

# The Use of Scaffolding to Enhance Students' Ability in Solving Geometry Problems

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**Abstract**—This study aims to describe the students' difficulties in solving geometry problems and type of scaffolding that will be beneficial to support them. The scaffolding is expected to enhance students' level in solving problem based on SOLO Taxonomy. To achieve the goal, the descriptive design study was employed as the approach of conducting the research. The overall participants were 36 ninth grade of junior high school students in Banyuwangi, Indonesia. One student whose initial level of problem solving was Unistructural (first problem) and Multi-structural (second problem) had chosen to be the subject of this present study. The data were gathered from students' written work, observation and interview. It was analyzed qualitatively using descriptive method. It was found that some difficulties encountered by the subject in solving geometry problems were caused by the lack of conceptual understanding, inability to see mathematical connections and less creativity to find alternative methods. Furthermore, the types of scaffolding that relevant for supporting the student were by explaining (showing and telling), reviewing (looking, touching and verbalizing; parallel modeling; probing and prompting questions; students explaining and justifying) and developing conceptual thinking (making connection). In the end the student's level improved to Relational (first problem) and Extended Abstract (second problem)

**Keywords**—Geometry Problems, Students' Difficulties, SOLO Taxonomy, Scaffolding

## I. INTRODUCTION

Geometry is one of important branches of mathematics. By learning geometry, the students will be able to identify the shape and space around them since basic geometry objects usually occurs or resembles naturally in daily life [1]. It also provides an opportunity for students to develop their mathematical reasoning, such as using deductive and inductive thinking, making and proving conjectures, defining and classifying objects [2].

Not only for solving mathematical related problems, geometry is a great medium to develop problem solving abilities in different branches of knowledge [3]. Mastery in basic geometry will help students to be success in higher mathematics classroom [4]. Therefore, a teacher should pay attention of students' level of understanding in geometry to support them in attaining the optimum results.

The students' level of understanding in geometry can be measured by SOLO Taxonomy. It classifies the students' cognitive development level based on certain topics which is evaluated in students' work in particular task. There are five level in SOLO Taxonomy, presented in hierarchical ascending order, namely Pre-Structural, Unistructural, Multi-Structural, Relational and Extended Abstract [5]. The complete scheme of SOLO Taxonomy can be seen in Fig.1.

Although it is awared that good understanding in geometry will be helpful for students, inreality, the teaching and learning of geometry is not easy [1] and a lot of students faced difficulties on it [6],[7]&[8]. There are a number of studies that address the difficulties in learning geometry, but fewer attempted to figure out how to solve theproblem.

One strategy that will be beneficial to support students in learning is by providing scaffolding. Scaffolding is an effective tools to help students improving their learning quality [9]. It is a support given by the teacher especially for those who encounter difficulties in understanding a concept or solving a problem. Scaffolding usually be given by providing a task which initially exists beyond the students' ability that is adjusted with the students' Zone of Proximal Development ( ZPD) to enable a new skill or knowledge acquired [10].

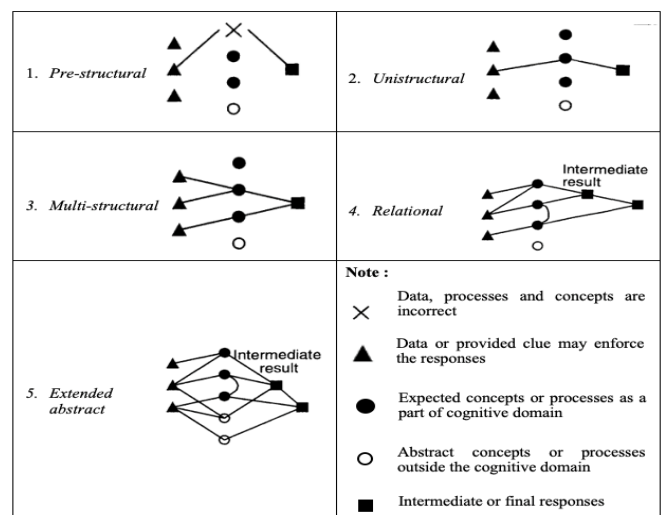


Fig. 1. Scheme of Responses in SOLO Taxonomy [11]

Scaffolding can be given in many types of activities, for instance clue cards, handout, advices or reinforcements, guidance, strategies, questions, stories, story pictures and telling [10]; inquiry and problem based learning [12]; real or authentic tasks [13]; talk and do [14]; and learning approaches [15]. In geometry domain, the students' misconception should be overcome by using scaffolding aimed to transform the abstract concept to be concrete in students' mind. To enable that, the teacher may use learning media [16]. Some types of scaffolding that can be used in geometry classroom are dynamic learning media [17], flow-chart proofs [18], media with conceptual building exploration [19], and GeoGebra [20].

In the previous study, we discussed about the students' general abilities in solving geometry problems based on SOLO Taxonomy [21] and the use of scaffolding that effective in supporting the students in pre-structural level to solve geometry level [22]. The present study reveal further by discussing the students' difficulties and type of appropriate scaffolding for students in unistructural and multi-structural to be enhanced into relational and extended abstract levels based on SOLO Taxonomy.

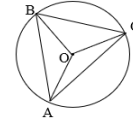
## II. METHODS

The present work is a descriptive study, part of the larger study to identify the students' level of geometry skills using SOLO Taxonomy and scaffolding that may be helpful to enhance their levels. The participants of the study were the ninth-grade students of a junior high school in Banyuwangi, Indonesia, with total of 36 students. For the present discussion, one student with level of Unistructural (in solving first problem) and Multi-structural (in solving second problem) was chosen. There was only one student who has that particular category in the study.

The data were gathered from students' written work, observation and interview. There were three steps in the study. In the first step the student was interviewed to identify the difficulties encountered in solving geometry problems. From the revealed difficulties, scaffoldings were provided based on the appropriate levels [23]. In the second step, the student worked to solve the similar problem (Fig.3). Last, the student was interviewed to understand their scheme of thinking and determine their level of SOLO Taxonomy after received scaffolding. The data from interview were analyzed using transcription, segmentation, coding, classification and conclusion making techniques [24].

The instrument employed in the study was geometry test consists of two problems (see Fig.2 and Fig.3), interview sheet and scaffolding sheet. The first problem was aimed to test the students' ability up to the level of Relational, while the second problem can measure up to Extended Abstract level based on SOLO Taxonomy. All of the instruments used in the study have passed the validation test from experts to ensure that the test is valid to measure the students' ability in geometry.

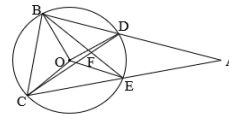
1. Consider the following picture.



Given O is the center of the circle,  $\angle BAC = 54^\circ$  and  $\angle ACO = 20^\circ$ .

- Find the measure of  $\angle BOC$ !
- Find the measure of  $\angle AOB$ !

2. Consider the following picture.

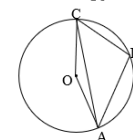


Given O is the center of the circle,  $\angle CFB = 70^\circ$ ,  $\angle CAB = 25^\circ$ , and  $\angle EOC = 120^\circ$ .

- Find the measure of  $\angle COB$  and  $\angle DOE$ !
- Find the measure of  $\angle BCD$ !

Fig. 2. Test Before Scaffolding

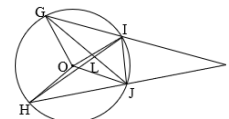
1. Consider the following picture.



Given O is the center of the circle,  $\angle OAC = 20^\circ$  and  $\angle BCA = 35^\circ$ .

- Find the measure of  $\angle AOC$ !
- Find the measure of  $\angle CAB$ !

2. Consider the following picture.



Given O is the center of the circle,  $\angle HLG = 80^\circ$ ,  $\angle HKG = 35^\circ$ , and  $\angle JIH = 52.5^\circ$ .

- Find the measure of  $\angle HOG$  and  $\angle IOJ$ !
- Find the measure of  $\angle GOI$ !

Fig. 3. Test After Scaffolding

## III. RESULT AND DISCUSSION

Based on the test and interview, it was found the subject encountered difficulties in making connection between mathematical ideas. The student understood the concept of central and inscribed angles, but hardly connecting the information given in the problem to choose and make strategy to solve it. The similar result also noticed by Haviger et.al [8].

In addition, the student was not fully understood the concept of exterior and interior angles of circle (Problem 2a). The student also was not able to apply the concept of angles in triangle to find the measure of  $\angle BOC$  in solving the Problem 1a and linear equation with one variable to find the measure of  $\angle CAB$  and  $\angle DOE$  in solving the Problem 2a. (Fig.2.).

The second reason that contributes to the students difficulties in solving geometry problem was misconception [25]&[26]. The misconception in geometry concepts can be distinguished by conceptual dan procedural misconceptions. The first misconception is the inability in identifying and making connection among the concepts while the second is related to the errors in writing symbols, rules and algorithm in solving the problem [16]. Based on the classification, the student was encountered the conceptual misconception.

Another source of student's struggling in solving geometry problem was the inability to find alternatives. The lack of understanding and the inability to make connection among the concepts lead to the dullness in creativity of finding alternative strategies to solve the problem.

After projecting the student's difficulties, the researchers provided them with scaffoldings. The types of scaffolding applied were Level 2: explaining (showing and telling), reviewing (looking, touching and verbalizing; parallel modeling; probing and prompting questions; students explaining and justifying) and Level 3: developing conceptual thinking (making connection). Those scaffoldings, with the addition of looking, touching and verbalizing in the present study, were successfully support the students in solving geometry problems [22]. The following discussion provides brief explanation of the type of scaffoldings.

In Showing and Telling, the researcher controlled the process by providing structured one-direction discussion or only explain how certain strategy works [23]. Therefore, in this part, the researchers merely explain the concepts that was missed by the students, in this case the exterior and interior angles of the circle. This type of scaffolding were used by Baxter and Williams to solve the lack of student's questions during the lesson [10]. On the other hand, Looking, Touching and Verbalizing done by re-examine the problem and refer to the information provided. The student was asked to re-explain what does the problem asked them to do. Afterwards, the student should re-think what kind of strategies that will be appropriate to apply [23].

In Probing and Prompting, questions were used to dig the information of student's knowledge and enhance student's creativity to think more effectively [27]. The probing and prompting question made to enable student in making connection between mathematical ideas. Furthermore, Parallel Modeling done by asking the student to solve similar but simpler problem. It helped the student to cope the problem easier [28].

Explaining and Justifying aimed to enable student to reflect on their own errors in solving the problem by explaining their strategy to others, in this case to the researcher. By hearing student's strategy, one can understand student's understanding, misconception and struggles in certain topic or problem [23][27]. In line with that, asking students to verbally explain what they know will put them as primary agent of knowledge construction for themselves [14].

In addition, Making Connections done by the asking the student to apply the concept of triangle and exterior-interior angles to solve the first problem and linear equation system with two variables to solve the second problem. Making Connections is beneficial in strategies construction to solve the problems [23].

After receiving the scaffolding, the student work to solve the problem in Fig.3. The student was able to solve both problems. For Problem 1a, the student found the measure of  $\angle AOC$  by using the given information of  $\angle ACO$ , the concept of isosceles triangle and the summary of interior angles of triangle  $AOC$ . From those, the student inferred that  $AOC$  is an isosceles triangle since it has two equal sides  $\overline{OA}$

and  $\overline{OC}$  (the radius of circle) and therefore,  $\angle OAC = \angle ACO$ .

In Problem 1b, the subject used  $\angle AOC$  to find its reflection angle,  $\angle COA$ , since there is concept of one full angle in circle. The information from  $\angle AOC$  be used to determine the measure of  $\angle ABC$  by implementing the concept of central and inscribed angles concept. Then, the student employed the angles of  $\angle ABC$  and  $\angle BCA$  to measure the angle of  $\angle CAB$  by using the total interior angles of triangle. The student's scheme responses in solving the first problem can be seen in Fig.4.

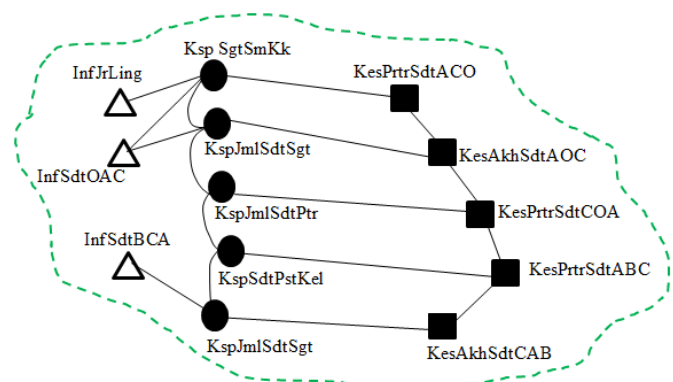


Fig. 4. Scheme Responses for Problem 1

TABLE 1. CODE DESCRIPTION FOR FIG. 4.

No	Code	Note
1	InfJrLing	Information about radius of the circle
2	InfSdtOAC	Information about $\angle OAC$
3	InfSdtBCA	Information about $\angle BCA$
4	Ksp SgtSmKk	The concept of isosceles triangle
5	KspJmlSdtSgt	The concept of the number of angles in a triangle
6	KspJmlSdtPtr	The concept of the number of angles of a full circle
7	KspSdtPstKel	The concept of the relationship between the central angle and the circumferential angle facing the same arc
8	KesPrtrSdtACO	Intermediate conclusions obtained about $\angle ACO$
9	KesAkhSdtAOC	The final conclusions obtained about $\angle AOC$
10	KesPrtrSdtCOA	Intermediate conclusions obtained about $\angle COA$
11	KesPrtrSdtABC	Intermediate conclusions obtained about $\angle ABC$
12	KesAkhSdtCAB	The final conclusions obtained about $\angle CAB$
13	---	Level SOLO Relational

Based on Fig.4, it can be observed that the student was in Relational Level when solving the first problem. The student was able to use all the information provided in the problem and apply all concepts or processes correctly. The student also competent to draw a relevant conclusion

For the Problem 2a, the student figured out the angles of  $\angle HOG$  and  $\angle IOJ$  by using information of angles  $\angle HLG$  and  $\angle HKG$ . Here, the student applied the concept of interior and exterior angles of a circle combined with the concept of linear equation system with two variables. In addition, the subject was able to find alternative method to determine the angle  $\angle HOG$  and  $\angle IOJ$ .

In Problem 2b, the subject applied the information of  $\angle HLG$  and the concept of opposite angles to find  $\angle ILJ$ . Then, the information of  $\angle ILJ$ ,  $\angle JIH$  and the concept of interior angles of a triangle were implemented to find  $\angle GJI$ . Later, it was used to determine the measure of  $\angle GOI$  using the concept of central and inscribed angles concept.

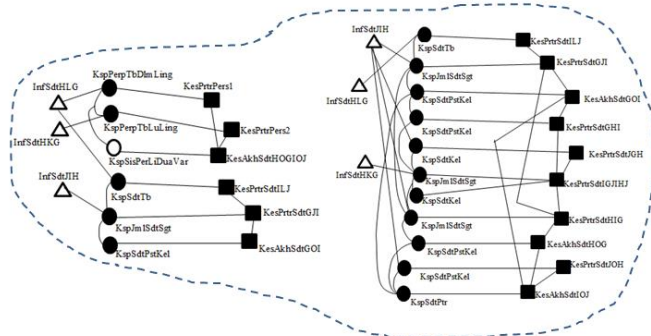


Fig. 5. Scheme Responses for Problem 2

TABLE 2. CODE DESCRIPTION FOR FIG. 5.

No	Code	Note
1	InfSdtHKG	Information about $\angle HKG$
2	InfSdtHLG	Information about $\angle HLG$
3	InfSdtJIH	Information about $\angle BCA$
4	KspPerpTbDlmLing	The concept of the angle of intersection between two chords in a circle
5	KspPerpTbLuLing	The concept of a large angle of intersection between two chords outside the circle
6	KspSisPerLiDuaVar	The concept of SPLDV
7	KspSdtTb	The concept of opposite angle
8	KspSdtKel	The concept of the circumferential angles facing the same arc
9	KesPrtrPers1	Intermediate conclusions obtained about $\angle HOG + \angle IOJ = 160$
10	KesPrtrPers2	Intermediate conclusions obtained about $\angle HOG - \angle IOJ = 70$
11	KesAkhSdtHOGIOJ	The final conclusions obtained about $\angle HOG$ and $\angle IOJ$
12	KesPrtrSdtILJ	The final conclusions obtained about $\angle ILJ$
13	KesPrtrSdtGJI	The final conclusions obtained about $\angle GJI$
14	KesAkhSdtGOI	The final conclusions obtained about $\angle GOI$
15	KesPrtrSdtGHI	Intermediate conclusions obtained about $\angle GHI$
16	KesPrtrSdtJGH	Intermediate conclusions obtained about $\angle JGH$
17	KesPrtrSdtIGJIHJ	Intermediate conclusions obtained about $\angle IGJ$ and $\angle IJH$
18	KesPrtrSdtHIG	Intermediate conclusions obtained about $\angle HIG$
19	KesPrtrSdtJOH	Intermediate conclusions obtained about $\angle JOH$
20	---	Level SOLO <i>Extended Abstract</i>

According to Fig.5., it can be observed that the student was in Extended Abstract level in solving the second problem. The student was able to gather all provided information, employ concepts or processes correctly and draw the conclusion. Also, the student was able to apply relevant concept outside geometry, i.e. linear equation system with two variables, to solve the problem and make generalization.

## IV. CONCLUSION

From this study, it is revealed that the student in Unistructural (first problem) and Multi-structural (second problem) levels encountered difficulties in geometry due to the inability of seeing the connection between mathematical ideas. The other reasons were blamed to the lack of understanding in mathematical concepts, especially interior and exterior angles, and finding the alternative strategies to solve the problems.

After receiving scaffolding, the student' level of solving geometry problem was enhanced to Relational (first problem) and Extended Abstract (second problem). The types of effective scaffolding used were Level 2, including Explaining (showing and telling) and Reviewing (looking, touching and verbalizing, parallel modeling, probing and prompting questions, explaining and justifying) and Level 3: developing conceptual thinking (making connections). Even though the research question was answered, further study in this topic is needed to elaborate more types of scaffolding that will be beneficial in supporting students with different levels of SOLO Taxonomy.

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