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# From Less to More Sophisticated Solutions: A Sociomathematical Norms to Develop Students' Self-Efficacy

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***Abstract*** **—** This paper is a part of a larger study which aimed to design a learning trajectory in pre-algebra. As a design research, it follows three steps of Preliminary Studies, Teaching Implementation and Retrospective Analysis. To enhance the quality of learning, we adjusted the socio-mathematical norms in the teacher's lesson plan. One aspect that will be discussed in the present study is the order of the students' solution during group and class discussion. The subjects of the study were 32 fifth grade students of a state elementary school in Palembang. We gathered the data about teacher and students' behavior during the lesson by using observation and interview method. To cross-check the data, the students' written work was also considered. The collected data were analyzed qualitatively by using constant comparative method. The results showed that the students were actively engaged in learning and brave to share their ideas if the teacher gave them opportunities. In addition, the students' self-efficacy was fostered if the teacher connected the ideas of students, instead of directly judging them as correct or not. This enhanced students' self-efficacy. Hence, the students were actively engaged in learning and no one left behind. Consequently, it is recommended to establish the socio-mathematical norms in conducting Mathematics lesson.

***Keywords*** **—** socio-mathematical norms, social norms, self-efficacy

## I. INTRODUCTION

Realistic Mathematics Education (RME) is an approach to provide a meaningful mathematics classroom which is connected to human activity [1]. According to [2], it should connect to what people can imagine as reality, close to people's experience and contribute to the human being and civilization. To achieve the goals of making sense of mathematics classroom, RME is conceptualized in five tenets of teaching and learning including phenomenological exploration, the support of vertical instruments, students' own constructions and productions, interactivity and intertwinement with the learning's strands [3]. Within these characteristics, RME is one of constructivism-based learning which aimed to engage the students in active learning setting.

Despite of all good intention of RME, the success of learning does not merely depend on the type of learning model employed by the teacher. In most cases, the model is good, the supports from students' worksheet and learning media is good, but the implementation is not optimum, if not misleading [4]. In the constructivism type of learning setting, such as RME, the learning trajectory will only working if the students' knowledge construction played majority role. In other words, the students are the center of learning. Also, the teacher's responsibility according to RME point of view is different with traditional classroom. The RME teacher should not be the one who transmit the knowledge to students, but the facilitator who enable the students in learning.

The interaction between teacher and students in the classroom is called with social norms. In general term, a social norms is the pattern of social interaction of certain group [5]. In the most of traditional class, the norms are teacher explain the materials, the students write their note and do exercise and the teacher will judge the correctness of the students' work [6]. It usually followed by another norm that the students feel insecure about themselves and keep asking for teacher agreement before continue their learning [7]. The aforementioned norms of learning will not be suitable in a RME classroom because students' construction tenet will be dismissed.

To optimize the implementation of RME, the social norms in the classroom should be adjusted. Basically, there are three types of students' interaction that usually managed in the classroom which are in pairs, in group of four and a whole class discussion. This setting enable students to increase their participation, reduce hesitant in sharing ideas and activating the silence students without neglecting the dominant one [8].

However, in the beginning of the study we found that social norms itself is not sufficient to bring the best of RME to accommodate the students' participation. From the observation, it was found that the students were not confidence in share their thoughts. It indicates by the students' nervousness in answering the teacher's question and the tendency to change their original answer when their friends have different points of view. Frequently, since most of the students were not confident to present their ideas in front of the class, the teacher asked the student who has the best answer to do the task. The rest of students usually erased their answer afterwards and copy what was written on the white board. It showed the lack of students' self-efficacy. Self-efficacy in mathematics is students' belief on their ability in solving specific mathematics task [9]. The development of self-efficacy is determined by the type of interaction they received [10]. Therefore, the teacher is the crucial pioneer in the development of students' self-efficacy.

In mathematics classroom, students' different points of view and different speed of learning are very likely to happen. As every student are unique, we cannot push everyone to come in similar answer. A study of [11] showed that the students tend to have anxiety in solving mathematical problems and it contributes to the difficulties of them in learning mathematics. Hence, the role of teacher is extremely significant in this case, to create a supportive learning environment such that the students feel welcome in learning even though they have different point of view and level of mastery in certain mathematical concept. This specific norm is called with socio-mathematical norms.

According to Kang & Kim [12], socio-mathematical norms is "the consideration of a mathematically acceptable explanation in conjunction with an understanding of what has been mathematically different" (p. 2735). It takes form of agreement used by teacher and students to distinguish the ideas based on different category [5]. One of them is based on its sophisticated or level of abstraction which will be the focus of the present study. Reflect to the aforementioned background, the research question addressed in this study is how can socio-mathematical norms develop students' efficacy in learning Mathematics?

## II. METHOD

This study is a part of a Design Research which aimed to design a learning trajectory in pre-algebra. To enhance the quality of learning, we adjusted the socio-mathematical norms in the teacher's lesson plan. One aspect that will be discussed in the present study is the order of the students' solution during group and class discussion to improve students' self-efficacy.

To achieve the goals the steps of Design Research consists of Preliminary Studies, Teaching Implementation and Retrospective Analysis were employed [13]. During the Teaching Implementation phase, we conducted the study in a real classroom to supports the transferability of the results to the common school practices [14], [15]. The subject of the study was the fifth grade students of a state elementary school in Palembang. We gathered the data about teacher and students' behavior related to socio-mathematical norms in ordering solutions based on its sophisticated strategy during the lesson by observation and interview method. To cross-check the data, the students' written work was also considered. The collected data were analyzed qualitatively using constant comparative method.

## III. RESULTS AND DISCUSSION

We will discuss the results from one of the lesson during the teaching implementation phase. The students were discussing the pattern of square numbers embodied in square shape. The context was the formation for martial art performance called *Pencak Silat* for a cultural event in Indonesia.

The teacher showed the picture of the first three formations and asked the students to draw the 4th formation and find the next $10^{th}$, $15^{th}$ and $100^{th}$ formation. The first three formations is given as in Fig. 1.

Fig. 1. The first three formation.

There are different methods used by the students. Of course, the ideal responses expected on this study is that the students come to conclusion that a square number pattern is related to squaring the number itself. For instance, the first number should be the square of 1, the second number is the square of 2 or in general the nth number is the square of *n*.

However, the teacher should be aware that not all students will directly see the relation among the numbers. If the teacher wants to conduct a constructivist learning and encourage all students to participate, the key answer cannot be given in the beginning of the exploration. Yet, the teacher can organize the students' strategy in the ascending order. It means, we started from less to more sophisticated answer. The following example is given to illustrate the types of strategy (in order) for square number problem.

### *A. Drawing Strategy*

The students continue drawing the formation and count how many dots in the n position

### *B. Addition Strategy*

On this stage, the students not merely depend on the illustration they made but see the growing pattern of how many numbers of dots added in each formation.

Fig. 2. Adding 1, 3, 5 and so on.

Some students might see the pattern in different point of view and realize that the number of dancers is increase as much as 2 times of the number of formation and then minus it one by one

### *C. Parts Strategy*

On this level, the students recognize the relation between the numbers in each row and column. They will see that for instance in the 2nd formation there will be 2 dancers in each row and since there are 2 rows and the total number of dancers will be 2 + 2 = 4

*D. Multiplication Strategy*

The students are able to see the relation between the "length" and the "width" of the square to determine the total dots fulfilled the square. They will realize that there is a relation between the number of dancers in a row and a column with the total number of dancers.

*E. Power Rule Strategy*

On this stage, the students aware that multiplication they employed involving the same number. In other word, it is squaring process, the power of two rules.

Before the current meeting, the teacher merely focus on correct answer with the most sophisticated strategy to be presented in the classroom discussion and neglected the students with less sophisticated strategy. This situation leads to the students' hesitant to share their ideas.

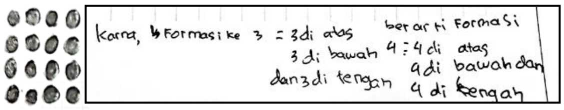
Therefore, before we conducted the current meeting, we discussed the vocal point of less to more sophisticated answer socio-mathematical norms with the teacher. It means, the teacher has to give the equal chance to the students to diversity of answers in the classroom. Therefore, the classroom discussion was started by the Drawing Strategy continued by Addition Strategy and move forward to the Parts Strategy which leads to Multiplication Strategy and concluded in Power Rule Strategy.

By ordering the students' type of answer and give them the chance to explain it, the students' participation in discussion increased. The students were enthusiast in learning and brave to share their ideas, both in group and in front of the classroom. Also, the students' believe toward their possible success in learning will be fostered if the teacher connect the ideas of students, instead of directly judge them as correct or not. Since one to other strategies are related in a growing order, the students will actively engage in the discussion. The discovery of squaring the number formula become the finding of all of the group or classroom members.

As to solve the given problem, a group of students were using drawing strategy. They said they will keep drawing the formation with the dots and create a square shape as in the worksheet until the 100th formation. Even though the teacher let the students to share their ideas, the teacher need to lead the discussion to help the students develop their thinking. After heard the students plan to keep drawing until the 100th formation, the teacher asked the students to use their drawing to observe the pattern.

Another group of students solve the problem by looking at the number of dots added in every next formation. The students were considering the growing pattern on each drawing. They noticed that it grows from 3 to 5, so in the next figure they should add 7.This addition strategy is a step higher then drawing one by one. But still it takes longer time since the number of previous term should be calculated before the current term. In other words, the Addition Strategy leads to a recursive formula.

After the presentation of the addition strategy ideas, the teacher asked the students to check the parts of the square, i.e. how many dots in the first and last row and the column. The students with Part Strategy explained their ideas as can be seen in the following Fig. 3




Translation:
Third formation consists of 3 above, 3 below and 3 in the middle; so in Forth formation consists of 4 above, 4 below and 4 in the middle.

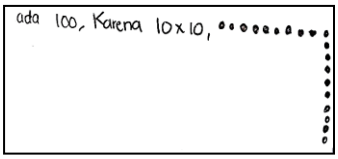
Fig. 3. Parts of the square.

The group of students who were not using Part Strategy need further support. Hence, the teacher elevated their thinking by combining their former strategy and drawing and the ideas of Part Strategy. The discussion between Teacher (T) and the students (S1, S2, S3, S4) can be observed in Fragment 1.

Fragment 1: Look at the Parts of the Square

| | | | |
|---|---|---|---|
| (1) | T | : | Observe the formation shape. |
| (2) | | | Look at the fifth formation. |
| (3) | | | Count the number of dancers in the row. |
| (4) | S1 | : | 5 dancers. |
| (5) | T | : | 5 dancers let's draw the formation. |
| (6) | (*S1, S2, S3, S4 illustrate the formation*) | | |
| (7) | T | : | How many row we have? |
| (8) | S1 | : | five. |
| (9) | T | : | What is the total number of dancers? |
| (10) | S1 | : | 25 dancers in total |

In Fragment 1, the teacher encourages the students to move forward and look at different strategy to see the structure of the square number. The Parts Strategy by consider the number in rows and columns were emerged before the students able to solve it with multiplication. Fig. 4 showed the students movement from Parts Strategy into Multiplication of the number of rows and the number columns.




Translation:
100 since 10 × 10

Fig. 4. Multiplication Strategy.

Here, the students used properties of square. They observed that the total number of dancers in a square formation is equal to the result of squaring the number of dancers in a side of the square. This strategy will lead the students to generate a general formula, which means they can find the number of dancers in any formation without the need of finding the number of dancers in the previous formation.

Shifting from addition to multiplication or in other words from recursive to general formula is a crucial movement in development of algebraic thinking which is the goal of this study. Hence, the learning trajectory was design to enable the students come to this step. However, students thinking ability are differ in varied ways. Even though it seems very clear for a student, it can completely abstract for the other. The following Fragment 2 showed how the student was rejected after proposing Squaring Strategy right after his friend got insight about multiplication

Fragment 2: Multiplication to Power Rule

| | | | |
|---|---|---|---|
| (1) | S1 | : | Oh! Multiplication! It means 10 multiply by 10! |
| (2) | S2 | : | Agree! |
| (3) | S2 | : | Squaring... |
| (4) | S1 | : | What do you mean by square? |
| (5) | T | : | Can you explain it S1? |
| (6) | | : | 10 by 10 equals 100, (but) what is the reason? |
| (7) | S4 | : | Let's looking for different strategy! |
| (8) | S2 | : | There are 10 rows … |
| (9) | S4 | : | There are 10 dancers in a row. |
| (10) | | | Multiply the number of row with 10. |
| (11) | T | : | A row has 10 dancers and then multiply it? |
| (12) | S4 | : | in the 10th formation, there are 10 rows. |
| (13) | | | Each row has 10 people. It means 10 by 10. |
| (14) | T | : | Okay, multiply 10 by 10. |
| (15) | | | Next, how many dancers in the 15th formation? |
| (16) | S4 | : | 15 by 15 |
| (17) | | | Oh, it means square number! |

The Fragment 2 showed the students' rapid movement from the addition strategy they employed before to the parts to multiplication and finally to the square strategy. In the beginning, the rest of students were not understand the term "squaring" proposed by a student. In the case like this, the teacher play significant role to bridge the gap between students' strategies. Instead of directly expressing agreement or disagreement towards particular strategy, the teacher should give the chance for the students to evaluate it by their own analysis. The teacher can propose several challenging questions to emphasize the important ideas they missed from different perspectives.

If in that important moment, the teacher cut their arguments and merely explain what does it means by squaring the number, the "squaring" idea will only be owned by the student who call for it first. But then, since the teacher followed the students' way of thinking, asked them to tell their strategy step by step, in the end the students concluded it by themselves. If we apply the similar strategy in different topics on mathematics, the vocal points remain the same. First of all, we need to categorize each solution into several categories according to its correctness, completeness and complexness. Second, the differentiation in point (1) will help us to distinguish the majority of ideas in the classroom.

By doing so, the similar answer will be in the same group of responses and one representative can present the work. The incorrect solution or correct but incomplete or correct but less sophisticated ideas can open the classroom discussion. Later, the "missing part" of the given solution will be the opportunities for the next group to start their ideas. It continuously develop until the most sophisticated solution presented in front of the class. The teacher also needs to aware with the creative solution of the students which use different point of view or in different way of thinking with the majority of the classroom. The unique answer should be discussed as well to encourage further discussion about the most effective solution.

The implementation of less to more sophisticated solution socio-mathematical norms will encourage the students to try longer and keep their mind busy with the task. It contributes to the believe construction that the task is doable and they can solve the problems, which is the characteristics of self-efficacy [16]. For the long-term goal, the increase in self-efficacy is expected to increase the students' mathematical ability [17]. Therefore, it is recommended for the teachers to consider the socio-mathematical norms in conducting mathematics lesson.

## IV. Conclusion

Socio-mathematical norms can be used to develop students' self-efficacy since it enables students to participate in the classroom discussion. On the other hand, it does not aim to let the students stop in less sophisticated ideas. It connects different strategies of the students in the ascending order. Hence, every student will participate in the discussion and develop their complete understanding. They will also be encouraged to employ more sophisticated mathematical ideas. Those important aspects can improve students' self-efficacy in learning mathematics.

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