

New Network Model of the Project Scheduling Problem With Hybrid Precedence Relations and Maximum Activity Duration

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ABSTRACT

This paper is a systematic study of the network model with hybrid precedence relations and maximum activity duration constraint in the unlimited resource case. Most traditional activity networks involve only four basic precedence relations, which makes it difficult to describe many complex practical situations clearly. Therefore, this paper proposes a new network model which can eliminate cycles in the traditional network and correctly reflect real precedence relations among activities. Activities with the maximum duration constraint are also considered in this paper. Furthermore, an effective time parameter algorithm is developed for new activity networks. To illustrate the usefulness of proposed network model and the effectiveness of its algorithm, a numerical example was solved using the new network algorithm and CPLEX Optimizer respectively. More importantly, the discovery of special activities ignored by traditional activity networks, such as passive activities, may further improve the theory of project scheduling.

Keywords: Activity networks, project scheduling, hybrid precedence relations, maximum time constraints

1. INTRODUCTION

Project scheduling is an important aspect of project management. Reasonable scheduling planning is an important guarantee to reduce cost, improve quality and shorten the project duration. Project scheduling problem (PSP) has been studied more and more widely since 1960s, and critical path method (CPM) was also widely used in PSP [1-3]. As is known to all, CPM is mainly applied in project scheduling under two assumptions. One is that there are sufficient resources and the other is that the precedence relation constraint between activities is only of the finish-to-start type with zero time lags.

In practice, however, resources are often an important constrained condition in project scheduling and it is often necessary to consider other kinds of time constraints such as generalized precedence relations (GPRs) among activities. Kerbosch and Schell [4] made a systematic study of GPRs for the first time. Elmaghraby and Kamburowski [5] developed the structure of GPRs network model that made outstanding contributions in the representation of GPRs. From then on, the network with GPRs has attracted the attention of many researchers and it is well known that there are four basic types of GPRs: Start-to-Start (SS), Start-to-Finish (SF), Finish-to-Start (FS), and Finish-to-Finish (FF). Furthermore, GPRs can be divided into four basic types and combinatorial types (hybrid precedence relations) that are combined by two types of basic GPRs. In practice project management, especially some complex large-scale engineering projects,

the requirements of advanced technology and technical level are constantly improved, which makes precedence relations between activities become more and more complex. Therefore, hybrid precedence relations are very common. For example, when underground concrete pouring works are carried out in areas with high ground water table, it is necessary to take 5 days to lower water table (activity i) before the underground concrete pouring works (activity j) can be started, and after the concrete pouring work is completed, the drainage work still needs to be continued for at least 7 days before it can be finished. This situation can be represented by a constraint “ $SS_{ij}(5) \& FF_{ji}(7)$ ”. However, to the best of our knowledge, traditional project networks currently have a wide range of research on the four basic GPRs, while there is almost no research related to PSP with hybrid precedence relations, especially reverse dual precedence relations. For this problem, PSP with hybrid precedence relations are studied in this paper.

For the uncertain PSP, existing research mainly focuses on activity duration uncertainties [6-8]. For the deterministic PSP, its variants that consider activity preemption, multiple activity execution modes, and multiple objectives have been extensively studied. However, the literature that considers maximum activity duration constraints in the deterministic PSP that is not equal to the minimum duration is quite rare. In the current project progress management research, the maximum or minimum activity duration constraint caused by factors which are related parties, contracts, and the characteristics of the resources is ignored. For example, the bonding process cannot exceed 24 hours, otherwise the adhesive will fail. In other words,

in an actual project, no matter how the resources are allocated, some activities need to be completed in a predetermined time (such as a 24-hour stress test) or cannot exceed a certain time. Elmaghraby and Kamburowski [5] proposed that the project activity has the lower bound of duration and the upper bound of duration. Therefore, the activity duration cannot be extended or shortened indefinitely.

It is well known that the traditional GPRs network model may contain some cycles that make the network graph not concise enough to be understood and it is very difficult to represent the complex relations among activities of project due to the inflexible structure. The representation of GPRs has always been a basic but difficult problem, and many researchers have improved the traditional GPRs network graph [9-10]. Therefore, a systemic new network model which can be used to solve the project scheduling problem with hybrid precedence relations and maximum activity duration (PSP-HPR-MAD) should be created. This improved model is also the main contribution of this paper. The aim is to solve the PSP-HPR-MAD more accurately and quickly. In this research, it is assumed that resources are unlimited and do not affect the time analysis. In light of the research objectives, the rest of this paper is organized as follows. Section 2 introduces the passive activity. Section 3 introduces the PSP-HPR-MAD network model and its algorithms. Section 4 gives a numerical example to detail how the approach proposed works, and the results between network model and CPLEX Optimizer are compared in section 5. Finally, Section 6 provides conclusions and presents direction for future research.

2. PASSIVE ACTIVITY

Generally, the duration of activities is independent of each other, and the duration of different activities does not affect each other. However, when the shortest duration of an activity is determined by resource-independent constraints such as hybrid precedence relations and some constraints may not be considered in the initial estimates, the duration of the activity has nothing to do with itself and can only be changed passively. Therefore, the shortest duration at this time can also be called passive duration. The existence of passive duration proves that not only are the duration of the activity interacting with each other but hybrid precedence relationships can also affect the activity duration. This article refers to activities with passive duration as passive activities. Figure 1 is a typical example.

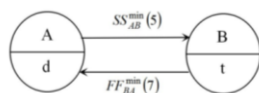


Figure 1 An example of passive activity

The finish time of activity A is not only constrained by the reverse hybrid precedence relation, but also by the duration of activity B. At this point, the shortest duration

of activity A should be " $5+t+7$ ". If activity B is delayed, the duration of activity A will also increase. In the same way, the shortest duration of activity A will also change with time lags of the precedence relations. If the originally estimated duration of activity A is less than " $5+t+7$ ", then the initial plan will not be feasible. As a result, there may be an error in the solution of the project scheduling algorithm. However, traditional GPRs networks do not take into account passive activities on either the representation method or the calculation of time parameters when searching for the optimal network plan, which limits the application scope of the traditional GPRs network and weakens its performance. Fortunately, these special activities are discovered and studied in the new network model of this paper.

3. THE PSP-HPR-MAD MODEL

3.1. Improvement of Representation

In this section, a new representation method of the activity network is formed based on the improvement of the traditional network model. As shown in Figure 2, in the new drawing method, activities are represented by nodes and arcs, and precedence relations are only represented by arcs. The labels under the arc denote the activity duration and the content above the arc indicates the activity name whereas the labels on the precedence relation arc encode time lags. It is worth noting that the minimal time lag (or the lower limit of duration) and the maximal time lag (or the upper limit of duration) between nodes are represented on the traditional GPRs network graph by forward and backward arcs respectively. The positive and negative value express the minimal time lag (or the lower limit on the activity duration) and the maximal time lag (or the upper limit on the activity duration) respectively. According to this representation, many cycles may occur in the network graph. Therefore, in this paper, a new expression that uses the symbol "[,]" and two positive numbers to denote the minimum and maximum time value respectively is proposed to replace the traditional expression. In particular, if the activity duration (or the time lag) is fixed, it is represented only by a positive number. If the minimum time lag between two activities is zero and there is no maximum time lag constraint, then the logical relationship label can be omitted.

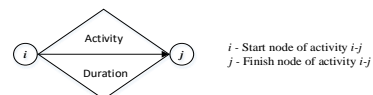


Figure 2 The new representation of an activity

3.2. Time Parameters

The following indices are the main parameters in the new network.

- ET_i : earliest occurrence time of node i .
- LT_i : latest occurrence time of node i .
- RL_{ij} : minimum time lags of the logical relationship or the shortest duration of activity $i-j$.
- NF_i : the float time of node i .
- FF_i : the free float time of node i .
- SF_i : the safety float time of node i refers to the maximum spare time that a node has without affecting the latest occurrence time of all immediate preceding nodes.
- P_j : the predecessor node sets of j .
- S_i : the successor node sets of i .

The calculation formulas of the main time parameters are as follows.

$$ET_j = \max(ET_i + RL_{ij}) \quad i \in P_j \quad (1)$$

$$LT_i = \min(LT_j - RL_{ji}) \quad j \in S_i \quad (2)$$

$$NF_i = LT_i - ET_i \quad (3)$$

$$FF_i = \min(ET_j - RL_{ij} - ET_i) \quad j \in P_i \quad (4)$$

$$SF_j = \min(LT_j - RL_{ij} - LT_i) \quad i \in S_j \quad (5)$$

3.3. Algorithm

The following algorithm defines how to develop a feasible project scheduling scheme and compute the optimal total duration of a network diagram.

Step 1. Define an initial project network with a source node and sink node based on the project information.

Step 2. Use the improved classical forward and backward recursive algorithm to compute the time parameters of each node and exclude infeasible networks. The detailed steps are as follows.

Step 2.1. Calculate the earliest occurrence time of each node recursively forward along the direction of the arc with the minimum time value (minimum activity duration or minimum time lags).

If the maximum time constraint is encountered, then the earliest time of the immediate preceding node needs to be checked and adjusted. The adjustment rule that is computed backwards is

$$ET'_j = \max(ET_j, ET_k - RL_{jk}^{\max}) \quad k \in S_j \quad (6)$$

After the adjustment, the time parameter value of the successor node should be verified again along the arc direction to ensure its correctness.

Step 2.2. Recursively calculate the latest occurrence time of each node backwards along the arc with the minimum time value.

If the maximum time constraint is encountered, then the latest time of the immediate successor node needs to be checked and adjusted, and the adjustment rule that is computed forwards is

$$LT'_k = \min(LT_k, LT_j + RL_{jk}^{\max}) \quad j \in P_k \quad (7)$$

Likewise, the time parameter value of the predecessor node should be verified again.

Step 3. Check the initial network diagram to determine if there are activities with passive duration.

Step 3.1. If there is an activity with passive duration, adjust the initial duration of the activity to the passive duration. After adjustment, determine the optimal duration of each activity. At this time, the length of the longest path on the network is the minimum completion time of the project.

Step 3.2. If there is no activity with passive duration, the earliest occurrence time of the total end node is the minimum completion time of the project.

Step 4. Determine the final feasible and optimal project network.

4. A NUMERICAL EXAMPLE

To explain how the new network model algorithm works, an example is provided. As shown in Figure 3. By applying the algorithm described in the section 3, we obtain the time parameters of each activity node depicted in Figure 4. The time parameter values of nodes 3, 4 and 15 are corrected twice, and the calculation and adjustment process is as follows.

$$ET_3 = 0$$

$$ET'_3 = \max(ET_3, ET_4 - d_A^{\max}) = \max(0, 10 - 5) = 5$$

$$ET_{15} = 25$$

$$ET'_{15} = \max(ET_{15}, ET_{16} - d_H^{\max}) = \max(25, 36 - 6) = 30$$

$$LT_4 = 19$$

$$LT'_4 = \min(LT_4, LT_3 + d_A^{\max}) = \min(19, 6 + 5) = 11$$

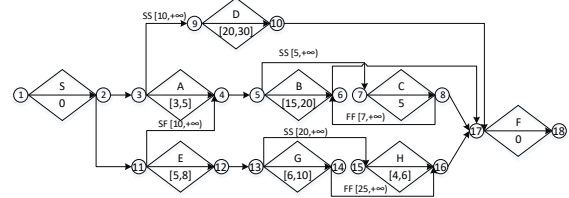


Figure 3 An example of project

According to the calculation results, it can be found that activity B is a passive activity in this project, so its shortest duration needs to be adjusted. Figure 4 is the final adjusted network graph, and the minimum project duration is 36.

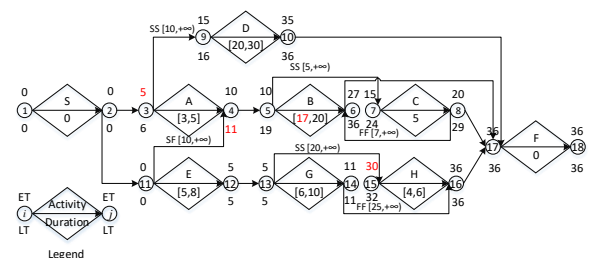


Figure 4 The results obtained by the network model

5. METHOD COMPARISON

In this section, the example project in previous section is also solved by IBM CPLEX 12.9 CP Optimizer. The accuracy of the new network graph algorithm is verified by comparing the consistency of the results. Figure 5 shows the scheduling scheme of the project under optimal results obtained by the CPLEX.

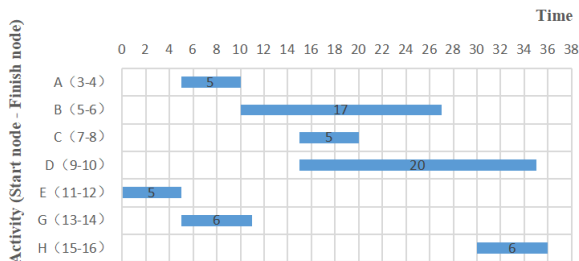


Figure 5 The solution results obtained by the CPLEX

By the results we can observe that the minimum completion time obtained by the CPLEX is exactly the same as the calculation result of the network graph. In particular, the earliest starting time of each node in the optimal scheduling scheme also corresponds to the results in the final network graph in section 4. This confirms that the proposed network graph algorithm is correct and feasible, and the new network model is worth being implemented in PSP-HPR-MAD.

6. CONCLUSION

This paper focuses on the improvement of the traditional network model with GPRs and enriches current theories of planning and scheduling for projects. The network model of PSP-HPR-MAD can not only eliminate positive cycles in the traditional network, but also express the complicated precedence relations more clearly. Therefore, the utility of the network model is enhanced and its application scope is also expanded. The discovery of passive activities indicates that the current network plan optimization theory still has some shortcomings. If this phenomenon is ignored, it will not only directly affect the accuracy of the solution of the PSP, but also cause the unreasonable resource allocation and schedule delay of the actual project. From the comparison of the numerical results solved by the network graph and CPLEX solver respectively, we could clearly see that the new network model and its algorithm could effectively solve the PSP-HPR-MAD. In addition, the new network model makes up for the shortcomings of the traditional network model, so it is more in line with the needs of the actual large-scale projects and has a wider application space. It will help project managers to develop more reasonable scheduling for projects.

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