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Analysis of Students' Mathematical Investigation Based on the Variation of Mathematical Abilities

Sri Subarinah*
Mathematics Education Study Program
Universitas Mataram
Mataram, Indonesia
s.subarinah@gmail.com

Nurul Hikmah
Mathematics Education Study Program
Universitas Mataram
Mataram, Indonesia

Syahrul Azmi
Mathematics Education Study Program
Universitas Mataram
Mataram, Indonesia

***Abstract*—**The present work is a descriptive study aims to describe the students' mathematical investigation based on their mathematical abilities. In general, there are four cognitive steps in mathematical investigation, i.e. Specializing, Conjecturing, Justifying and Generalizing. The data were gathered from students' written work in solving mathematical problems and interview. The participants were 32 first year students of mathematics education study program in a university in Mataram, Indonesia. The participants were classified into three mathematical ability categories namely high, moderate and low. The data were analyzed using mixed method. The results confirmed that the subject with high and moderate mathematical abilities were able to perform four steps mathematical investigation, while those of low category cannot make the Generalizing stage. Furthermore, compare to moderate students, students with high mathematical abilities have more constructive way of thinking. The results indicate that the prospective teachers' mathematical investigation skills need to be improved through the series of lesson embedded in various courses in mathematics education study program.

***Keywords*—** Cognitive process, mathematical investigation, mathematical abilities

I. INTRODUCTION

In every school level, mathematics should be taught by letting the students to experience mathematical investigation [1]. It encourages the students to scientific activities, from gathering data, observing phenomenon, identifying patterns, constructing and proofing conjectures and generalizing the result [2]. Mathematical investigation will support the development of students' curiosity, argumentative skills and self-confidence. It contributes to the students' active participation in the classroom [3] in posing or responding to their teacher or peer's questions.

The cognitive process in mathematical investigation consist of four stages, which are Specializing, Conjecturing, Justifying and Generalizing [4]&[5]. In every stage, the students need to develop creative, analytical, synthetical and critical thinking.

The previous studies revealed that mathematics investigation enhanced the students' activities, creativities, productivities and problem-solving abilities [6]. It is very beneficial for students, especially since the setting of learning is usually in cooperative group [7]. Furthermore, mathematical investigation can be started from early age of students [8], in which the teacher can provide a meaningful support by connecting the problem solving with the mathematical representation, reasoning and manipulating. One type of activity that encourage mathematical investigation is through number pattern [9].

Our previous studies focused on junior high school students' cognitive process in mathematical investigation [10]–[16]. The results showed that the cognitive processes of the subjects (which were varied in term of the level of mathematical abilities and gender) had similarities and differences. It inspired us to conduct the study with the students of mathematics education study program, since they will be future mathematics' teachers, mostly at secondary level.

Generally, there still numerous problems in mathematical abilities and mathematical thinking abilities of the prospective teachers. It can be seen in the previous studies that revealed the lack of mathematical content knowledge and basic skills of the mathematics education study program [17] & [18].

Reflecting on the aforementioned background, it is clear that the prospective teachers have to develop their mathematical thinking, especially their mathematical investigation. Therefore, this study will focus on analyzing how well mathematical investigation of the prospective mathematics teachers. The result of this study will be a useful guidance for teacher training institution as consideration for further decision related to the teacher training program.

II. METHODS

The present study was descriptive study with the aim to describe the mathematical investigation skills of the prospective mathematics teachers. The participants of the study were 32 first-year students of mathematics education study program in a university in Mataram, Indonesia.

The data were gathered from students' written work in solving mathematical problem and interview. First, the students' mathematical abilities were classified based on their mathematical score in national final examination, the test they took in their final year as senior high school students. Afterwards, the students work on problem solving test. The students' written works were analyzed using four steps of mathematical investigation, i.e. Specializing, Conjecturing, Justifying and Generalizing [4]&[5]. For each step, students' difficulties were highlighted. From the written work analysis, some students were selected to be interviewed further.

The instrument for the test was initially validated using expert judgement. The collected data were analyzed using

mixed method, qualitatively and quantitatively [19]. The quantitative part was done while evaluating students' written work. The qualitative part was also attached in this section and the results were confirmed during the interview.

III. RESULTS AND DISCUSSION

We classified the students' mathematical abilities into three groups: high, moderate and low. From the analysis of students' written work in problem-solving test related to number pattern, it was found that students with high mathematical abilities were able to do all four steps in mathematical investigation.

In Specializing step, the students investigated some specific examples, made drawings and important notes, simplified assumptions and created systematic lists to investigate specific cases. Continued in Conjecturing step, the students created approximation, developed code system, formulated hypothesis, tried to solve the problem and focused only into one problem aspect. It can be seen when the students found the number of squares in a $n \times n$ square, when n equals 1, 2, 3, 4, 5 and continue their conjecture to the n equals 8, without making the drawing of it.

In Justifying step, the students used inductive reasoning to solve the problem and used their conjectures to solve further part of the problems. In the Generalizing step, the students made the general formula by transforming the pattern representation they got previously in Conjecturing step. However, here, the general formula was not in its simplest form. The students also try to check the truth of their generalization by considering the formula with the pictures represent the pattern.

The support of visualization is found as effective tools to enable students in making generalization [20] & [21]. In those studies, the students constructed their conjectures by seeing the structure of the visualized pattern and generalized their ideas by relating the predicted numbers into the template or the general form of the given shape. The following figure is an example of how students used the structure of visualization to create a generalization.

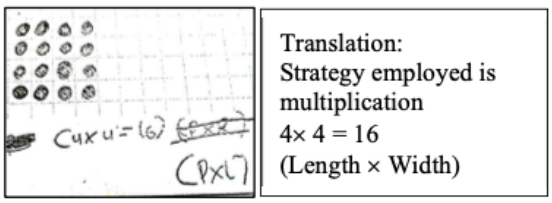

Fig.1. Example of Students' Generalization with the Help of Visual Support [20] (p. 4)

The students with moderate mathematical abilities were able to do the four steps of mathematical investigation as well. In Specializing step, the students created some specific examples, tried some specific examples, made pictures of specific cases, highlighted important notes, simplified assumptions and created a systematical list to examine the particular case. It can be inferred from students' work when they draw the squares and found the number of squares in a 3 × 3, 4 × 4 and 5 × 5 squares.

In Conjecturing step, the students were able to create approximation, develop code system, formulate hypothesis and focus only in one aspect before jump to another part of the problem. Here, the students were able to find the number squares in a 3 × 3, 4 × 4, 5 × 5 squares and able to continue the pattern to find the number of squares in a larger $n \times n$ square without making the drawing.

In Justifying step, the students used representation and inductive reasoning to solve another part of the problem. Here, the students tried to relate the number of unit square in a pattern with what they have if they draw the square as they did in Specializing and Conjecturing step. In Generalizing step, the students tried to create a general formula from the pattern. Here, they can create a formula of the pattern, but not in their simplest form.

The students with low mathematical abilities were able to fulfil three from four steps of mathematical investigation. In the Specializing step, the students check some specific examples, create pictures, make notes, simplify assumptions and create a systematical list to evaluate a particular case. Here, the students were able to determine the number of squares. However, some students made mistakes by only counting the unit squares in the given squares.

In the Conjecturing step, the students were able to create an approximation, but not develop a code system yet. They tried to make hypothesis, tried to solve the problem and focused only in one part of the problem before move to other parts. Here, the students were able to find the number of squares in a 3 × 3, 4 × 4 and 5 × 5 squares. They also try to continue the pattern to get the number of squares in 8 × 8 square without making the drawing. However, the result was incorrect.

In Justifying step, the students used their inductive reasoning to solve the unknown part of the problem. They also evaluated their conjecture to count the number of squares in 8 × 8 square and some of them noticed that they made mistakes, while the rest got the support for their conjectures.

In Generalizing step, the students were not be able to formulate the general formula of the given pattern. During the interview, it was found that they have difficulties to see the connection between the number of squares and the size of the square. This crucial step is not easy. The previous study found that the students did not always succeed in generalization of the pattern, if they did not achieve the level of algebraic thinking yet [22]&[23].

To support the students' mathematical investigation, the educator may consider the use of enacting students in posing questions to enhance students' ability in making conjecture and generalizing their ideas [24]&[25]. Researches showed the use of questions were beneficial to stimulate students in thinking and participating in the classroom.

Furthermore, reflecting to the results of the study, educators need to provide the students with guidance, for instance by using scaffolding [24], [25] & [26]. As the variation of the pattern, the educators also can support the students to investigate simpler patterns, i.e. the one with repeating patterns [22] or growing patterns with constant difference [21], before moving to the growing patterns. The well-ordered patterns will help students to be ready in seeing

the specialization in the patterns that help to create conjecture, justify the correctness of their conjecture and generalize it.

IV. CONCLUSION

The students with high and moderate mathematical abilities were able to do four steps in mathematical investigation, which are Specializing, Conjecturing, Justifying and Generalizing. In doing Generalization, they were able to construct a formula for the pattern which is related to the previous term of the pattern, not establishing the simplest form of the formula yet. Meanwhile, the students with low mathematical abilities were not be able to perform Generalization step and some of them also did mistakes in Specializing and Conjecturing steps.

The results of this study can be used as the feedback for lecturers in higher education to plan and implement lessons that encourage students to improve their mathematical investigation skills. This also contributes to a meaningful starting point, what kind of pattern investigation activities that will be challenging for students in higher education. For further study, the prior knowledge of mathematical abilities should not merely be measured by their scores in national examination.

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