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Research Article

Novel Cross-Entropy Based on Multi-attribute Group Decision-Making with Unknown Experts' Weights Under Interval-Valued Intuitionistic Fuzzy Environment

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ABSTRACT

This paper studies the multi-attribute group decision-making problems with unknown experts' weights under interval-valued intuitionistic fuzzy environment. First, in order to provide more flexibilities for decision-makers in actual decision-making problems, a novel cross-entropy measure with parameter of interval-valued intuitionistic fuzzy set (IVIFS) based on J-divergence is proposed. The novel cross-entropy measure can obtain more flexible and practical optimal ranking results by adjusting the parameter. Then, by using the designed cross-entropy measure, two models are established to obtain experts' weights, which consider the influence of experts' experience and professional knowledge on experts' weights. Finally, two examples are provided to illustrate the effectiveness and applicability of optimizing the group decision-making approach.

1. INTRODUCTION

Decision-making is generally considered a process in which human beings make choices among several alternatives [1]. In real life, due to the increasingly complex socioeconomic environment, it is impossible for a single decision-maker (DM) or expert to consider all relevant aspects of the decision-making problem without difficulty [2–6]. Therefore, many practical decisions are often made by multiple DMs or experts, which leads to abundant research concerning the topic of multi-attribute group decision-making (MAGDM) problems.

In MAGDM problems, DMs or experts should provide their preferences for alternative attributes to achieve a collective decision. Because of the uncertainty of the problem and the fuzziness of human thinking, it is difficult for DMs to evaluate alternatives with real numbers. There exist some hesitation and uncertainty inherent in DMs' judgments. Then, the intuitionistic fuzzy set (IFS) is defined in [7], which is just a strong tool to deal with hesitation and uncertainty [8]. Then the interval-valued intuitionistic fuzzy set (IVIFS) is proposed [9]. Since the membership and nonmembership of IVIF are described by intervals, IVIFS is better than IFS in representing fuzziness and uncertainty [10]. Since its appearance in the literature, the IVIFS and its extension theory has attracted increasing attention, many fuzzy MAGDM approaches have been presented [11–14].

In addition, how to obtain comprehensive weights of experts in MAGDM problems under IVIF environment is also the focus of many scholars. For example, a nonlinear optimization model is adopted [15], which minimizes the differences between individual and group opinions. Soon after, a method to derive experts' weights based on the distance between each matrix and the average matrix decision matrix is introduced [16]. Then, another method which depends on the difference between each matrix and the ideal group decision matrix is proposed [17]. However, the above methods for obtaining experts' weights do not consider the experience and expertise of experts. The experts' experience and the richness of professional knowledge determine the accuracy and reliability of experts' weights, and finally determine the feasibility of the sequencing scheme.

Since the entropy in the theory of information and developed the cross-entropy is introduced in [18], the fuzzy cross-entropy is defined to evaluate the relation between two sets or objects [19]. It is used to measure the divergence between two probability distributions or two random variables. After that, in [20], an effective application of the fuzzy cross-entropy MAGDM problems is showed. In [21], an integration of the cross-entropy under IF environments to solve MAGDM problems is proposed. A novel cross-entropy measure under IVIF environments to solve MAGDM problems is presented in [22]. An extended fuzzy cross-entropy measure of belief values based on a belief degree using available evidence is proposed

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to decision-making problems [23], and so on [24,25]. However, the above cross-entropy measures do not consider the psychology of DMs. The reliability and accuracy of decision results can be improved by describing decision psychology of DMs in decision-making methods [26].

Motivated by all mentioned above, in this paper, MAGDM problems with unknown experts' weights under IVIF environments are studied. The main contributions of this paper can be summarized as follows: (i) A novel cross-entropy measure with a free parameter of IVIFS is proposed based on J-divergence. Compared with the previous works [27,28], the proposed cross-entropy measure with parameter of IVIFS may provide more opportunities for DMs in actual decision-making, and it is more flexible and practical in order to obtain the optimal ranking results by adjusting the parameter. (ii) A new method in this paper is presented to obtain experts' weights, in which two programming models are constructed by considering the influence of experts' experience and professional knowledge on experts' weights.

The rest of the paper is organized as follows: Section 2 presents the preliminary concepts of IFS and IVIFS. In Section 3, the unknown expert weights are computed. Section 4 defines new cross-entropy measure under IVIF environments and studies its properties. In Section 6, an approach integrated previously proposed methods for MAGDM under IVIF environments is developed to select an optimal scheme. Two examples are given to illustrate the effectiveness and applicability of proposed methods in Section 6. Finally, Section 7 gives conclusion.

2. BASIC CONCEPTS OF IFS AND IVIFS

To describe the fuzzy nature of things more detailed and comprehensive, Atanassov initiated the concept of IFSs by extending FSs of Zadeh. This section is devoted to reviewing some basic notions of IFSs and IVIFSs.

Definition 1. [7] Suppose that $X = \{x_1, x_2, \cdots, x_n\}$ is a fixed set. An IFS A on X can be defined as

$$A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) | x_i \in X\}$$

where the function $\mu_A(x_i) \in [0,1]$ and $\nu_A(x_i)) \in [0,1]$ represent the membership degree and nonmembership degree, respectively. $\pi_A(x_i) \in [0,1]$ satisfying $\pi_A(x_i) + \mu_A(x_i) + \nu_A(x_i) = 1$ means hesitation degree. Especially, if $\mu_A(x_i) = \nu_A(x_i)$ holds for any $i = 1, 2, \cdots, n$, the given IFS A is degraded to an ordinary FS.

In 1989, Atanassov and Gargov introduced IVIFS as a further generalization of IFS and gave the following definition.

Definition 2. [9] Let $X = \{x_1, x_2, \ldots, x_n\}$ be fixed, an IVIFS A in X is defined by

$$\begin{aligned} A &= \{< x_i, \mu_A(x_i), \nu_A(x_i) > | x_i \in X\} \\ &= \{< x_i, [\mu_A^L(x_i), \mu_A^U(x_i)], [\nu_A^L(x_i), \nu_A^U(x_i)] > | x_i \in X\} \end{aligned}$$

where $0 \leqslant \mu_A^L(x_i) \leqslant \mu_A^U(x_i) \leqslant 1$, $0 \leqslant \nu_A^L(x_i) \leqslant \nu_A^U(x_i) \leqslant 1$, $\mu_A^U(x_i) + \nu_A^U(x_i) \leqslant 1$ for all $x_i \in X$.

Similarly, $\mu_A(x_i)$ and $\nu_A(x_i)$ represent the membership and nonmembership degree of an element to an IVIFS. Then corresponding interval-valued hesitation degree related to A can be computed as follows: $\pi_A = [\pi_A^L(x_i), \pi_A^U(x_i)] = [1 - \mu_A^U(x_i) - \nu_A^U(x_i), 1 - \mu_A^L(x_i) - \nu_A^L(x_i)]$.

Another two important concepts for IVIFSs are deviation degree and cross-entropy measure which have been applied to many fields.

Definition 3. [29] Deviation degree $g(a, b)$ between any two interval-valued intuitionistic fuzzy numbers (IVIFNs) $a = ([\mu_a^L, \mu_a^U], [\nu_a^L, \nu_a^U])$ and $b = ([\mu_b^L, \mu_b^U], [\nu_b^L, \nu_b^U])$ is defined as follows:

$$\begin{aligned} g(a, b) = &|\mu_a^L - \mu_b^L| + |\mu_a^U - \mu_b^U| \\ &+ |\nu_a^L - \nu_b^L| + |\mu_a^U + \nu_b^U - \mu_b^U - \nu_b^U| \\ &+ |\nu_a^U - \nu_b^U| + |\mu_a^L + \nu_b^L - \mu_b^L - \nu_b^L|, \end{aligned}$$

that is,

$$\begin{aligned} g(a, b) = &|\mu_a^L - \mu_b^L| + |\mu_a^U - \mu_b^U| + |\nu_a^L - \nu_b^L| \\ &+ |\nu_a^U - \nu_b^U| + |\pi_a^L - \pi_b^L| + |\pi_a^U - \pi_b^U|. \end{aligned}$$

Definition 4. [20] For two IVIFNs A and B, $CD^\gamma(A, B)$ with parameter γ is a cross-entropy measure, which should satisfy the following conditions:

(i) For any $\gamma \geqslant 1$, $0 \leqslant CD^\gamma(A, B) \leqslant 1$.

(ii) $CD^\gamma(A, B) = CD^\gamma(A, B)$.

(iii) For IVIFNs A, B and C, if $A \leqslant B \leqslant C$ then $CD^\gamma(A, B) \leqslant CD^\gamma(A, C)$, $CD^\gamma(B, C) \leqslant CD^\gamma(A, C)$.

Cross-entropy measure is often used to measure the discrimination information, in the light of Shannon's inequality in [30].

Definition 5. [9] Supposing α, α_1, and α_2 are three IVIFNs, then

$$\begin{aligned} (1)\quad \alpha_1 \oplus \alpha_2 = &([\mu_{\alpha_1}^L + \mu_{\alpha_2}^L - \mu_{\alpha_1}^L \mu_{\alpha_2}^L, \mu_{\alpha_1}^U + \mu_{\alpha_2}^U \\ &- \mu_{\alpha_1}^U \times \mu_{\alpha_2}^U], [\upsilon_{\alpha_1}^L \upsilon_{\alpha_2}^L, \upsilon_{\alpha_1}^U \upsilon_{\alpha_2}^U]) \\ (2)\quad \alpha_1 \otimes \alpha_2 = &([\mu_{\alpha_1}^L \mu_{\alpha_2}^L, \mu_{\alpha_1}^U \mu_{\alpha_2}^U], [\upsilon_{\alpha_1}^L + \upsilon_{\alpha_2}^L \\ &- \upsilon_{\alpha_1}^L \upsilon_{\alpha_2}^L, \upsilon_{\alpha_1}^U + \upsilon_{\alpha_2}^U - \upsilon_{\alpha_1}^U \upsilon_{\alpha_2}^U]) \\ (3)\quad \lambda\alpha = &([1 - (1 - \mu_\alpha^L)^\lambda, 1 - (1 - \mu_\alpha^U)^\lambda], \\ &[(\mu_\alpha^L)^\lambda, (\mu_\alpha^U)^\lambda]) \\ (4)\quad \lambda\alpha = &([(\mu_\alpha^L)^\lambda, (\mu_\alpha^U)^\lambda], [1 - (1 - \mu_\alpha^L)^\lambda, \\ &1 - (1 - \mu_\alpha^U)^\lambda]) \end{aligned}$$

In order to compare two IVIFNs α_1 and α_2, based on the concepts of score function $s(\alpha)$ and accuracy function $h(\alpha)$, an approach is proposed in [31], where $s(\alpha) = \frac{1}{2}(\mu_\alpha^L + \mu_\alpha^U - \nu_\alpha^L - \nu_\alpha^U)$ and $h(\alpha) = \frac{1}{2}(\mu_\alpha^L + \mu_\alpha^U + \nu_\alpha^L + \nu_\alpha^U)$.

If $s(\alpha_1) < s(\alpha_2)$, then $\alpha_1 < \alpha_2$.

If $s(\alpha_1) = s(\alpha_2)$, then (i) if $h(\alpha_1) = h(\alpha_2)$ then $\alpha_1 = \alpha_2$; (ii) if $h(\alpha_1) < h(\alpha_2)$, then $\alpha_1 < \alpha_2$; (iii) if $h(\alpha_1) > h(\alpha_2)$, then $\alpha_1 > \alpha_2$.

Definition 6. [32] An n dimension interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operator: $R^n \to R$ is a mapping, in which has a weighting vector $w = (w_1, w_2, \cdots, w_n)$, satisfying $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, then

$$\begin{aligned} IVIFWA(\alpha_1, \dots, \alpha_n) &= w_1\alpha_1 \oplus w_2\alpha_2 \oplus \cdots \oplus w_n\alpha_n \\ &= \left(\left[1 - \prod_{i=1}^{n}(1-\mu_{\alpha_i}^L)^{w_i}, 1 - \prod_{i=1}^{n}(1-\mu_{\alpha_i}^U)^{w_i} \right], \right. \\ &\left. \left[\prod_{i=1}^{n}(\nu_{\alpha_i}^L)^{w_i}, \prod_{i=1}^{n}(\nu_{\alpha_i}^U)^{w_i} \right] \right). \end{aligned}$$

3. A MODEL-BASED METHOD TO DETERMINE THE WEIGHTS OF EXPERTS UNDER IVIF ENVIRONMENTS

Because the IVIFS can give a more reasonable mathematical skeleton to process inaccurate facts or imperfect information, in this section, we develop a comprehensive algorithm to integrate two models for obtaining appropriate experts' weights. Each expert's weight in our method is combined with two parts, the experience and the expertise of expert.

3.1. The Weighting Vector of Expert Determined by Experience Degree of Expert

Suppose that $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \cdots, \lambda^{(t)})$ is the expert's weighting vector. From Definition 2.3, it is obvious to observe that $([0,0],[1,1])$ and $([1,1],[0,0])$ are the smallest and the largest IVIFNs, respectively. Therefore, we define that $R^L = (r_{ij}^L)_{n\times m} = ([0,0],[1,1])_{n\times m}$ as the lowest experience of expert decision matrix, and $R^U = (r_{ij}^U)_{n\times m} = ([1,1],[0,0])_{n\times m}$ as the highest experience of expert's decision matrix. In view of that a certain IVIFN has more experience when it has a larger divergence from $([1,1],[0,0])$ or $([0,0],[1,1])$. So we can devise relative divergence measure as the expression $\sum_{i=1}^{n}\sum_{j=1}^{m}|CE^{\gamma}(r_{ij}^{(k)}, r^L) - CE^{\gamma}(r_{ij}^{(k)}, r^U)|$ to represent experience of an individual IVIF decision matrix. Obviously, the expert who gives the individual decision matrix with higher experience should be assigned a bigger weight.

By considering the experience of expert to compute the optimal expert's weighting vector, it can be formed as the following model:

$$(M-1)\begin{cases} max \quad F(\lambda_1) = \dfrac{1}{mn}\sum_{k=1}^{t}\sum_{i=1}^{n}\sum_{j=1}^{m}\lambda_1^{(k)} \\ \quad \times \left|CE^{\gamma}\left(r_{ij}^{(k)}, r^L\right) - CE^{\gamma}\left(r_{ij}^{(k)}, r^U\right)\right| \\ s.t. \ \sum_{k=1}^{t}\left(\lambda_1^{(k)}\right)^2 = 1, \ \lambda_1^{(k)} \geqslant 0, \ k = 1, \cdots, t. \end{cases}$$

where $\lambda_1^{(k)}$ is weight of the kth expert.

To solve model (M-1), the Lagrange function is constructed as

$$L(\lambda_1, \zeta) = \frac{1}{mn}\sum_{k=1}^{t}\sum_{i=1}^{n}\sum_{j=1}^{m}\lambda_1^{(k)}|CE^{\gamma}(r_{ij}^{(k)}, r^L) - CE^{\gamma}(r_{ij}^{(k)}, r^U)| + \frac{1}{2}\zeta\left(\sum_{k=1}^{t}(\lambda_1^{(k)})^2 - 1\right) \tag{1}$$

where ζ is the Lagrange multiplier.

By differentiating Eq. (1) with respect to $\lambda_1^{(k)}$ $(k = 1, 2, \cdots, t)$ and ζ, we set these partial derivatives equal to zero, then the following equations are obtained:

$$\begin{cases} \dfrac{\partial L}{\partial \lambda_1^{(1)}} = \dfrac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|CE^{\gamma}\left(r_{ij}^{(1)}, r^L\right) - CE^{\gamma}\left(r_{ij}^{(1)}, r^U\right)\right| + \zeta\lambda_1^{(1)} = 0 \\ \quad \vdots \\ \dfrac{\partial L}{\partial \lambda_1^{(t)}} = \dfrac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|CE^{\gamma}\left(r_{ij}^{(t)}, r^L\right) - CE^{\gamma}\left(r_{ij}^{(t)}, r^U\right)\right| + \zeta\lambda_1^{(t)} = 0 \\ \dfrac{\partial L}{\partial \zeta} = \dfrac{1}{2}\sum_{k=1}^{t}\left(\lambda_1^{(k)}\right)^2 - \dfrac{1}{2} = 0 \end{cases}$$

To solve above equations, we can get a simple and exact formula for determining the weights of experts as follows:

$$\lambda_1^{(k)} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left|CE^{\gamma}\left(r_{ij}^{(k)}, r^L\right) - CE^{\gamma}\left(r_{ij}^{(k)}, r^U\right)\right|}{\sqrt{\sum_{k=1}^{t}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\left|CE^{\gamma}\left(r_{ij}^{(k)}, r^L\right) - CE^{\gamma}\left(r_{ij}^{(k)}, r^U\right)\right|\right)^2}}$$

By standardizing $\lambda_1^{(k)} (k = 1, 2, \cdots, t)$ within the $[0,1]$, we have

$$\lambda_1^{*(k)} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left|CE^{\gamma}\left(r_{ij}^{(k)}, r^L\right) - CE^{\gamma}\left(r_{ij}^{(k)}, r^U\right)\right|}{\sum_{k=1}^{t}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|CE^{\gamma}\left(r_{ij}^{(k)}, r^L\right) - CE^{\gamma}\left(r_{ij}^{(k)}, r^U\right)\right|} \tag{2}$$

Therefore, we can get the expert's weighting vector $\lambda_1^* = (\lambda_1^{*(1)}, \lambda_1^{*(2)}, \cdots, \lambda_1^{*(t)})$.

Further, from the viewpoint of similarity degree between pairwise individual decision matrices in [33] and [34], another optimization model based on the cross-entropy and the similarity measures for ensuring weights of experts can be constructed here.

3.2. The Weighting Vector of Expert Determined by Expertise of Expert

Let $R^{(k)} = (r_{ij}^{(k)})_{n\times m}$ $(k = 1, 2, \cdots, t)$ be IVIF decision matrices provided by experts $e^{(k)}$ on evaluating alternatives g_i $(i = 1, 2, \cdots, n)$, where $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)}) = ([\mu_{ij}^{L(k)}, \mu_{ij}^{U(k)}], [\nu_{ij}^{L(k)}, \nu_{ij}^{U(k)}])$ is IVIFNs. Here, $[\mu_{ij}^{L(k)}, \mu_{ij}^{U(k)}]$ indicates the alternatives g_i satisfy the attributes G_j $(j = 1, 2, \cdots, m)$, while $[\nu_{ij}^{L(k)}, \nu_{ij}^{U(k)}]$ indicates the alternatives g_i do not satisfy the attributes G_j and $e^{(k)}$ $(k = 1, 2, \cdots, t)$ represent the kth expert. Then, a similarity measure with free parameter between $r_{ij}^{(k)}$ and $m_{ij} = [\mu_{ij}^{L'}, \mu_{ij}^{U'}], [\nu_{ij}^{L'}, \nu_{ij}^{U'}]$ is defined as [29]

$$S\left(r_{ij}^{(k)}, m_{ij}\right) = \begin{cases} 1 - \dfrac{CD^{\gamma}\left(r_{ij}^{(k)}, m_{ij}\right)}{\sum_{k=1}^{t} CD^{\gamma}\left(r_{ij}^{(k)}, m_{ij}\right)}, & if\, r_{ij}^{(k)} \neq m_{ij}, \\ 1, & if\, r_{ij}^{(k)} = m_{ij}, \end{cases}$$

where $\mu_{ij}^{L'} = \frac{1}{t}\sum_{k=1}^{t}\mu_{ij}^{L(k)}$, $\mu_{ij}^{U'} = \frac{1}{t}\sum_{k=1}^{t}\mu_{ij}^{U(k)}$, $\nu_{ij}^{L'} = \frac{1}{t}\sum_{k=1}^{t}\nu_{ij}^{L(k)}$, $\nu_{ij}^{U'} = \frac{1}{t}\sum_{k=1}^{t}\nu_{ij}^{U(k)}$ and $k = 1, 2, \cdots, t; i = 1, 2, \cdots, n; j = 1, 2, \cdots, m$.

Note that the closer the value of $r_{ij}^{(k)}$ to the mean rating m_{ij} is, the higher the similarity degree is. Consequently, the experts' weights are also higher. All in all, it means that the more expertise of an expert has, the more the weight of his judgment get. This can avoid the unduly high or low evaluation values induced by experts because of limited expertise.

Accordingly, for obtaining the expert's weighting vector, we concern similarity degree between individual decision matrices and expert evaluation matrices.

$$(M-2)\begin{cases} max\, F(\lambda_2) = \sum_{k=1}^{t}\sum_{i=1}^{n}\sum_{j=1}^{m}\dfrac{S\left(r_{ij}^{(k)}, m_{ij}\right)\lambda_2^{(k)}}{mn} \\ s.t.\ \sum_{k=1}^{t}\left(\lambda_2^{(k)}\right)^2 = 1, \lambda_2^{(k)} \geqslant 0, k = 1, \ldots, t \end{cases}$$

where $\lambda_2^{(k)}$ is weight of the kth expert. To solve model (M-2), the following Lagrange function has been constructed:

$$L\left(\lambda_2, \zeta\right) = \frac{1}{mn}\sum_{k=1}^{t}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}S\left(r_{ij}^{(k)}, m_{ij}\right)\right)\lambda_2^{(k)} + \frac{1}{2}\zeta\left(\sum_{k=1}^{t}\left(\lambda_2^{(k)}\right)^2 - 1\right) \tag{3}$$

where ζ is the Lagrange multiplier.

For differentiating Eq. (3) with respect to $\lambda_2^{(k)}$ $(k = 1, 2, \cdots, t)$ and ζ, these partial derivatives are set equal to zero, the following equations are obtained:

$$\begin{cases} \dfrac{\partial L}{\partial \lambda_2^{(1)}} = \dfrac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}S\left(r_{ij}^{(1)}, m_{ij}\right) + \zeta\lambda_2^{(1)} = 0 \\ \vdots \\ \dfrac{\partial L}{\partial \lambda_2^{(t)}} = \dfrac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}S\left(r_{ij}^{(t)}, m_{ij}\right) + \zeta\lambda_2^{(t)} = 0 \\ \dfrac{\partial L}{\partial \zeta} = \dfrac{1}{2}\sum_{k=1}^{t}\left(\lambda_2^{(k)}\right)^2 - \dfrac{1}{2} = 0 \end{cases}$$

By solving above equations, a simple and exact formula for determining the weights of experts can be get as follows:

$$\lambda_2^{(k)} = \frac{\frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}S(r_{ij}^{(k)}, m_{ij})}{\sqrt{\sum_{k=1}^{t}\left(\frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}S(r_{ij}^{(k)}, m_{ij})\right)^2}}$$

By normalizing $\lambda_2^{(k)}, (k = 1, 2, \cdots, t)$ within $[0, 1]$, we can have

$$\lambda_2^{*(k)} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}S\left(r_{ij}^{(k)}, m_{ij}\right)}{\sum_{k=1}^{t}\sum_{i=1}^{n}\sum_{j=1}^{m}S\left(r_{ij}^{(k)}, m_{ij}\right)} \tag{4}$$

Then we can get the weighting vector of expert that is $\lambda_2^* = (\lambda_2^{*(1)}, \lambda_2^{*(2)}, \cdots, \lambda_2^{*(t)})$.

In practice, models (M-1) and (M-2) can be integrated for computing experts' weights in MAGDM under IVIF environment. The steps are summarized as follows:

Step 1. Calculate the fuzziness degree between individual decision matrix $R^{(}k)$) of kth and the lowest/highest experience decision matrix, then get the weighting vector of expert, $\lambda_1^* = (\lambda_1^{*(1)}, \lambda_1^{*(2)}, \cdots, \lambda_1^{*(t)})$ by using Eq. (2).

Step 2. Compute the similarity degree between each individual decision matrix and matrices of expert evaluation, then get weighting vector of expert, $\lambda_2^* = (\lambda_2^{*(1)}, \lambda_2^{*(2)}, \cdots, \lambda_2^{*(t)})$ through Eq. (4).

Step 3. Let $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(t)})$ be the ultimate weighting vector of expert, which can be comprehensively determined as follows:

$$\lambda^{(k)} = \frac{\phi}{\varphi + \phi}\lambda_1^{*(k)} + \frac{\varphi}{\varphi + \phi}\lambda_2^{*(k)}$$

where φ and ϕ are the parameters that can reflect the attitudinal characteristics of experts, $0 \leqslant \varphi, \phi \leqslant 1, \phi + \varphi = 1$. Normally, φ and ϕ are set to 0.5.

4. CROSS-ENTROPY MEASURE OF IVIFS

In this section, a new cross-entropy measure with parameter of IVIFS is constructed to support the proposed expert weight-determining method. We consider A and B of various fuzzy sets defined in one-element universal set $X \in \{x_1\}$. Based on the [35,36], we can obtain the J-divergence of IFS A and B as follows:

$$J_\gamma(A,B) = \frac{-1}{\gamma-1}\left\{\left(\frac{\mu_A+\mu_B}{2}\right)^\gamma - \frac{1}{2}\left(\mu_A^\gamma+\mu_B^\gamma\right) + \left(\frac{\nu_A+\nu_B}{2}\right)^\gamma - \frac{1}{2}\left(\nu_A^\gamma+\nu_B^\gamma\right) + \left(\frac{\pi_A+\pi_B}{2}\right)^\gamma - \frac{1}{2}\left(\pi_A^\gamma+\pi_B^\gamma\right)\right\}$$

where $\gamma \in (1,2]$.

By analogy information and based on the above, an information measure which is a divergence measure of two IVIFSs A and B can be defined as

$$CD^\gamma(A,B) = -\frac{1}{\gamma-1}\left[\left(\frac{m_A+m_B}{2}\right)^\gamma - \frac{1}{2}\left(m_A^\gamma+m_B^\gamma\right)\right] \quad (5)$$

where $\gamma \in (1,2]$, $m_A = \frac{\mu_A^L+\mu_A^U+2-\nu_A^L-\nu_A^U}{4} = \frac{2\mu_A^L+2\mu_A^U+2+\pi_A^L+\pi_A^U}{4}$, $m_B = \frac{\mu_B^L+\mu_B^U+2-\nu_B^L-\nu_B^U}{4} = \frac{2\mu_B^L+2\mu_B^U+2+\pi_B^L+\pi_B^U}{4}$.

Theorem 1. *The divergence measure $CD^\gamma(A,B)$ ($\gamma \in (1,2]$) satisfies the conditions of Definition 2.4.*

Proof: Obviously, $CD^\gamma(A,B) = CD^\gamma(B,A)$, by Jensen's inequality, we have $CD^\gamma(A,B) \geqslant 0$. And we can obtain $CD^\gamma(A,B)$ is convex if $\gamma \in (1,2]$. The proof is provided in Appendix. Thus, for all $\gamma \in (1,2]$, $CD^\gamma(A,B)$ increases as $\| A-B \|_1$ increases, where $\| A-B \|_1 = g(A,B)$. Then, the $CD^\gamma(A,B)$ attains its maximum at $A = ([0,1],[0,0],[0,0])$, $B = ([0,0],[0,1],[0,0]$ or $A = ([0,0],[0,1],[0,0])$, $B = ([0,0],[0,0],[0,1])$ or $A = ([0,1],[0,0],[0,0])$, $B = ([0,0],[0,0],[0,1])$. This means that the $CD^\gamma(A,B)$ attains its maximum at $m_A = 0.75$ and $m_B = 0.25$. Thus, in order to prove $CD^\gamma(A,B) < 1$ at $m_A = 0.75$ and $m_B = 0.25$, an auxiliary function is defined as

$$f(\gamma) = \gamma - 1 + \left(\frac{1}{2}\right)^\gamma - \frac{1}{2}\left[\left(\frac{3}{4}\right)^\gamma + \left(\frac{1}{4}\right)^\gamma\right], \gamma \in (1,2].$$

Then, we have for all $\gamma \in (1,2]$,

$$f'(\gamma) = 1 - \frac{ln2}{2}\left(\frac{3}{4}\right)^\gamma + ln2\left[\left(\frac{3}{4}\right)^\gamma + \left(\frac{1}{4}\right)^\gamma - \left(\frac{1}{2}\right)^\gamma\right] > 0.$$

Thereby, we can derive that $f(\gamma) > f(1) = 0$. Therefore, $CD^\gamma(A,B) < 1$ at $m_A = 0.75$ and $m_B = 0.25$, which means that the value of $CD^\gamma(A,B)$ is $0 < CD^\gamma(A,B) < 1$ for $\forall\gamma \in (1,2]$. In the third condition of Definition 2.5, if $A \leqslant B \leqslant C$ then $\mu_A^L \leqslant \mu_B^L \leqslant \mu_C^L$, $\mu_A^U \leqslant \mu_B^U \leqslant \mu_C^U$, $\nu_A^L \geqslant \nu_B^L \geqslant \nu_C^L$ and $\nu_A^U \geqslant \nu_B^U \geqslant \nu_C^U$. By the same argument, we can obtain that $\| A-B \|_1 \leqslant \| A-C \|_1$, and $\| B-C \|_1 \leqslant \| A-C \|_1$. Therefore, $CD^\gamma(A,B)$ is a cross-entropy measure.

Remark 1. If A and B are two IVIFSs in $X = \{x_1, x_2, \cdots, x_n\}$, the associated cross-entropy measures $CD^\gamma(A,B)$ $\gamma \in (1,2]$ is defined by

$$CD^\gamma(A,B) = \frac{1}{n}\sum_{i=1}^{n} CD^\gamma(A_i,B_i).$$

Here $A_i = (x_i, [\mu_{A_i}^L(x_i), \mu_{A_i}^U(x_i)], [\nu_{A_i}^L(x_i), \nu_{A_i}^U(x_i)])$ and $B_i = (x_i, [\mu_{B_i}^L(x_i), \mu_{B_i}^U(x_i)], [\nu_{B_i}^L(x_i), \nu_{B_i}^U(x_i)])$, the Remark 1 is a more powerful evidence to illustrate $CD^\gamma(A,B)$ is a cross-entropy measure between IVIFSs.

Remark 2. Flexibility is an indispensable and important element in modern decision-making. Each individual is different, and DM tend to consider the problem from his own perspective. By adjusting parameter $\gamma \in (1,2]$, the cross-entropy measure $CD^\gamma(A,B)$ provides more flexibilities and can be applied to different DMs and different decision-making environments.

5. INTEGRATED IVIF GROUP DECISION-MAKING APPROACH WITH UNKNOWN EXPERT WEIGHTS

Suppose that the information about the weighting vector of attribute is given in advance, that is, $W = (w_1, w_2, \ldots, w_m)$. Then, an approach for solving MAGDM problems under IVIF environments with unknown experts' weights is proposed. The steps are summarized as follows:

Step 1. Get the collective IVIF decision matrices $\tilde{R}^{(k)} = (\tilde{r}_i^{(k)})_{n\times 1}$ by using IVIFWA operator to aggregate each individual IVIF decision matrix $R^{(k)} = (r_{ij}^{(k)})_{n\times m}$.

Step 2. Obtain the weighting vector of expert $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \cdots, \lambda^{(t)})$ by using our method proposed in Section 3.

Step 3. Aggregate all the collective IVIF decision matrix $\tilde{R}^{(k)} = (\tilde{r}_i^{(k)})_{n\times 1}$ into an overall group IVIF decision matrix $\hat{R} = (\hat{r}_i)_{n\times 1}$ by IVIFWA operator.

Step 4. From Definition 2.5, the values of the score function $s(\hat{r}_i)$ and the accuracy function $h(\hat{r}_i)$ can be calculated.

Step 5. Obtain the priority of alternatives according to the above values of $s(\hat{r}_i)$ and $h(\hat{r}_i)$.

6. ANALYSIS OF NUMERICAL EXAMPLES

In this section, some examples are illustrated to demonstrate the applicability of the proposed method. Meanwhile, the comparative analyses are also conducted to show the superiority of the proposed method.

6.1. Example 1

Suppose that A_i ($i = 1, 2, \cdots, 5$) are known patterns, d_i ($i = 1, 2, \cdots, 5$) stand for corresponding five decision alternatives, respectively. From [37], the patterns are given by the following IVIFSs in $X = \{x_1, x_2, x_3, x_4\}$. A_1 = {(x_1, [0.4, 0.5], [0.3, 0.4]), (x_2, [0.4, 0.6], [0.2, 0.4]), (x_3, [0.3, 0.4], [0.4, 0.5]), (x_4, [0.5, 0.6], [0.1, 0.3])}, A_2 = {(x_1, [0.5, 0.6], [0.2, 0.3]), (x_2, [0.6, 0.7], [0.2, 0.3]), (x_3, [0.5, 0.6], [0.3, 0.4]), (x_4, [0.4, 0.7], [0.1, 0.2])}, A_3 = {(x_1, [0.3, 0.5], [0.3, 0.4]), (x_2, [0.1, 0.3], [0.5, 0.6]), (x_3, [0.2, 0.5], [0.4, 0.5]), (x_4, [0.2, 0.3], [0.4, 0.6])}, A_4 = {(x_1, [0.2, 0.5], [0.3, 0.4]), (x_2, [0.4, 0.7], [0.1, 0.2]), (x_3, [0.4, 0.5], [0.3, 0.5]), (x_4, [0.5, 0.8], [0.1, 0.2])}, A_5 = {(x_1, [0.3, 0.4], [0.1, 0.3]), (x_2, [0.7, 0.8], [0.1, 0.2]), (x_3, [0.5, 0.6], [0.2, 0.4]), (x_4, [0.6, 0.5], [0.1, 0.2])}.

Assume that t is an unknown sample, which is considered as the positive ideal solution of this problem.

$$t = \{(x_1, [0.5, 0.6], [0.1, 0.3]), (x_2, [0.7, 0.8], [0.1, 0.2]),$$
$$(x_3, [0.5, 0.6], [0.2, 0.4]), (x_4, [0.6, 0.8], [0.1, 0.2])\}$$

From Eq. (5), we can obtain the cross-entropy between each pattern $A_i (i = 1, \cdots, 5)$ and t, as listed in Table 1.

By using the new cross-entropy measure Eq. (5), we can classify pattern t to one of the decision alternatives d_i $(i = 1, 2, \cdots, 5)$. Then, Figure 1 shows the effect of parameter γ on the cross-entropy between known pattern A_i and unknown pattern t. From Figure 1, we can obtain that the value of $CD^\gamma(A_i, t)$ increases with the increase of free parameter γ. Further, we can find that the ranking order of cross-entropy measure $CD^\gamma(A_i, t)$ is not changed with parameter γ, which is $CD^\gamma(A_3, t) > CD^\gamma(A_1, t) > CD^\gamma(A_4, t) > CD^\gamma(A_2, t) > CD^\gamma(A_5, t)$ for all $\gamma \in (1, 2]$. This means that the pattern t belongs to the decision alternative d_5. These results agree with the ones obtained in [37].

Next, an example from [12], in which the weights of the attribute are known while the weights of the experts are unknown. Based on the aforementioned example, we show the method introduced in Section 4 is effective.

6.2. Example 2

Now, the information quality assessments of four social networks, Weibo (G1), QQ (G2), WeChat (G3), and Zhihu (G4) are taken into account. After a thorough investigation and evaluation, five attributes are constructed as g_1: reliability of information acquisition, g_2: timeliness of information acquisition, g_3: information availability, g_4: rapidity of information dissemination, g_5: information availability, where $w = (0.13, 0.17, 0.2, 0.33, 0.17)$ is the weighting vector of attribute. The matrices $R^{(k)} = (r_{ij}^{(k)})$ are provided

Table 1 Cross-entropy between each pattern A_i and t.

γ	1.2	1.6	2
$CD^\gamma(A_1, t)$	0.0053	0.0058	0.0060
$CD^\gamma(A_2, t)$	0.0008	0.0009	0.0010
$CD^\gamma(A_3, t)$	0.0263	0.0253	0.0277
$CD^\gamma(A_4, t)$	0.0025	0.0031	0.0032
$CD^\gamma(A_5, t)$	0.0008	0.0009	0.0001

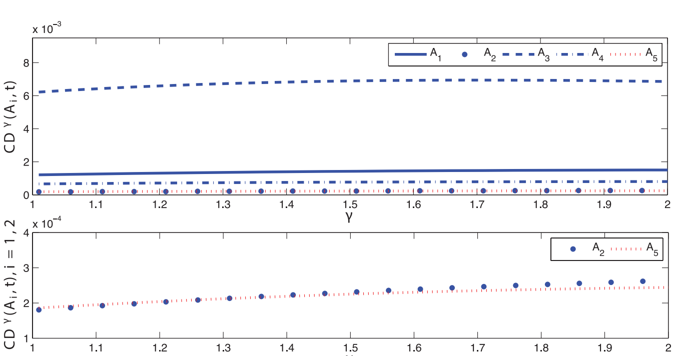

Figure 1 Values of in example 1.

by experts $e^{(k)} (k = 1, 2, 3)$ and let γ and ϕ are set to 1.5 and 0.5, then we can obtain Tables 2–4.

The weighting vector of the attribute is known in advance and then the individual weighting vector of the expert is unknown. The following steps are given to select the optimal decision alternatives:

Step 1. Construct the aggregated overall individual IVIF decision matrices based on the opinions of experts.

Aggregate IVIF matrixes $R^{(k)}$ into an overall individual IVIF decision matrix $\tilde{R}^{(k)}$ by utilizing IVIFWA operator, then we can get Table 5 in which each row expresses an overall individual IVIF decision matrix $\tilde{R}^{(k)}$.

Step 2. Determine weighting vector of expert.

By utilizing Eqs. (2) and (4), the weighting vector of expert can be obtained as follows:

$$\lambda = \left(\lambda^{(1)}, \lambda^{(2)}, \lambda^{(3)}\right) \tag{6}$$
$$= (0.3472, 0.3360, 0.3167)$$

Step 3. Obtain the overall group decision matrix.

Combined with the IVIFWA operator and Eq. (6), the overall individual decision matrix $\tilde{R}^{(k)}$ can be aggregated into an overall group decision matrix $\hat{R}$ in Table 6.

Step 4. By Definition 2.3, the scores of values in $\hat{R}$ can be calculated.

$s(\hat{r}_1) = 0.5625$, $s(\hat{r}_2) = 0.3580$, $s(\hat{r}_3) = 0.4763$, $s(\hat{r}_4) = 0.3584$.

Step 5. Rank the alternatives.

On the base of the scores of $s(\hat{r}_i)$ $(i = 1, 2, 3, 4)$, the ranking of the alternatives G_i $(i = 1, 2, 3, 4)$ can be obtained: $G_1 > G_3 > G_4 > G_2$. Consequently, it is obvious that the alternative G_1 is the most appropriate supplier among the alternatives.

Comparative analysis: To further shown the superiority of our approach in comparison with other group decision-making methods, for example, in [10,23,38,39], some simulation results are depicted in Table 7.

From Table 7, we can get the alternative ranking result given by the method in [23,38] is the same with that given by our method. And the ranking results obtained by the proposed method in this paper are different from those obtained by methods [10,39]. On the basis of the above simulation results, we can find that the proposed method in this paper has some advantages over methods in [10,23,38,39].

(1) Each expert has the same weight in [39] and the weight to each expert is assigned in advance [10]. Both two methods neglected the determination of experts' weights. To overcome the shortcoming, experts' weights for each attribute are derived in [23,38] by using the similarity degree and proximity degree. But they do not consider the influence of experts' experience and professional knowledge on expert weights. By contrast, a new method in this paper is presented to obtain experts' weights, in which two programming models are constructed by considering the influence of experts' experience and professional knowledge on experts' weights. Moreover, the proposed method can provide more opportunities for

Table 2 Decision matrix $R^{(1)}$ is provided by expert $e^{(1)}$.

	G_1	G_2	G_3	G_4
g_1	([0.45,0.70], [0.10,0.25])	([0.40,0.65], [0.20,0.30])	([0.60,0.80], [0.15,0.20])	([0.65,0.75], [0.10,0.20])
g_2	([0.60,0.85], [0.05,0.10])	([0.45,0.65], [0.20,0.30])	([0.70,0.75], [0.15,0.25])	([0.35,0.60], [0.10,0.30])
g_3	([0.65,0.80], [0.05,0.15])	([0.30,0.55], [0.35,0.45])	([0.35,0.50], [0.30,0.40])	([0.55,0.70], [0.15,0.25])
g_4	([0.45,0.60], [0.25,0.35])	([0.55,0.75], [0.10,0.25])	([0.70,0.75], [0.10,0.20])	([0.40,0.70], [0.15,0.25])
g_5	([0.35,0.60], [0.35,0.40])	([0.30,0.55], [0.20,0.40])	([0.60,0.65], [0.25,0.35])	([0.55,0.75], [0.05,0.20])

Table 3 Decision matrix $R^{(2)}$ is provided by expert $e^{(2)}$.

	G_1	G_2	G_3	G_4
g_1	([0.50,0.65],[0.05,0.30])	([0.45,0.60],[0.25,0.35])	([0.30,0.75],[0.05,0.20])	([0.55,0.70],[0.10,0.25])
g_2	([0.65,0.80],[0.05,0.20])	([0.45,0.85],[0.05,0.10])	([0.55,0.70],[0.10,0.25])	([0.30,0.65],[0.15,0.30])
g_3	([0.45,0.85],[0.10,0.15])	([0.40,0.60],[0.25,0.35])	([0.60,0.65],[0.20,0.30])	([0.55,0.70],[0.15,0.25])
g_4	([0.70,0.80],[0.05,0.15])	([0.55,0.75],[0.10,0.20])	([0.60,0.65],[0.05,0.30])	([0.35,0.70],[0.15,0.25])
g_5	([0.50,0.70],[0.10,0.25])	([0.55,0.60],[0.25,0.40])	([0.80,0.85],[0.05,0.10])	([0.20,0.65],[0.20,0.30])

Table 4 Decision matrix $R^{(3)}$ is provided by expert $e^{(3)}$.

	G_1	G_2	G_3	G_4
g_1	([0.50,0.55],[0.15,0.35])	([0.30,0.60],[0.20,0.30])	([0.45,0.75],[0.10,0.25])	([0.65,0.85],[0.05,0.15])
g_2	([0.70,0.85],[0.05,0.10])	([0.55,0.60],[0.25,0.30])	([0.60,0.75],[0.15,0.20])	([0.20,0.50],[0.40,0.45])
g_3	([0.55,0.65],[0.30,0.35])	([0.40,0.60],[0.10,0.30])	([0.70,0.75],[0.20,0.25])	([0.50,0.80],[0.05,0.20])
g_4	([0.80,0.90],[0.05,0.10])	([0.60,0.80],[0.10,0.20])	([0.55,0.75],[0.15,0.20])	([0.20,0.45],[0.35,0.50])
g_5	([0.65,0.85],[0.10,0.15])	([0.20,0.55],[0.30,0.40])	([0.50,0.60],[0.30,0.35])	([0.45,0.80],[0.05,0.20])

Table 5 A overall individual interval-valued intuitionistic fuzzy set (IVIF) decision matrix $\tilde{R}^{(k)}$.

	$\tilde{R}^{(1)}$	$\tilde{R}^{(2)}$	$\tilde{R}^{(3)}$
G_1	([0.5393,0.7416],[0.1041,0.2083])	([0.5959,0.7821],[0.0643,0.1881])	([0.6888,0.8214],[0.0921,0.1612])
G_2	([0.4321,0.6573],[0.1770,0.3207])	([0. 4942,0.7109],[0.1398,0.2397])	([0.4656,0.6763],[0.1542,0.2750])
G_3	([0.6197,0.7059],[0.1634,0.2614])	([0.6094,0.7173],[0.0738,0.2291])	([0.5741,0.7294],[0.1691,0.2364])
G_4	([0.4896,0.7016],[0.1103,0.2412])	([0.3954,0.6839],[0.1493,0.2660])	([0.3853,0.6842],[0.1367,0.3004])

Table 6 The overall group decision matrix $\hat{R}$.

Alternative	$\hat{R}$
G_1	([0.6114,0.7835],[0.0846,0.1853])
G_2	([0.4650,0.6828],[0.1560,0.2759])
G_3	([0.6020,0.7175],[0.1251,0.2418])
G_4	([0.4256,0.6900],[0.1312,0.2676])

Table 7 Ranking results of different methods for Example 2.

Methods	Ranking Results
Method in this paper	$G_1 > G_3 > G_4 > G_2$
Method in [23]	$G_1 > G_3 > G_4 > G_2$
Method in [10]	$G_4 > G_1 > G_2 > G_3$
Method in [38]	$G_1 > G_3 > G_4 > G_2$
Method in [39]	$G_2 > G_1 > G_3 > G_4$

DMs in actual decision-making by proposing a novel cross-entropy with parameter, which is comprehensive and flexible.

(2) The alternatives ranking method in [39] is to calculate the distances between IVIF positive ideal solution and schemes, but this method is not robust for different IVIF positive ideal solutions. The alternatives ranking method in [38] is directly transformed the IVIF matrix into an interval matrix, which could result in information loss. The ranking method in [10] is the same as the ranking method in [40]. And the ranking order of alternatives is generated according to an order relation of IVIFVs in [23]. However, the above ranking methods cannot provide more opportunities for DMs in actual decision-making, so these methods also cannot be applied to different DMs and different decision-making environments. By contrast, the method in this paper can be applied to different DMs and different decision-making environments by adjusting the parameters. Furthermore, it has some desirable properties and advantages over existing ones.

Discussion of the influence of parameters: It is necessary to discuss whether and how the ranking results change when the values of these parameters ϕ and γ are different. Considering that different values of the parameter ϕ in can be set, three special cases are calculated as examples including $\phi = 0$, $\phi = 0.5$ and $\phi = 1$, which represent the minority, compromise and majority principles, respectively. Accordingly, the computation results are listed in Table 8.

It can be seen from Table 8 that the ranking order of alternatives is not the same for different decision principles of DMs when the value of parameters ϕ or γ is fixed. First, without loss of generality, we assume $\gamma = 1.5$. Based on the minority principle ($\phi = 0.0$), the ranking order is $G_1 > G_3 > G_4 > G_2$. Based on the compromise principle ($\phi = 0.5$), the ranking order is $G_1 > G_3 > G_4 > G_2$. Based on the majority principle ($\phi = 1.0$), the ranking order is

Table 8 Computation results with different values of parameters ϕ and γ.

Methods	Ranking Results
$\gamma = 1.1, \phi = 0.0$	$G_1 > G_3 > G_4 > G_2$
$\gamma = 1.1, \phi = 0.5$	$G_1 > G_3 > G_4 > G_2$
$\gamma = 1.1, \phi = 1.0$	$G_1 > G_3 > G_2 > G_4$
$\gamma = 1.5, \phi = 0.0$	$G_1 > G_3 > G_4 > G_2$
$\gamma = 1.5, \phi = 0.5$	$G_1 > G_3 > G_4 > G_2$
$\gamma = 1.5, \phi = 1.0$	$G_1 > G_3 > G_2 > G_4$
$\gamma = 1.9, \phi = 0.0$	$G_1 > G_3 > G_2 > G_4$
$\gamma = 1.9, \phi = 0.5$	$G_1 > G_3 > G_2 > G_4$
$\gamma = 1.9, \phi = 1.0$	$G_1 > G_3 > G_2 > G_4$

$G_1 > G_3 > G_2 > G_4$. Then, we can discover that the optimal alternative will not change with ϕ under the fixed parameter γ. Next, without loss of generality, we assume $\phi = 0.5$. Then, it is obtained that the ranking order is $G_1 > G_3 > G_4 > G_2$ when $\gamma = 1.1$ or $\gamma = 1.5$ and the ranking order is $G_1 > G_3 > G_2 > G_4$ under $\gamma = 1.9$. We can find that the ranking order of alternatives G_4 and G_2 will change with the parameter γ. Through the analysis above, we can find that although the ranking results will change with the parameters ϕ and γ, the optimal alternative is not changed. This means that the proposed approach in this paper has the stable robustness.

7. CONCLUSION

In this paper, MAGDM problems with unknown experts' weights under IVIF are investigated by fully considering the experts' experience and professional knowledge. Then, a novel cross-entropy measure with parameter of IVIFS based on J-divergence is proposed to handle the MAGDM problems. Next, in order to further show the effectiveness of the proposed method, comparative case studies have been carried out. Compared with the existing representative cross-entropy measures [22,23], the proposed cross-entropy has better discriminating ability. Furthermore, we can further study and extend it to other practical fuzzy decision-making environments, such as the fuzzy group decision-making support system under the interactive intelligent decision framework of green suppliers.

CONFLICT OF INTEREST

The authors declared that they have no conflicts of interest to this work.

AUTHORS' CONTRIBUTIONS

All authors contributed to the work. All authors read and approved the final manuscript.

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APPENDIX

A proof of Theorem 4.1

$$CD^{\gamma}(A,B) = -\frac{1}{\gamma-1}\left[\left(\frac{m_A+m_B}{2}\right)^{\gamma} - \frac{1}{2}\left(m_A^{\gamma}+m_B^{\gamma}\right)\right]$$

where $\gamma \in (1,2]$, $m_A = \frac{\mu_A^L+\mu_A^U+2-\nu_A^L-\nu_A^U}{4}$, $m_B = \frac{\mu_B^L+\mu_B^U+2-\nu_B^L-\nu_B^U}{4}$. For the operational rule's convenience, let

$$\begin{aligned} \mu_A^L &= x_1, \mu_A^U = x_2, \nu_A^L = x_3, \nu_A^U = x_4 \\ \mu_B^L &= x_5, \mu_B^U = x_6, \nu_B^L = x_7, \nu_B^U = x_8. \end{aligned}$$

The $CD^{\gamma}(A,B)$ can be simplified as follow:

$$\begin{aligned} & f(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8) \\ & = -\frac{1}{\gamma-1}\left[\left(\frac{m_A+m_B}{2}\right)^{\gamma} - \frac{1}{2}\left(m_A^{\gamma}+m_B^{\gamma}\right)\right] \end{aligned}$$

where $m_A = \frac{x_1+x_2+2-x_3-x_4}{4}$, $m_B = \frac{x_5+x_6+2-x_7-x_8}{4}$. For the calculation easily, we suppose $a_1 = \frac{\gamma}{64}(\frac{m_A+m_B}{2})^{\gamma-2}$, $b_1 = \frac{\gamma}{32}m_A^{\gamma-2}$, and $b_2 = \frac{\gamma}{32}m_B^{\gamma-2}$, then we can obtain the Hesse matrix as follow:

$$C = \begin{bmatrix} A_1 - B_1 & A_1 \\ A_1 & A_1 - B_2 \end{bmatrix}$$

where

$$A_1 = \begin{bmatrix} -a_1 & -a_1 & a_1 & a_1 \\ -a_1 & -a_1 & a_1 & a_1 \\ a_1 & a_1 & -a_1 & -a_1 \\ a_1 & a_1 & -a_1 & -a_1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -b_1 & -b_1 & b_1 & b_1 \\ -b_1 & -b_1 & b_1 & b_1 \\ b_1 & b_1 & -b_1 & -b_1 \\ b_1 & b_1 & -b_1 & -b_1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -b_2 & -b_2 & b_2 & b_2 \\ -b_2 & -b_2 & b_2 & b_2 \\ b_2 & b_2 & -b_2 & -b_2 \\ b_2 & b_2 & -b_2 & -b_2 \end{bmatrix}.$$

Obviously, C is a symmetric matrix. According to the properties of symmetric positive definite matrix, the matrix C change into a matrix D after inverse transformation as follow:

$$D = \begin{bmatrix} -a_1+b_1 & a_1 & 0 & \cdots & 0 \\ a_1 & -a_1+b_2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0. \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Though the above discussion, D is a positive semi-definite matrix if and only if $-a_1+b_1 \geqslant 0, -a_1+b_2 \geqslant 0$ and $-a_1(b_1+b_2)+b_1b_2 \geqslant 0$. Firstly, since $\gamma \in (1,2]$ and $m_A m_B > 0$, then we have $2m_A^{\gamma-2} \geqslant (m_A+m_B)^{\gamma-2}$. Thus $-a_1+b_1 \geqslant 0$. Similarly, we can prove that $-a+b_2 \geqslant 0$. For the proof of $-a_1(b_1+b_2)+b_1b_2 \geqslant 0$, we made the following assumptions:

$$g(m_A,m_B) = -\frac{1}{\gamma-1}\left[\left(\frac{m_A+m_B}{2}\right)^{\gamma} - \frac{1}{2}\left(m_A^{\gamma}+m_B^{\gamma}\right)\right].$$

From the Theorem 1 in [41], we can obtain $g(m_A,m_B)$ is convex function, namely,

$$\begin{aligned} \left|D_g\right| &= \begin{vmatrix} g_{m_A m_a} & g_{m_A m_B} \\ g_{m_A m_B} & g_{m_B m_B} \end{vmatrix} \\ &= \begin{vmatrix} -a_1+b_1 & -a_1 \\ -a_1 & -a_1+b_2 \end{vmatrix} \geqslant 0 \end{aligned}$$

Thereby,

$$\mid D_g \mid = -a_1(b_1+b_2)+b_1b_2 \geqslant 0$$

Therefore, $f(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8)$ is convex function, which completes the proof.