as 0.15, 0.25, 0.35 to represent the experts' preferences and sort the alternatives. The function is shown in Figure 3.

Table 6 Independent sample t-test of subjective and objective weight.

	Levene's Test		t-test		
	F	significance	t	significance	Mean Difference
w and a^1	0.416	0.537	0.000	1.000	0.000
w and a^2	0.086	0.777	0.000	1.000	0.000
w and a^3	1.054	0.335	0.000	1.000	0.000

1. The threshold is 0.15
   By Eqs. (24) and (25), based on the preference function $f(\cdot)$ with the threshold value $b = 0.15$, we can get the preference matrices of grades H_1 and $\overline{H}$ as follows:

$$P_1 = \begin{bmatrix} 0 & 0.67 & 1 & 1 \\ -0.67 & 0 & 0.33 & 0.33 \\ -1 & -0.33 & 0 & 0 \\ -1 & -0.33 & 0 & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & -0.2 & 1 & 1 \\ 0.2 & 0 & 1 & 1 \\ -1 & -1 & 0 & -0.26 \\ -1 & -1 & 0.26 & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & -0.13 & -0.04 & -0.73 \\ 0.13 & 0 & -0.26 & -0.6 \\ 0.04 & 0.26 & 0 & -0.33 \\ 0.73 & 0.6 & 0.33 & 0 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & -0.26 & -1 & -1 \\ 0.26 & 0 & -1 & -1 \\ 1 & 1 & 0 & -0.33 \\ 1 & 1 & 0.33 & 0 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0 & -0.06 & -0.73 & -0.33 \\ 0.06 & 0 & -0.67 & -0.26 \\ 0.73 & 0.67 & 0 & 0.4 \\ 0.33 & 0.26 & -0.4 & 0 \end{bmatrix}$$

$$P_{\overline{H}} = \begin{bmatrix} 0 & 0 & 0.06 & -0.06 \\ 0 & 0 & 0.06 & -0.06 \\ -0.06 & -0.06 & 0 & -0.13 \\ 0.06 & 0.06 & 0.13 & 0 \end{bmatrix}$$

By assigning the preference of $\overline{H}$ to the best evaluation grade H_1 and the worst evaluation grade H_5, respectively, we can get the best expected preference matrix $Q_B = \left(Q^n_{it,B}\right)_{I\times I}$ and the worst expected preference matrix $Q_W = \left(Q^n_{it,W}\right)_{I\times I}$ as

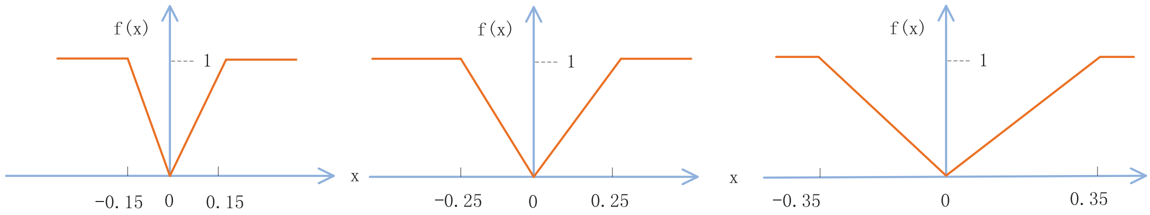

Figure 3 Preference functions with the threshold being 0.15, 0.25 and 0.35.

$$Q_B = \begin{bmatrix} 0 & -0.37 & -1.25 & -1.2 \\ 0.37 & 0 & -1.3 & -1.06 \\ 1.25 & 1.3 & 0 & -0.08 \\ 1.2 & 1.06 & 0.08 & 0 \end{bmatrix},$$

$$Q_W = \begin{bmatrix} 0 & -0.37 & -1.19 & -1.26 \\ 0.37 & 0 & -1.24 & -1.12 \\ 1.19 & 1.24 & 0 & -0.21 \\ 1.26 & 1.12 & 0.21 & 0 \end{bmatrix}$$

Based on the generated expected preference matrices, the best net flow and the worst net flow can be calculated by Eqs. (18–20). Then, by Eq. (26), we can obtain the average net flow of alternatives as $\varphi_1 = -5.64$, $\varphi_2 = -3.98$, $\varphi_3 = 4.69$ and $\varphi_4 = 4.93$. By the PROMETHEE-based ranking method, the ranking of the risk to catch lung cancer of these four patients is $x_4 > x_3 > x_2 > x_1$, which implies that the patient x_4 has the highest risk of lung cancer.

2. The threshold is 0.25
By Eqs. (24) and (25), based on a preference function $f(\cdot)$ with the threshold value $b = 0.25$, we can get the preference matrices of grades H_1 and $\overline{H}$ as follows:

$$P_1 = \begin{bmatrix} 0 & 0.4 & 0.6 & 0.6 \\ -0.4 & 0 & 0.2 & 0.2 \\ -0.6 & -0.2 & 0 & 0 \\ -0.6 & -0.2 & 0 & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & -0.12 & 1 & 1 \\ 0.12 & 0 & 1 & 1 \\ -1 & -1 & 0 & -0.16 \\ -1 & -1 & 0.16 & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & -0.08 & -0.24 & -0.44 \\ 0.08 & 0 & -0.16 & -0.36 \\ 0.24 & 0.16 & 0 & -0.2 \\ 0.44 & 0.36 & 0.2 & 0 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & -0.16 & -1 & -1 \\ 0.16 & 0 & -1 & -1 \\ 1 & 1 & 0 & -0.2 \\ 1 & 1 & 0.2 & 0 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0 & -0.04 & -0.44 & -0.2 \\ 0.04 & 0 & -0.4 & -0.16 \\ 0.44 & 0.4 & 0 & 0.24 \\ 0.2 & 0.16 & -0.24 & 0 \end{bmatrix}$$

$$P_{\overline{H}} = \begin{bmatrix} 0 & 0 & 0.04 & -0.04 \\ 0 & 0 & 0.04 & -0.04 \\ -0.04 & -0.04 & 0 & -0.08 \\ 0.04 & 0.04 & 0.08 & 0 \end{bmatrix}$$

By assigning the preference of $\overline{H}$ to the best evaluation grade H_1 and the worst evaluation grade H_5, respectively, we can get the best expected preference matrix $Q_B = \left(Q^n_{it,B}\right)_{I\times I}$ and the worst weighted preference matrix $Q_W = \left(Q^n_{it,W}\right)_{I\times I}$ as

$$Q_B = \begin{bmatrix} 0 & -0.23 & -1.06 & -0.92 \\ 0.23 & 0 & -0.98 & -0.84 \\ 1.06 & 0.98 & 0 & -0.05 \\ 0.92 & 0.84 & 0.05 & 0 \end{bmatrix},$$

$$Q_W = \begin{bmatrix} 0 & -0.23 & -1.02 & -0.96 \\ 0.23 & 0 & -0.94 & -0.88 \\ 1.02 & 0.94 & 0 & -0.13 \\ 0.96 & 0.88 & 0.13 & 0 \end{bmatrix}$$

Based on the generated expected preference matrices, the best net flow and the worst net flow can be calculated by Eqs. (18–20). Then, by Eq. (26), we can obtain the average net flow of alternatives as $\varphi_1 = -4.42$, $\varphi_2 = -3.18$, $\varphi_3 = 3.82$ and $\varphi_4 = 3.80$. By the PROMETHEE-based ranking method, the ranking of the risk to catch lung cancer of these four patients is $x_3 > x_4 > x_2 > x_1$, which implies that the patient x_3 has the highest risk of lung cancer.

3. The threshold is 0.35
By Eqs. (24) and (25), based on a preference function $f(\cdot)$ with the threshold value $b = 0.35$, we can get the preference matrices of grades H_1 and $\overline{H}$ as follows:

$$P_1 = \begin{bmatrix} 0 & 0.29 & 0.43 & 0.43 \\ -0.29 & 0 & 0.14 & 0.14 \\ -0.43 & -0.14 & 0 & 0 \\ -0.43 & -0.14 & 0 & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & -0.09 & 1 & 0.91 \\ 0.09 & 0 & 1 & 1 \\ -1 & -1 & 0 & -0.11 \\ -0.91 & -1 & 0.11 & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & -0.06 & -0.17 & -0.31 \\ 0.06 & 0 & -0.11 & -0.26 \\ 0.17 & 0.11 & 0 & -0.14 \\ 0.31 & 0.26 & 0.14 & 0 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & -0.11 & -1 & -0.86 \\ 0.11 & 0 & -0.89 & -0.74 \\ 1 & 0.89 & 0 & -0.14 \\ 0.86 & 0.74 & 0.14 & 0 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0 & -0.03 & -0.31 & -0.14 \\ 0.03 & 0 & -0.29 & -0.11 \\ 0.31 & 0.29 & 0 & 0.17 \\ 0.14 & 0.11 & -0.17 & 0 \end{bmatrix}$$

$$P_{\overline{H}} = \begin{bmatrix} 0 & 0 & 0.03 & -0.03 \\ 0 & 0 & 0.03 & -0.03 \\ -0.03 & -0.03 & 0 & -0.06 \\ 0.03 & 0.03 & 0.06 & 0 \end{bmatrix}$$

By assigning the preference of $\overline{H}$ to the best evaluation grade H_1 and the worst evaluation grade H_5, respectively, we can get the best expected preference matrix $Q_B = \left(Q^n_{it,B}\right)_{I\times I}$ and the worst expected preference matrix $Q_W = \left(Q^n_{it,W}\right)_{I\times I}$ as

$$Q_B = \begin{bmatrix} 0 & -0.17 & -0.9 & -0.71 \\ 0.17 & 0 & -0.76 & -0.55 \\ 0.9 & 0.76 & 0 & -0.03 \\ 0.71 & 0.55 & 0.03 & 0 \end{bmatrix},$$

$$Q_W = \begin{bmatrix} 0 & -0.17 & -0.87 & -0.74 \\ 0.17 & 0 & -0.73 & -0.58 \\ 0.87 & 0.73 & 0 & -0.09 \\ 0.74 & 0.58 & 0.09 & 0 \end{bmatrix}$$

Based on the generated expected preference matrices, the best net flow and the worst net flow can be calculated by Eqs. (18–20). Then, by Eq. (26), we can obtain the average net flow of alternatives as $\varphi_1 = -3.56$, $\varphi_2 = -2.28$, $\varphi_3 = 3.14$ and $\varphi_4 = 2.70$. By the PROMETHEE-based ranking method, the ranking of the risk to catch lung cancer of these four patients is $x_3 > x_4 > x_2 > x_1$, which implies that the patient x_3 has the highest risk of lung cancer.

From the above results, the gap between x_3 and x_4 becomes larger as the threshold increases, and when the threshold is equal to 0.15, x_4 is greater than x_3. This reflects the fact that experts are sensitive to preferences. This means that the advantage of x_3 is smaller relative to that of x_4. According to the pairwise comparison matrix between alternatives, 0.15 is too small and 0.3 is a suitable threshold. So, we chose a threshold of 0.3 in the comparative analysis.

5. CONCLUSIONS

How to deal with the uncertainty of decision problems is a vital problem. In this study, we proposed a Deng-entropy-based ER approach to solve uncertain MEMCDM problems. Since the existing methods cannot well model uncertain evaluations, there is a need to propose a new representation method to model uncertainty. By assigning the remaining belief to the set of focal elements of given evaluation grades, this study introduced a novel method to model the uncertainty of evaluation information accurately. Moreover, except for the subjective weights given by experts, the objective weights were also considered to reduce the subjective uncertainty. Considering this, we calculated the certainty of evaluation information to obtain the objective weights by the Deng entropy, and then obtained comprehensive weights considering both the subjective weights and objective weights. To aggregate the uncertain evaluations of each alternative, the ER approach was used twice on the premise that experts are independent of each other before giving evaluations. Furthermore, given that the utility value-based ranking method used in the original ER approach only considered the absolute importance of evaluation grades and ignored the pairwise comparison relations between alternatives, we proposed a PROMETHEE-based ranking method to rank alternatives in the ER approach, which takes into account both the utility values of the evaluation grades and the pairwise comparison relations between alternatives. Finally, our proposed method was used to screen the high-risk lung cancer patients. The results deduced from the proposed method were consistent with those derived by other existing methods, which verified the rationality of our method.

In this paper, although we considered both the objective and subjective weights of criteria and experts, the reliability of experts was ignored. Scholars have considered experts' reliability and subjective weights together in decision-making methods, but ignored the objective weights. In the future, we will take into account how to determine and deal with the subjective weights, objective weights and the reliability of experts.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHORS' CONTRIBUTIONS

H.C. Liao, Z.Y. Ren and R. Fang proposed the original idea and conceived the study. H.C. Liao and Z.Y. Ren were responsible for developing the method. H.C. Liao, Z.Y. Ren and R. Fang were responsible for collecting and analyzing the data. H.C. Liao, Z.Y. Ren and R. Fang were responsible for data interpretation. H.C. Liao and Z.Y. Ren wrote the first draft of the article. H.C. Liao, Z.Y. Ren and R. Fang revised the paper.

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