

Look Same but Different: Misconceptions in Linear Combinations

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Abstract

This article describes misconceptions in a linear combination, answer with the correct conclusion but of workmanship that contains an improper process. In special cases, namely a symmetric matrix, the matrix is formed of scalar multiplication on a linear combination that generates an answer, which is the same as the matrix is formed without performing scalar multiplication. Although obtained the correct final answer, but it contains misconceptions, and seems different in the case of the matrix is not symmetric.

Keywords: misconceptions, combinations, linear, matrix

1. Introduction

Linear Algebra is a subject that has a broad scope of concepts, may include vector, geometry, trigonometry, calculus and so forth. One with the other concepts are interrelated and mutually also underlie. Problems arise when the concepts of the material prerequisites are not interpreted correctly. In previous research [1], [2], Something very fundamental about the concept of representation by symbols, can not be understood by students. As a result, the concept of which has a value of truth can be considered one student simply because of a different symbol or writing sequences of different symbols. The teacher needs to understand students' misconceptions [3]–[5].

Misconceptions in mathematics are a disturbing process of forming a new concept. There are research in mathematics learning's misconceptions [6]–[8]. Misconceptions can not be realized because of the particular case, understanding that contains misconceptions get the correct final answer. The process through which it passes too impressed really. Linear combinations can be concept underlies the concept of linear independence, the vector construct, base, base changes, and the transition matrix [9], Thus a linear

combination into the material prerequisites for the concept of linear independence, the vector construct, base, base changes, and the transition matrix. The inability to understand concepts in a linear combination resulted in the emergence of misconceptions even the wrong concept. Misconceptions as a result of the lack of proper understanding in forming the coefficient matrix, lead to different results [2], On this basis then the researchers wanted to analyze further if the coefficient matrix is formed is symmetric, then it makes no difference coefficient matrix formed by scalar multiplication in the formation of a linear combination, as well as the coefficient matrix formed by without scalar multiplication.

2. Method

Descriptive qualitative research conducted at the University of PGRI Jakarta Indraprasta Campus B Address: Jl. Middle Kingdom No. 80, Ex. Gedong, district. Pasar Rebo, East Jakarta 13760. Selection of descriptive qualitative study, intended to study the data obtained can be analyzed in-depth, namely misconceptions experienced by the subject (5th-semester student) relating to understanding the coefficient matrix

in its application to the topic of linear combinations. Subjects were selected from the 5th-semester mathematics education student who took advanced courses in linear algebra. Snowball sampling in this study, the process of selecting subjects stop having acquired subjects who had misconceptions in a linear combination. One subject chosen this treatment is given a written test and interview form description. Mechanical examination of the validity of data (validity) is carried out after obtaining data from research subjects and by triangulation techniques, the results of the analysis of the subject and the written test results of the analysis of interviews with the subject. The validity of the data presented can ensure that researchers following the actual conditions in this regard misconceptions linear combinations. Data analysis is done through data reduction, data display, verification, and final stage draw conclusions.

3. Result and Discussion

The same matrix form, but is derived from the process and the different mathematical concept is as a result of the properties that form the matrix coefficient:

$$A = A^T, \tag{1}$$

The misconception came from understanding the form of a linear vector system that is perceived as a linear combination of the form completion system of linear equations. If we have already seen that a linear system with equations and variables mn

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases} \tag{2}$$

Can be written in matrix form as, where is the coefficient matrix, is the vector in of variables, and is the vector in of constants. If we use the column vectors of the coefficient matrix, then the matrix equation can be written in the equivalent form

$$A\vec{x} = \vec{b} \quad A \in \mathbb{R}^{m \times n}, \vec{x} \in \mathbb{R}^n, \vec{b} \in \mathbb{R}^m$$

$$x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \dots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \tag{3}$$

This last equation is called the vector form of a linear system. This equation can also be written as

$$x_1A_1 + x_2A_2 + \dots + x_nA_n = \vec{b}, \tag{4}$$

Where denotes the column vector of the matrix. Observe that this equation is consistent Whenever the vector can be written as a linear combination of the column vectors of $AA_i; iA\vec{b}$ [10]. Based on the equation (4), can be obtained by the coefficient matrix A, as a result of the shape. The coefficient matrix A is $A\vec{x} = \vec{b}$.

$$[A_1 \ A_2 \ \dots \ A_n] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \tag{5}$$

In different contexts, the matrix coefficients obtained from a linear combination. In the case of linear freedom to investigate, for example, take the system of linear equations,

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{cases} \tag{6}$$

Then, must be shown

$$k_1(a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n) + k_2(a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n) + \dots + k_n(a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n) = 0$$

With, as only a single settlement.

$$k_1 = k_2 = \dots = k_n = 0 \tag{7}$$

Based on the equation (7), koeffisien matrix can be obtained as:

$$\begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \vdots & \vdots & \dots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix} \tag{8}$$

Clear that the matrix coefficients obtained from equation (4) and equation (7) is not the same. Through this instrument, which is intended to measure a student is the understanding of the linear combination of linear independence in solving the problem.

Based on student work, it can be obtained a description of the initial allegation on how to relate the concept of a linear combination of the linear independence issues. Based on snowball sampling techniques, obtained the written test answer the following subject.

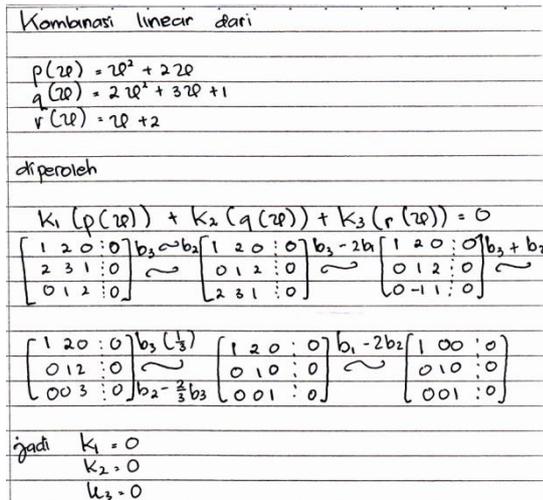


Fig. 1. Results of the subject's answer

The initial findings, researchers suspect, the subject experienced a misconception (fig.1). The misconception that question is the equation (9), (10) and (11) immediately viewed as a system of linear equations zero maker function.

Or,

$$A\vec{x} = \mathbf{0} \quad (13)$$

So, we expect the subject directly form a matrix as the coefficient matrix, namely: A

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Note that if through the steps correctly, equation (12), can be transformed into:

$$k_1(x^2 + 2x) + k_2(2x^2 + 3x + 1) + k_3(x + 2) = 0 \quad (14)$$

In the right concept, conducted prior to the scalar multiplication, and, thus obtained $k_1k_2k_3 (k_1k_2k_3 \in \mathbb{R})$
 $k_1x^2 + 2k_1x + 2k_2x^2 + 3k_2x + k_2 + k_3x + 2k_3 = 0x^2 + 0x + 0 \quad (15)$

So that the linear equation system will look after the grouping value, and, x^2 konstantanya obtained,
 $k_1x^2 + 2k_2x^2 = 0 \quad (16)$

$$2k_1x + 3k_2x + k_3 = 0 \quad (17)$$

$$k_2 + 2k_3 = 0 \quad (18)$$

Based on the equation (16), (17), (18) it is formed the matrix of coefficients of the linear combination is right. The coefficient matrix is named, so that

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Turns out, it appears that,

$$A = B \quad (19)$$

Equation (19) is what is meant in this study as look the same but different. Equation (19) can be formed only in special cases only. A case in point is the coefficient matrix that forms a symmetric matrix, or in the language of mathematics is written as, as mentioned earlier (in equation (1)). $A^T = A$

Based on the initial allegations of researchers through the results of answers to subjects with misconceptions. Location of indication of misconceptions, the subject is too early using the concept of systems of linear equations in formulating solving solutions. so that polynomial in equation (9), (10), (11) directly analogous to a system of linear equations, matrices obtained settlement (Fig. 1)

with elementary row operations as $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

The coefficient matrix that forms a matrix symmetrical thus, workmanship containing misconceptions gives the impression of the correct answer. Answers containing misconceptions (Fig. 1), both forms of the coefficient matrix and the conclusion finally give the correct answer. Researchers provide analysis, based Matrix A and Matrix, a subject that may be stuck in a situation see the formation of a linear combination only as a system of linear equations. Lack of skills in algebra, importance to understand and manage mathematical structures [11] [12].

Subject fooled by the form of a linear combination consisting of polynomials. Although in this case, the subject obtaining the correct conclusion about linear independence, but the process of the formation of the linear combination through a process containing misconceptions. B

The misconception in the process of forming the matrix of coefficients performed by the subject. Coefficient polynomial equation (9), (10), (11) directly formed into rows in the coefficient matrix. Because the coefficient matrix that is formed is symmetric, so that no visible difference, Matrix A together with Matrix B. Misconceptions found in the initial analysis of this subject comes from the work and then deepened further through interviews. Besides deepening the data obtained, this interview is also to clarify on the preliminary findings in the analysis of written answers to the subject. In a later interview subjects script labeled S, and researchers labeled P.

The main concept, which asked the researcher,

P: " They ask what the matter? "

S: " Determine linear independence, looking for a linear combination, so that the result is zero, and it also should be zero Pak all three. "k

Based on the interview script, the subject has understood the main issues to be determined solution. The main concept for solving the problems has been ruled by the subject well. Then go into the concept of a linear combination,

P "Linear combination, This is based on your answers, which one? "

S: '(while pointing his job) "

$$k_1 (p(x)) + k_2 (q(x)) + k_3 (r(x)) = 0$$

P: " Matrix in elementary row operations, it came from? "

S: "Yes, right Systems of Linear Equations (SPL) Sir, yes stay formed line of each equation, sir. "

P "Instead there is a multiplier?"

S "Just wrote Mr result (reworked), The results

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}, \text{ Just right sir "}$$

Based on further deepening of the concept of linear combination, the polynomial form is already a direct system of linear equations that can be used in the coefficient matrix. This is why the subjects had misconceptions. Analogous polynomial form directly as a system of linear equations, making processes in the kingdom subjects had misconceptions.

Then the researchers reassuring findings of the onset of misconceptions by providing other forms of problems. The shape of the problems that do not contain a symmetric matrix. These problems are,

Are the following polynomials are linearly independent?

$$a(x) = x^2 + 3x + 1 \tag{20}$$

$$b(x) = 3x^2 + 2x + 2 \tag{21}$$

$$c(x) = 5x + 3 \tag{22}$$

The subject then provides answers shown in Figure 2.

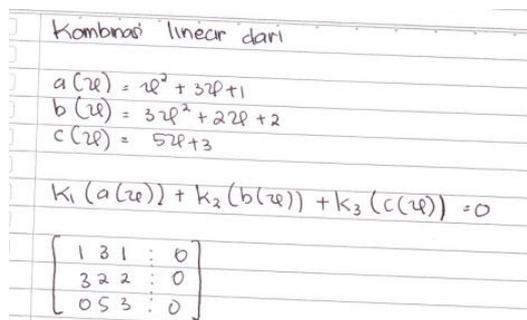


Fig. 2. Results of Additional Questions Answers by Subject

Based on Figure 1, the coefficient matrix is obtained subject. And if through the correct process coefficient

obtained matrix. $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \\ 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 3 & 2 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

The characteristics of the unique symmetric matrix make it easy to find the solution of systems of linear equations. Complex calculations can be simple by first forming a symmetric matrix via elementary row operations [6].

The conceptual understanding must be an important goal for the algebra course [14][15]. Based on the analysis of the results of the written work of the subject and the results of analysis of interviews with the subject, misconceptions experienced by subjects in a linear combination of material, ie in the process of forming the coefficient matrix, the subject immediately assume that the coefficients of each polynomial is known as the row vectors in the matrix coefficients, regardless of the scalar's multiplier. This indicates the subject to the start of implementing the settlement systems of linear equations, which should be multiplied first by the scalar.

4. Conclusion

Misconceptions experienced by subjects in a linear combination of material, ie in the process of forming the coefficient matrix, the subject immediately assumes that the coefficients of each polynomial are known as the row vectors in the matrix coefficients, regardless of the scalar's multiplier. In special cases, coefficients matrix in the form of the asymmetric matrix, no visible difference, look the same, even produce the correct conclusion, but the actual process through which contain misconceptions.

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