

Research Article

Multi-Criteria Group Decision-Making Using Spherical Fuzzy Prioritized Weighted Aggregation Operators

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ARTICLE INFO

Article History

Received 10 Jun 2020

Accepted 02 Sep 2020

Keywords

Spherical fuzzy numbers

Score function

Spherical fuzzy prioritized weighted aggregation operators

Idempotency

Spherical fuzzy decision matrix

ABSTRACT

Spherical fuzzy sets, originally proposed by F.K. Gündoğdu, C. Kahraman, Spherical fuzzy sets and spherical fuzzy TOPSIS method, J. Intell. Fuzzy Syst. 36 (2019), 337–352, can handle the information of type: yes, no, abstain and refusal, owing to the feature of broad space of admissible triplets. This remarkable feature of spherical fuzzy set to manage the uncertainty and vagueness distinguishes it from other fuzzy set models. In this research article, we utilize spherical fuzzy sets and prioritized weighted aggregation operators to construct some spherical fuzzy prioritized weighted aggregation operators, including spherical fuzzy prioritized weighted averaging operator and spherical fuzzy prioritized weighted geometric operator. We discuss some properties which are satisfied by these operators. Further, we establish an algorithm to the multi-criteria group decision-making problem by utilizing the aforesaid operators. To elaborate the applicability of proposed operators in decision-making, we apply the algorithm to a numerical example which is related to the appointment for the post of Finance Manager. Finally, to demonstrate the authenticity of presented operators, we conduct a comparison with existing methods.

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1. INTRODUCTION

Multi-criteria group decision-making (MCGDM) is a procedure of solving practical problems in different areas, in which the most accurate solution is provided after examining the alternatives over multiple criteria. Due to the imprecision and ambiguity in many decision making (DM) problems, Zadeh [1] offered a very useful tool, called fuzzy set (FS), for managing the DM problems. In an FS, Zadeh discussed only the membership degree (MD) of an item. FS theory has numerous applications in different fields, such as management sciences, computer sciences, engineering, decision theory, and so on. The remarkable idea of FS has been successfully applied and explored by many researchers. Atanassov [2] generalized the notion of FS to intuitionistic FS (IFS) by introducing an MD (μ) and a nonmembership degree (NMD) (ν) with the condition $\mu + \nu \leq 1$. Yager [3] strengthened the notion of IFS by presenting the idea of Pythagorean FS (PyFS) and stretched out IFS by introducing a new condition $\mu^2 + \nu^2 \leq 1$. IFS and PyFS discuss only the MD and the NMD of items in a FS, but, the human opinions are may be of abstinence and refusal type. To overcome this issue, Cuong [4,5] presented the concept of picture FS (PFS), an extension of IFS. PFS has qualities to represent the human opinions of type: yes, abstain, no and refusal. A PFS gives three degrees of an element, named, MD (μ), abstinence degree (AD) or neutral degree (γ) and NMD (ν) with the constraint $\mu + \gamma + \nu \leq 1$. The concept of PFS is applicable in different fields such as, fuzzy inference, DM, clustering, and so on. Gündoğdu and Kahraman [6] originated the idea of spherical FS (SFS), an extension of PFS, which has broadened the space of MD (μ), AD (γ) and NMD (ν) in the interval $[0, 1]$ with the condition $0 \leq \mu^2 + \gamma^2 + \nu^2 \leq 1$. For further study, one may refer to [7,8]. Kahraman *et al.* [9] applied the spherical fuzzy TOPSIS method to find the solution of a problem which is related to the selection of hospital location. Akram *et al.* [10] studied the notion of SFS by providing the solution of a DM problem and Akram [11] presented a DM method based on spherical fuzzy graphs.

Aggregation operators (AOs) have great importance and significance in solving MCGDM problems. The AOs are useful to convert the whole data into a single value. To handle IF information, Xu [12] proposed some averaging operators in IF environment. Apart from this, Xu and Yager [13] defined geometric operators based on IFSs, including weighted, ordered weighted and hybrid. Li [14] proposed generalized ordered weighted averaging (GOWA) operators. In order to define IF ordered weighted distance (IFOWD) operators, Zeng and Sua [15] combined distance measures and AOs. Yager [16] presented some averaging and geometric AOs under PyF weighted, ordered weighted and weighted power circumstances. Later, Peng and Yaun [17] investigated some basic properties of PyF AOs. The generalized PyF AOs

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were developed by Liu *et al.* [18]. Researchers introduced several AOs within different circumstances [19–24]. Ashraf and Abdullah [25] presented spherical AOs with a DM application and Ashraf *et al.* [26] developed spherical fuzzy Dombi AOs. For further study on SFSs and T-SFSs, the readers are referred to [27–32]. Liu *et al.* [33] developed specific types of q-rung picture fuzzy Yager AOs for DM. Recently, Ma *et al.* [34] presented a group DM method using complex Pythagorean fuzzy information.

The earlier concepts related to FSs and its extensions were constructed on the fact that the priority level of criteria and decision makers was same. However, to deal the issue of prioritization, researchers presented certain prioritized AOs in different circumstances, based on the fact that the priority level of different criteria are different. Yu [35] presented prioritized AOs under IF environment with their application. Yu [36] also developed IF generalized prioritized weighted averaging and geometric AOs and their application. Arora and Garg [37] developed a group DM method on prioritized linguistic AOs and discussed its fundamental properties. Garg [38] discovered a multi-criteria DM method based on prioritized Muirhead mean AO under neutrosophic set environment. For further study on prioritized AOs, one may refer to [39–44]. From previous study, we have noticed that there is no investigation on prioritized weighted AOs in spherical fuzzy environment. Therefore, motivated by the already established prioritized AOs and SFS, we develop some spherical fuzzy prioritized weighted AOs, namely, spherical fuzzy prioritized weighted averaging (SFPWA) operator and spherical fuzzy prioritized weighted geometric (SFPWG) operator. Following are the major contributions and the objectives of this article:

- To develop prioritized weighted AOs under spherical fuzzy environment, which deals the information having prioritization relationship in the data. Therefore, to cope with such data, SFPWA operator and SFPWG operator are successfully established.
- A new score function to score the alternatives is presented.
- To define certain elementary properties of the stated operators. Some properties including, idempotency, monotonicity and boundedness are defined and discussed with appropriate elaboration.
- An MCGDM algorithm, based on spherical fuzzy prioritized weighted AOs, is developed to solve the DM numerical problems.
- To demonstrate the applicability of the proposed work, a fully developed numerical example is solved.
- A comparison with already established methods is given to demonstrate the significance of the presented approach.

This article is organized as follows: Section 2 presents some basic definitions which are helpful to comprehend the presented AOs. Section 3 introduces the prioritized AOs, namely, SFPWA operator and SFPWG operator. Section 3 also presents some useful properties of these operators. Section 4 gives an algorithm of our proposed work and a numerical example. Section 5 comprises with a comparison with existing methods available in the literature and finally, Section 6 gives the conclusion.

2. PRELIMINARIES

Definition 2.1. [6] An SFS \mathcal{F} on a universe Y is represented as

$$\mathcal{F} = \{(g, \mathfrak{N}_{\mathcal{F}}(g), \mathfrak{h}_{\mathcal{F}}(g), \rho_{\mathcal{F}}(g)) | g \in Y\},$$

where $\mathfrak{N}_{\mathcal{F}}(g), \mathfrak{h}_{\mathcal{F}}(g), \rho_{\mathcal{F}}(g) \in [0, 1]$, $0 \leq \mathfrak{N}_{\mathcal{F}}^2(g) + \mathfrak{h}_{\mathcal{F}}^2(g) + \rho_{\mathcal{F}}^2(g) \leq 1$ for all $g \in Y$. We consider the triplet $(\mathfrak{N}_{\mathcal{F}}(g), \mathfrak{h}_{\mathcal{F}}(g), \rho_{\mathcal{F}}(g))$ as SFN and denote it by $\mathcal{F} = (\mathfrak{N}_{\mathcal{F}}, \mathfrak{h}_{\mathcal{F}}, \rho_{\mathcal{F}})$. Note that $\mathfrak{N}_{\mathcal{F}}, \mathfrak{h}_{\mathcal{F}}$ and $\rho_{\mathcal{F}}$ are the MD, AD and NMD of \mathcal{F} , respectively. Further $\pi_{\mathcal{F}}(g) = \sqrt{1 - (\mathfrak{N}_{\mathcal{F}}^2(g) + \mathfrak{h}_{\mathcal{F}}^2(g) + \rho_{\mathcal{F}}^2(g))}$ is the hesitancy degree of g in \mathcal{F} .

Definition 2.2. [25] Suppose Y is the universe. $\mathcal{F}_1 = (\mathfrak{N}_{\mathcal{F}_1}, \mathfrak{h}_{\mathcal{F}_1}, \rho_{\mathcal{F}_1})$ and $\mathcal{F}_2 = (\mathfrak{N}_{\mathcal{F}_2}, \mathfrak{h}_{\mathcal{F}_2}, \rho_{\mathcal{F}_2})$ are two SFNs on Y . Then some basic operations between \mathcal{F}_1 and \mathcal{F}_2 are defined as

1. $\mathcal{F}_1 \oplus \mathcal{F}_2 = (\mathfrak{N}_{\mathcal{F}_1}, \mathfrak{h}_{\mathcal{F}_1}, \rho_{\mathcal{F}_1}) \oplus (\mathfrak{N}_{\mathcal{F}_2}, \mathfrak{h}_{\mathcal{F}_2}, \rho_{\mathcal{F}_2}) = \left(\sqrt{\mathfrak{N}_{\mathcal{F}_1}^2 + \mathfrak{N}_{\mathcal{F}_2}^2 - \mathfrak{N}_{\mathcal{F}_1}^2 \mathfrak{N}_{\mathcal{F}_2}^2}, \mathfrak{h}_{\mathcal{F}_1} \mathfrak{h}_{\mathcal{F}_2}, \rho_{\mathcal{F}_1} \rho_{\mathcal{F}_2} \right);$
2. $\mathcal{F}_1 \otimes \mathcal{F}_2 = (\mathfrak{N}_{\mathcal{F}_1}, \mathfrak{h}_{\mathcal{F}_1}, \rho_{\mathcal{F}_1}) \otimes (\mathfrak{N}_{\mathcal{F}_2}, \mathfrak{h}_{\mathcal{F}_2}, \rho_{\mathcal{F}_2}) = \left(\mathfrak{N}_{\mathcal{F}_1} \mathfrak{N}_{\mathcal{F}_2}, \mathfrak{h}_{\mathcal{F}_1} \mathfrak{h}_{\mathcal{F}_2}, \sqrt{\rho_{\mathcal{F}_1}^2 + \rho_{\mathcal{F}_2}^2 - \rho_{\mathcal{F}_1}^2 \rho_{\mathcal{F}_2}^2} \right);$
3. $\gamma_{\mathcal{F}} = \left(\sqrt{1 - (1 - \mathfrak{N}_{\mathcal{F}}^2)^{\gamma}}, \mathfrak{h}_{\mathcal{F}}^{\gamma}, \rho_{\mathcal{F}}^{\gamma} \right), \gamma \geq 0;$
4. $\mathcal{F}^{\gamma} = \left(\mathfrak{N}_{\mathcal{F}}^{\gamma}, \mathfrak{h}_{\mathcal{F}}^{\gamma}, \sqrt{1 - (1 - \rho_{\mathcal{F}}^2)^{\gamma}} \right), \gamma \geq 0.$

Definition 2.3. [25] Suppose $\mathcal{F} = (\mathfrak{N}_{\mathcal{F}}, \mathfrak{h}_{\mathcal{F}}, \rho_{\mathcal{F}})$ is an SFN. The score function $S(\mathcal{F})$ of \mathcal{F} can be defined as

$$S(\mathcal{F}) = \frac{1}{3}(2 + \mathfrak{N}_{\mathcal{F}} - \mathfrak{h}_{\mathcal{F}} - \rho_{\mathcal{F}}), \quad \text{where } S(\mathcal{F}) \in [0, 1], \quad (1)$$

Definition 2.4. [26] Suppose $F_1 = (\mathfrak{N}_{F_1}, \mathfrak{h}_{F_1}, \rho_{F_1})$ and $F_2 = (\mathfrak{N}_{F_2}, \mathfrak{h}_{F_2}, \rho_{F_2})$ are two SFNs. For the comparison of F_1 and F_2

1. $S(F_1) > S(F_2) \Rightarrow F_1 > F_2$ (F_1 is superior to F_2);
2. $S(F_1) < S(F_2) \Rightarrow F_1 < F_2$ (F_1 is inferior to F_2).

The prioritized weighted average (PWA) operator was originally introduced by Yager [44], which was defined as follows:

Definition 2.5. [44] Let $C = \{C_1, C_2, \dots, C_n\}$ be a collection of criteria and there is a prioritization between the criteria which can be expressed by the linear ordering $C_1 > C_2 > \dots > C_n$, indicate C_j has higher priority than C_k if $j < k$. The value $C_j(y)$ is the performance of any alternative y under criteria C_j , and satisfies $C_j(y) \in [0, 1]$. If

$$PWA(C_j(y)) = \sum_{j=1}^n w_j C_j(y), \quad (2)$$

where $w_j = \frac{T_j}{\sum_{j=1}^n T_j}$, $T_j = \prod_{k=1}^{j-1} C_k(y)$ ($j = 2, \dots, n$), $T_1 = 1$. Then the operator is called PWA operator.

Definition 2.6. For two SFNs $F_1 = (\mathfrak{N}_{F_1}, \mathfrak{h}_{F_1}, \rho_{F_1})$ and $F_2 = (\mathfrak{N}_{F_2}, \mathfrak{h}_{F_2}, \rho_{F_2})$ on a universe Y ,

$$F_1 \subseteq F_2, \text{ if and only if, } \mathfrak{N}_{F_1} \leq \mathfrak{N}_{F_2}, \mathfrak{h}_{F_1} \leq \mathfrak{h}_{F_2} \text{ and } \rho_{F_1} \geq \rho_{F_2}.$$

3. SPHERICAL FUZZY PRIORITIfuzzy set ZED WEIGHTED AOs

In this section, we propose two operators, namely, SFPWA operator and SFPWG operator. Moreover, we discuss some pivotal properties of these operators. First, we introduce a new score function to score the alternatives.

Definition 3.1. Suppose $F = (\mathfrak{N}_F, \mathfrak{h}_F, \rho_F)$ is an SFN. The new score function to score the alternatives is defined as

$$S^*(F) = \frac{1}{3}(2 + \mathfrak{N}^2 - \mathfrak{h}^2 - \rho^2), \quad \text{where } S^*(F) \in [0, 1]. \quad (3)$$

On replacing the score function $S(F)$ by $S^*(F)$, the order relation between two SFNs F_1 and F_2 introduced by [25] is also valid.

3.1. SFPWA Operator

Definition 3.2. For a collection $F_j = (\mathfrak{N}_{F_j}, \mathfrak{h}_{F_j}, \rho_{F_j})$ ($j = 1, 2, \dots, n$) of SFNs, the SFPWA operator is defined by a mapping $SFPWA : F^n \rightarrow F$, where

$$SFPWA(F_1, F_2, \dots, F_n) = \bigoplus_{j=1}^n \left(\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j} F_j \right) = \left(\frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j} F_1 \oplus \frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j} F_2 \oplus \dots \oplus \frac{\mathcal{A}_n}{\sum_{j=1}^n \mathcal{A}_j} F_n \right),$$

where $\mathcal{A}_j = \prod_{k=1}^{j-1} S^*(F_k)$ ($j = 2, \dots, n$) with $\mathcal{A}_1 = 1$ and $S^*(F_k)$ is the score of $F_k = (\mathfrak{N}_{F_k}, \mathfrak{h}_{F_k}, \rho_{F_k})$.

Theorem 3.1. For a collection $F_j = (\mathfrak{N}_{F_j}, \mathfrak{h}_{F_j}, \rho_{F_j})$ ($j = 1, 2, \dots, n$) of SFNs, the aggregated value of these SFNs by applying SFPWA operator is again an SFN. The formula to calculate the aggregated value is given as follows:

$$SFPWA(F_1, F_2, \dots, F_n) = \bigoplus_{j=1}^n \left(\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j} F_j \right) = \left(\sqrt[n]{1 - \prod_{j=1}^n (1 - \mathfrak{N}_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}}, \prod_{j=1}^n (\mathfrak{h}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \prod_{j=1}^n (\rho_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}} \right), \quad (4)$$

where $\mathcal{A}_j = \prod_{k=1}^{j-1} S^*(F_k)$ ($j = 2, \dots, n$) with $\mathcal{A}_1 = 1$ and $S^*(F_k)$ is the score of $F_k = (\mathfrak{N}_{F_k}, \hbar_{F_k}, \rho_{F_k})$.

Proof. Theorem 3.1 can be easily prove by induction method. First, we prove that Equation (4) is true for $n = 2$. Therefore,

$$\frac{A_1}{\sum_{j=1}^n A_j}_{F_1} = \left(\sqrt{1 - (1 - \mathfrak{N}_1^2)^{\frac{A_1}{\sum_{j=1}^n A_j}}}, (\hbar_1)^{\frac{A_1}{\sum_{j=1}^n A_j}}, (\rho_1)^{\frac{A_1}{\sum_{j=1}^n A_j}} \right), \frac{A_2}{\sum_{j=1}^n A_j}_{F_2} = \left(\sqrt{1 - (1 - \mathfrak{N}_2^2)^{\frac{A_2}{\sum_{j=1}^n A_j}}}, (\hbar_2)^{\frac{A_2}{\sum_{j=1}^n A_j}}, (\rho_2)^{\frac{A_2}{\sum_{j=1}^n A_j}} \right).$$

Thus,

$$\begin{aligned} SFPWA(F_1, F_2) &= \frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j}_{F_1} \oplus \frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j}_{F_2} \\ &= \left(\sqrt{1 - (1 - \mathfrak{N}_1^2)^{\frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j}}}, (\hbar_1)^{\frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j}}, (\rho_1)^{\frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j}} \right) \oplus \left(\sqrt{1 - (1 - \mathfrak{N}_2^2)^{\frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j}}}, (\hbar_2)^{\frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j}}, (\rho_2)^{\frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j}} \right) \\ &= \left(\sqrt{1 - \prod_{j=1}^2 (1 - \mathfrak{N}_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}}, \prod_{j=1}^2 (\hbar_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \prod_{j=1}^2 (\rho_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}} \right). \end{aligned}$$

So, the Equation (4) is true for $n = 2$. For $n = l$, suppose that the Equation (4) holds.

$$SFPWA(F_1, F_2, \dots, F_l) = \bigoplus_{j=1}^l \left(\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j}_{F_j} \right) = \left(\sqrt{1 - \prod_{j=1}^l (1 - \mathfrak{N}_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j}}}, \prod_{j=1}^l (\hbar_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j}}, \prod_{j=1}^l (\rho_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j}} \right).$$

Now, we prove that Equation (4) is true for $n = l + 1$.

$$\begin{aligned}
 SFPWA(F_1, F_2, \dots, F_{l+1}) &= \bigoplus_{j=1}^{l+1} \left(\frac{\mathcal{A}_j}{\sum_{j=1}^{l+1} \mathcal{A}_j} F_j \right) = \bigoplus_{j=1}^l \left(\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j} F_j \right) \oplus \left(\frac{\mathcal{A}_{l+1}}{\sum_{j=1}^{l+1} \mathcal{A}_j} F_{l+1} \right) \\
 &= \left(\sqrt{1 - \prod_{j=1}^l (1 - \mathfrak{N}_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j}}}, \prod_{j=1}^l (\mathfrak{h}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j}}, \prod_{j=1}^l (\rho_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^l \mathcal{A}_j}} \right) \\
 &\quad \oplus \left(\sqrt{1 - (1 - \mathfrak{N}_{l+1}^2)^{\frac{\mathcal{A}_{l+1}}{\sum_{j=1}^{l+1} \mathcal{A}_j}}}, (\mathfrak{h}_{l+1})^{\frac{\mathcal{A}_{l+1}}{\sum_{j=1}^{l+1} \mathcal{A}_j}}, (\rho_{l+1})^{\frac{\mathcal{A}_{l+1}}{\sum_{j=1}^{l+1} \mathcal{A}_j}} \right), \\
 &= \left(\sqrt{1 - \prod_{j=1}^{l+1} (1 - \mathfrak{N}_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^{l+1} \mathcal{A}_j}}}, \prod_{j=1}^{l+1} (\mathfrak{h}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^{l+1} \mathcal{A}_j}}, \prod_{j=1}^{l+1} (\rho_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^{l+1} \mathcal{A}_j}} \right).
 \end{aligned}$$

Thus, the Equation (4) is true for $n = l + 1$. Hence Equation (4) holds for all $n \in N$.

Following are some properties satisfied by SFPWA operator. These properties can be proved by utilizing Theorem 3.1.

Theorem 3.2. (Idempotency) Suppose $F_j = (\mathfrak{N}_{F_j}, \mathfrak{h}_{F_j}, \rho_{F_j})$ ($j = 1, 2, \dots, n$) is a collection of SFNs with the condition $F_j = F$ (for all j). Then

$$SFPWA(F_1, F_2, \dots, F_n) = F.$$

Proof. Suppose that $F_j = F$, for all j . Using Equation (4), we have

$$\begin{aligned}
 SFPWA(F_1, F_2, \dots, F_n) &= \bigoplus_{j=1}^n \left(\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j} F_j \right) = \left(\frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j} F_1 \oplus \frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j} F_2 \oplus \dots \oplus \frac{\mathcal{A}_n}{\sum_{j=1}^n \mathcal{A}_j} F_n \right) \\
 &= \left(\sqrt{1 - \prod_{j=1}^n (1 - \mathfrak{N}_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}}, \prod_{j=1}^n (\mathfrak{h}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \prod_{j=1}^n (\rho_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}} \right)
 \end{aligned}$$

$$\begin{aligned}
&= \left(\sqrt{1 - \prod_{j=1}^n (1 - \aleph^2)^{\frac{A_j}{\sum_{j=1}^n A_j}}}, \prod_{j=1}^n (\hbar)^{\frac{A_j}{\sum_{j=1}^n A_j}}, \prod_{j=1}^n (\rho)^{\frac{A_j}{\sum_{j=1}^n A_j}} \right) \\
&= \left(\sqrt{1 - (1 - \aleph^2)^{\frac{\sum_{j=1}^n A_j}{\sum_{j=1}^n A_j}}}, (\hbar)^{\frac{\sum_{j=1}^n A_j}{\sum_{j=1}^n A_j}}, (\rho)^{\frac{\sum_{j=1}^n A_j}{\sum_{j=1}^n A_j}} \right) \\
&= (\sqrt{1 - (1 - \aleph^2)}, \hbar, \rho) \\
&= (\aleph, \hbar, \rho) \\
&= \mathcal{F}.
\end{aligned}$$

Theorem 3.3. (Monotonicity) Suppose $\mathcal{F}'_j = (\aleph'_{\mathcal{F}'_j}, \hbar'_{\mathcal{F}'_j}, \rho_{\mathcal{F}'_j})$ and $\mathcal{F}_j = (\aleph_{\mathcal{F}_j}, \hbar_{\mathcal{F}_j}, \rho_{\mathcal{F}_j})$ are two collections of SFNs, with $(j = 1, 2, \dots, n)$, $\aleph'_{\mathcal{F}'_j} \leq \aleph_{\mathcal{F}_j}$, $\hbar'_{\mathcal{F}'_j} \leq \hbar_{\mathcal{F}_j}$, and $\rho_{\mathcal{F}'_j} \geq \rho_{\mathcal{F}_j}$ for all j . Then

$$SFPWA(\mathcal{F}'_1, \mathcal{F}'_2, \dots, \mathcal{F}'_n) \subseteq SFPWA(\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_n).$$

Proof. Suppose $\mathcal{F}'_j = (\aleph'_{\mathcal{F}'_j}, \hbar'_{\mathcal{F}'_j}, \rho_{\mathcal{F}'_j})$ and $\mathcal{F}_j = (\aleph_{\mathcal{F}_j}, \hbar_{\mathcal{F}_j}, \rho_{\mathcal{F}_j})$ are two collections of SFNs with the following prioritized weight vectors,

$$\frac{\mathcal{A}'_{\mathcal{F}'_j}}{\sum_{j=1}^n \mathcal{A}'_{\mathcal{F}'_j}} = \left(\frac{\mathcal{A}'_{\mathcal{F}'_1}}{\sum_{j=1}^n \mathcal{A}'_{\mathcal{F}'_j}}, \frac{\mathcal{A}'_{\mathcal{F}'_2}}{\sum_{j=1}^n \mathcal{A}'_{\mathcal{F}'_j}}, \dots, \frac{\mathcal{A}'_{\mathcal{F}'_n}}{\sum_{j=1}^n \mathcal{A}'_{\mathcal{F}'_j}} \right), \quad \frac{\mathcal{A}_{\mathcal{F}_j}}{\sum_{j=1}^n \mathcal{A}_{\mathcal{F}_j}} = \left(\frac{\mathcal{A}_{\mathcal{F}_1}}{\sum_{j=1}^n \mathcal{A}_{\mathcal{F}_j}}, \frac{\mathcal{A}_{\mathcal{F}_2}}{\sum_{j=1}^n \mathcal{A}_{\mathcal{F}_j}}, \dots, \frac{\mathcal{A}_{\mathcal{F}_n}}{\sum_{j=1}^n \mathcal{A}_{\mathcal{F}_j}} \right),$$

respectively, where $\frac{\mathcal{A}'_{\mathcal{F}'_j}}{\sum_{j=1}^n \mathcal{A}'_{\mathcal{F}'_j}}, \frac{\mathcal{A}_{\mathcal{F}_j}}{\sum_{j=1}^n \mathcal{A}_{\mathcal{F}_j}} \in [0, 1]$ having the condition $\sum_{j=1}^n \frac{\mathcal{A}'_{\mathcal{F}'_j}}{\sum_{j=1}^n \mathcal{A}'_{\mathcal{F}'_j}} = 1$ and $\sum_{j=1}^n \frac{\mathcal{A}_{\mathcal{F}_j}}{\sum_{j=1}^n \mathcal{A}_{\mathcal{F}_j}} = 1$. Consider

$$SFPWA(\mathcal{F}'_1, \mathcal{F}'_2, \dots, \mathcal{F}'_n) = (\aleph', \hbar', \rho'),$$

$$SFPWA(\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_n) = (\aleph, \hbar, \rho).$$

Since $\aleph'_{f_j} \leq \aleph_{f_j}$, therefore $\sqrt{(\aleph'_{f_j})^2} \leq \sqrt{(\aleph_{f_j})^2}$. We first show that $\aleph' \leq \aleph$. So, we have

$$\begin{aligned} \sqrt{1 - (\aleph'_{f_j})^2} &\geq \sqrt{1 - (\aleph_{f_j})^2} \\ \sqrt{\prod_{j=1}^n \left(1 - (\aleph'_{f_j})^2\right)^{\frac{\mathcal{A}_{f'_j}}{n}}} &\geq \sqrt{\prod_{j=1}^n \left(1 - (\aleph_{f_j})^2\right)^{\frac{\mathcal{A}_{f_j}}{n}}} \\ \sqrt{1 - \prod_{j=1}^n \left(1 - (\aleph'_{f_j})^2\right)^{\frac{\mathcal{A}_{f'_j}}{n}}} &\leq \sqrt{1 - \prod_{j=1}^n \left(1 - (\aleph_{f_j})^2\right)^{\frac{\mathcal{A}_{f_j}}{n}}}. \end{aligned}$$

Thus, $\aleph' \leq \aleph$. Moreover, $\hbar' \leq \hbar$. Then,

$$\prod_{j=1}^n (\hbar'_{f_j})^{\frac{\mathcal{A}_{f'_j}}{n}} \leq \prod_{j=1}^n (\hbar_{f_j})^{\frac{\mathcal{A}_{f_j}}{n}}.$$

Similarly, we can show that $\rho' \geq \rho$. Thus, the theorem is proved.

Theorem 3.4. (Boundedness) Suppose $F_j = (\aleph_{f_j}, \hbar_{f_j}, \rho_{f_j})$ ($j = 1, 2, \dots, n$) is a collection of SFNs with $F_{\min} = \min(F_1, F_2, \dots, F_n)$ and $F_{\max} = \max(F_1, F_2, \dots, F_n)$. Then

$$F_{\min} \subseteq SFPWA(F_1, F_2, \dots, F_n) \subseteq F_{\max}.$$

Proof. Suppose that $F_{\min} = \min(F_1, F_2, \dots, F_n) = (\aleph^-, \hbar^-, \rho^+)$ and $F_{\max} = \max(F_1, F_2, \dots, F_n) = (\aleph^+, \hbar^+, \rho^-)$. Therefore,

$$\aleph^- = \min\{\aleph_j\}, \hbar^- = \min\{\hbar_j\}, \rho^- = \min\{\rho_j\}, \aleph^+ = \max\{\aleph_j\}, \hbar^+ = \max\{\hbar_j\}, \rho^+ = \max\{\rho_j\}.$$

The inequality for membership grade is given as follows:

$$\sqrt{1 - \prod_{j=1}^n \left(1 - (\aleph^-)^2\right)^{\frac{\mathcal{A}_j}{n}}} \leq \sqrt{1 - \prod_{j=1}^n \left(1 - (\aleph_j)^2\right)^{\frac{\mathcal{A}_j}{n}}} \leq \sqrt{1 - \prod_{j=1}^n \left(1 - (\aleph^+)^2\right)^{\frac{\mathcal{A}_j}{n}}}.$$

The inequality for neutral grade is given as follows:

$$\prod_{j=1}^n (\hbar^-)^{\frac{\mathcal{A}_j}{n}} \leq \prod_{j=1}^n (\hbar_j)^{\frac{\mathcal{A}_j}{n}} \leq \prod_{j=1}^n (\hbar^+)^{\frac{\mathcal{A}_j}{n}}.$$

In a similar manner, we can get the other inequalities.

3.2. SFPWG Operator

Definition 3.3. For a collection $F_j = (\mathfrak{N}_{F_j}, \mathfrak{h}_{F_j}, \rho_{F_j})$ ($j = 1, 2, \dots, n$) of SFNs, the SFPWG operator is defined by a mapping $SFPWG : F^n \rightarrow F$, where

$$SFPWG(F_1, F_2, \dots, F_n) = \bigotimes_{j=1}^n (F_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}} = \left(F_1^{\frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j}} \otimes F_2^{\frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j}} \otimes \dots \otimes F_n^{\frac{\mathcal{A}_n}{\sum_{j=1}^n \mathcal{A}_j}} \right),$$

where $\mathcal{A}_j = \prod_{k=1}^{j-1} S^*(F_k)$ ($j = 2, \dots, n$) with $\mathcal{A}_1 = 1$ and $S^*(F_k)$ is the score of $F_k = (\mathfrak{N}_{F_k}, \mathfrak{h}_{F_k}, \rho_{F_k})$.

Theorem 3.5. For a collection $F_j = (\mathfrak{N}_{F_j}, \mathfrak{h}_{F_j}, \rho_{F_j})$ ($j = 1, 2, \dots, n$) of SFNs, the aggregated value of these SFNs by applying SFPWG operator is again a SFN. The formula to calculate the aggregated value is given as follows:

$$SFPWG(F_1, F_2, \dots, F_n) = \bigotimes_{j=1}^n \left(F_j^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}} \right) = \left(\prod_{j=1}^n (\mathfrak{N}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \prod_{j=1}^n (\mathfrak{h}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \sqrt{1 - \prod_{j=1}^n (1 - \rho_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}} \right), \quad (5)$$

where $\mathcal{A}_j = \prod_{k=1}^{j-1} S^*(F_k)$ ($j = 2, \dots, n$) with $\mathcal{A}_1 = 1$ and $S^*(F_k)$ is the score of $F_k = (\mathfrak{N}_{F_k}, \mathfrak{h}_{F_k}, \rho_{F_k})$.

Proof. Similar to the proof of Theorem 3.1.

Theorem 3.6. (Idempotency) Suppose $F_j = (\mathfrak{N}_{F_j}, \mathfrak{h}_{F_j}, \rho_{F_j})$ ($j = 1, 2, \dots, n$) is a collection of SFNs with the condition $F_j = F$ (for all j). Then

$$SFPWG(F_1, F_2, \dots, F_n) = F.$$

Proof. Suppose that $F_j = F$, for all j . Using Equation (5), we have

$$\begin{aligned} SFPWG(F_1, F_2, \dots, F_n) &= \bigotimes_{j=1}^n \left(F_j^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}} \right) = \left(F_1^{\frac{\mathcal{A}_1}{\sum_{j=1}^n \mathcal{A}_j}} \otimes F_2^{\frac{\mathcal{A}_2}{\sum_{j=1}^n \mathcal{A}_j}} \otimes \dots \otimes F_n^{\frac{\mathcal{A}_n}{\sum_{j=1}^n \mathcal{A}_j}} \right) \\ &= \left(\prod_{j=1}^n (\mathfrak{N}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \prod_{j=1}^n (\mathfrak{h}_j)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \sqrt{1 - \prod_{j=1}^n (1 - \rho_j^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}} \right) \\ &= \left(\prod_{j=1}^n (\mathfrak{N})^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \prod_{j=1}^n (\mathfrak{h})^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}, \sqrt{1 - \prod_{j=1}^n (1 - \rho^2)^{\frac{\mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}} \right) \end{aligned}$$

$$\begin{aligned}
&= \left(\frac{\sum_{j=1}^n \mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}, \frac{\sum_{j=1}^n \mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}, \sqrt{1 - (1 - \rho^2)^{\frac{\sum_{j=1}^n \mathcal{A}_j}{\sum_{j=1}^n \mathcal{A}_j}}} \right) \\
&= (\mathfrak{N}, \mathfrak{h}, \sqrt{1 - (1 - \rho^2)}) \\
&= (\mathfrak{N}, \mathfrak{h}, \rho) \\
&= \mathcal{F}.
\end{aligned}$$

Theorem 3.7. (Monotonicity) Suppose $\mathcal{F}'_j = (\mathfrak{N}'_{f_j}, \mathfrak{h}'_{f_j}, \rho'_{f_j})$ and $\mathcal{F}_j = (\mathfrak{N}_{f_j}, \mathfrak{h}_{f_j}, \rho_{f_j})$ are two collections of SFNs, with $(j = 1, 2, \dots, n)$, $\mathfrak{N}'_{f_j} \leq \mathfrak{N}_{f_j}$, $\mathfrak{h}'_{f_j} \leq \mathfrak{h}_{f_j}$ and $\rho'_{f_j} \geq \rho_{f_j}$ for all j . Then

$$SFPWG(\mathcal{F}'_1, \mathcal{F}'_2, \dots, \mathcal{F}'_n) \subseteq SFPWG(\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_n).$$

Proof. Suppose $\mathcal{F}'_j = (\mathfrak{N}'_{f_j}, \mathfrak{h}'_{f_j}, \rho'_{f_j})$ and $\mathcal{F}_j = (\mathfrak{N}_{f_j}, \mathfrak{h}_{f_j}, \rho_{f_j})$ are two collections of SFNs with the following prioritized weight vectors,

$$\frac{\mathcal{A}_{f'_j}}{\sum_{j=1}^n \mathcal{A}_{f'_j}} = \left(\frac{\mathcal{A}_{f'_1}}{\sum_{j=1}^n \mathcal{A}_{f'_j}}, \frac{\mathcal{A}_{f'_2}}{\sum_{j=1}^n \mathcal{A}_{f'_j}}, \dots, \frac{\mathcal{A}_{f'_n}}{\sum_{j=1}^n \mathcal{A}_{f'_j}} \right), \frac{\mathcal{A}_{f_j}}{\sum_{j=1}^n \mathcal{A}_{f_j}} = \left(\frac{\mathcal{A}_{f_1}}{\sum_{j=1}^n \mathcal{A}_{f_j}}, \frac{\mathcal{A}_{f_2}}{\sum_{j=1}^n \mathcal{A}_{f_j}}, \dots, \frac{\mathcal{A}_{f_n}}{\sum_{j=1}^n \mathcal{A}_{f_j}} \right),$$

respectively, where $\frac{\mathcal{A}_{f'_j}}{\sum_{j=1}^n \mathcal{A}_{f'_j}}, \frac{\mathcal{A}_{f_j}}{\sum_{j=1}^n \mathcal{A}_{f_j}} \in [0, 1]$ having the condition $\sum_{j=1}^n \frac{\mathcal{A}_{f'_j}}{\sum_{j=1}^n \mathcal{A}_{f'_j}} = 1$ and $\sum_{j=1}^n \frac{\mathcal{A}_{f_j}}{\sum_{j=1}^n \mathcal{A}_{f_j}} = 1$. Consider

$$SFPWG(\mathcal{F}'_1, \mathcal{F}'_2, \dots, \mathcal{F}'_n) = (\mathfrak{N}', \mathfrak{h}', \rho'),$$

$$SFPWG(\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_n) = (\mathfrak{N}, \mathfrak{h}, \rho).$$

Since $\mathfrak{N}'_{f_j} \leq \mathfrak{N}_{f_j}$, We first show that $\mathfrak{N}' \leq \mathfrak{N}$. So, we have

$$\prod_{j=1}^n (\mathfrak{N}'_{f_j})^{\frac{\mathcal{A}_{f'_j}}{\sum_{j=1}^n \mathcal{A}_{f'_j}}} \leq \prod_{j=1}^n (\mathfrak{N}_{f_j})^{\frac{\mathcal{A}_{f_j}}{\sum_{j=1}^n \mathcal{A}_{f_j}}}.$$

Thus, $\mathfrak{N}' \leq \mathfrak{N}$. Moreover, $\rho' \geq \rho$. Therefore, $\sqrt{(\rho'_{f_j})^2} \geq \sqrt{(\rho_{f_j})^2}$. Then,

$$\sqrt{1 - (\rho'_{f_j})^2} \leq \sqrt{1 - (\rho_{f_j})^2}$$

$$\sqrt{\prod_{j=1}^n (1 - (\rho'_{f_j})^2)^{\frac{\mathcal{A}_{f'_j}}{\sum_{j=1}^n \mathcal{A}_{f'_j}}}} \leq \sqrt{\prod_{j=1}^n (1 - (\rho_{f_j})^2)^{\frac{\mathcal{A}_{f_j}}{\sum_{j=1}^n \mathcal{A}_{f_j}}}}$$

$$\sqrt{1 - \prod_{j=1}^n (1 - (\rho'_{f_j})^2)^{\frac{\mathcal{A}_{f'_j}}{\sum_{j=1}^n \mathcal{A}_{f'_j}}}} \geq \sqrt{1 - \prod_{j=1}^n (1 - (\rho_{f_j})^2)^{\frac{\mathcal{A}_{f_j}}{\sum_{j=1}^n \mathcal{A}_{f_j}}}}.$$

Similarly, we can show that $h' \leq h$. Thus, the theorem is proved.

Theorem 3.8. (Boundedness) Suppose $F_j = (\mathfrak{N}_{F_j}, \mathfrak{h}_{F_j}, \rho_{F_j})$ ($j = 1, 2, \dots, n$) is a collection of SFNs with $F_{\min} = \min(F_1, F_2, \dots, F_n)$ and $F_{\max} = \max(F_1, F_2, \dots, F_n)$. Then

$$F_{\min} \subseteq SFPWG(F_1, F_2, \dots, F_n) \subseteq F_{\max}.$$

Proof. Suppose that $F_{\min} = \min(F_1, F_2, \dots, F_n) = (\mathfrak{N}^-, \mathfrak{h}^-, \rho^+)$ and $F_{\max} = \max(F_1, F_2, \dots, F_n) = (\mathfrak{N}^+, \mathfrak{h}^-, \rho^-)$. Therefore,

$$\mathfrak{N}^- = \min\{\mathfrak{N}_j\}, \mathfrak{h}^- = \min\{\mathfrak{h}_j\}, \rho^- = \min\{\rho_j\}, \mathfrak{N}^+ = \max\{\mathfrak{N}_j\}, \mathfrak{h}^+ = \max\{\mathfrak{h}_j\}, \rho^+ = \max\{\rho_j\}.$$

The inequality for membership grade is given as follows:

$$\prod_{j=1}^n (\mathfrak{N}^-)^{\sum_{j=1}^n \frac{\mathcal{A}_j}{n}} \leq \prod_{j=1}^n (\mathfrak{N}_j)^{\sum_{j=1}^n \frac{\mathcal{A}_j}{n}} \leq \prod_{j=1}^n (\mathfrak{N}^+)^{\sum_{j=1}^n \frac{\mathcal{A}_j}{n}}.$$

The inequality for non-membership grade is given as follows:

$$\sqrt[n]{1 - \prod_{j=1}^n (1 - (\rho^-)^2)^{\sum_{j=1}^n \frac{\mathcal{A}_j}{n}}} \leq \sqrt[n]{1 - \prod_{j=1}^n (1 - (\rho_j)^2)^{\sum_{j=1}^n \frac{\mathcal{A}_j}{n}}} \leq \sqrt[n]{1 - \prod_{j=1}^n (1 - (\rho^+)^2)^{\sum_{j=1}^n \frac{\mathcal{A}_j}{n}}}.$$

In a similar manner, we can get the other inequalities.

4. MCGDM TECHNIQUE BASED ON SFPWA OPERATOR AND SFPWG OPERATOR

In this section, we propose an MCGDM technique by using SFPWA operator and SFPWG operator. Our presented approach is based on SFNs. Further, we present an algorithm of our proposed technique. At last, we present a numerical example to exhibit the validity and authenticity of our presented operators.

4.1. Mathematical Description of the MCGDM Problem

Let $\mathfrak{Y} = \{\mathfrak{Y}_1, \mathfrak{Y}_2, \dots, \mathfrak{Y}_m\}$ be a set of alternatives and $\mathcal{L} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_n\}$ be the set of criteria or attributes. The prioritization among the criteria can be represented by the ordering $\mathcal{L}_1 > \mathcal{L}_2 > \dots > \mathcal{L}_n$, where the criteria \mathcal{L}_p is preferential than the criteria \mathcal{L}_q , $p < q$. Suppose $\mathbb{D} = \{\mathbb{D}_1, \mathbb{D}_2, \dots, \mathbb{D}_l\}$ is the set of experts (decision makers) and the prioritization among the experts can be represented by the ordering $\mathbb{D}_1 > \mathbb{D}_2 > \dots > \mathbb{D}_l$, which shows that the decision maker \mathbb{D}_f is preferential than the decision maker \mathbb{D}_β , $f < \beta$. Also, let $\mathbb{D}^{(a)} = (E_{pq}^{(a)})_{m \times n} = (\mathfrak{N}_{pq}^{(a)}, \mathfrak{h}_{pq}^{(a)}, \rho_{pq}^{(a)})_{m \times n}$ be the spherical fuzzy decision matrix (SFDM) and $(E_{pq}^{(a)}) = (\mathfrak{N}_{pq}^{(a)}, \mathfrak{h}_{pq}^{(a)}, \rho_{pq}^{(a)})$ be the SFN assigned by the decision makers. Here, $\mathfrak{N}_{pq}^{(a)}$, $\mathfrak{h}_{pq}^{(a)}$ and $\rho_{pq}^{(a)}$ represents the membership grade, abstinence grade (neutral grade) and nonmembership grade of the alternatives with respect the criteria, satisfying the condition $(\mathfrak{N}_{pq}^{(a)})^2 + (\mathfrak{h}_{pq}^{(a)})^2 + (\rho_{pq}^{(a)})^2 \leq 1$ for all $(p = 1, 2, \dots, m)(q = 1, 2, \dots, n)$. Basically, there are two types of criteria, benefit type (larger value is better) and cost type (smaller value is better). So, we have to convert these criteria in the same type. Therefore, transform the SFDM $\mathbb{D}^a = (E_{pq}^a)_{m \times n}$ into the normalized SFDM $\mathbb{D}^{*a} = (M_{pq}^a)_{m \times n}$, where,

$$M_{pq}^a = \begin{cases} E_{pq}^a, & \text{for benefit type criteria,} \\ (E_{pq}^a)^c, & \text{for cost type criteria,} \end{cases}$$

where $(E_{pq}^a)^c$ denote the complement of (E_{pq}^a) , such that $(E_{pq}^a)^c = (\rho_{pq}^a, \mathfrak{h}_{pq}^a, \mathfrak{N}_{pq}^a)$, for all $(p = 1, 2, \dots, m)(q = 1, 2, \dots, n)$. The algorithm which is used in the given numerical example is developed in Table 1.

4.2. Numerical Example

A housing society wants to appoint competent and trustworthy Finance Manager. For this purpose, the society decided to invite three decision makers, namely, \mathbb{D}_1 : chartered accountant, \mathbb{D}_2 : owner of the housing society, \mathbb{D}_3 : finance executive of the housing society. Four candidates, namely, $\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3$ and \mathfrak{Y}_4 are considered for interview after preliminary screening. The prioritization among the decision makers is $\mathbb{D}_1 > \mathbb{D}_2 > \mathbb{D}_3 > \mathbb{D}_4$, indicates that the decision maker \mathbb{D}_1 is at high priority level than the other two. The appointment is free

from political or any other kind of influence. The interview panel made strict evaluation among the four candidates for the post of Finance Manager on the basis of the following four criteria:

- \mathcal{L}_1 : Communication skills.
- \mathcal{L}_2 : Past experience.
- \mathcal{L}_3 : Academic background.
- \mathcal{L}_4 : Competency.

The criteria \mathcal{L}_1 is at high priority level than the other criteria. Therefore, the prioritization among the criteria is $\mathcal{L}_1 > \mathcal{L}_2 > \mathcal{L}_3 > \mathcal{L}_4$. The decision values given by the experts are in the form of SFNs. Since, all the considered criteria are of benefit type so normalization is not needed.

Step 1. The decision values given by the experts are represented in Tables 2–4 in the form of SFDMs $\mathbb{D}^a = (E^a)_{4 \times 4}$ ($a = 1, 2, 3$).

Step 2. Calculate the values of \mathcal{A}_{pq}^a ($a = 1, 2, 3$).

$$\mathcal{A}_{mn}^{(1)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

Table 1 | Algorithm.

Algorithm	Steps to solve MCGDM problem by using SFPW aggregation operators
Step 1.	Organize the membership, neutral and non-membership values of the alternatives, assigned by the experts to each criteria in the form of SFDMs as: $\mathbb{D} = \begin{bmatrix} (\aleph_{11}, \hbar_{11}, \rho_{11}) & (\aleph_{12}, \hbar_{12}, \rho_{12}) & \cdots & (\aleph_{1n}, \hbar_{1n}, \rho_{1n}) \\ (\aleph_{21}, \hbar_{21}, \rho_{21}) & (\aleph_{22}, \hbar_{22}, \rho_{22}) & \cdots & (\aleph_{2n}, \hbar_{2n}, \rho_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\aleph_{m1}, \hbar_{m1}, \rho_{m1}) & (\aleph_{m2}, \hbar_{m2}, \rho_{m2}) & \cdots & (\aleph_{mn}, \hbar_{mn}, \rho_{mn}) \end{bmatrix}.$
Step 2.	To calculate the values of \mathcal{A}_{pq}^a ($a = 1, 2, \dots, l$), use $\mathcal{A}_{pq}^{(a)} = \prod_r^{a-1} S^*(M_{pq}^{(a)})$, ($a = 2, 3, \dots, l$), such that $\mathcal{A}_{pq} = 1$.
Step 3.	Aggregate the SF decision matrices $\mathbb{D}^{*(a)} = (M_{pq}^a)_{m \times n}$, ($a = 1, 2, \dots, l$) into the combine SFDM $\mathbb{D} = (M_{pq})_{m \times n}$, ($p = 1, 2, \dots, m$)($q = 1, 2, \dots, n$) by using SFPWA or SFPWG operator as follows: $M_{pq} = (\aleph_{pq}, \hbar_{pq}, \rho_{pq}) = \text{SFPWA}(M_{pq}^{(1)}, M_{pq}^{(2)}, \dots, M_{pq}^{(l)}) = \left(\sqrt{1 - \prod_{a=1}^l (1 - (\aleph_{pq}^a)^2)^{\frac{\mathcal{A}_{pq}^a}{\sum_{a=1}^l \mathcal{A}_{pq}^a}}}, \prod_{a=1}^l (\hbar_{pq}^a)^{\frac{\mathcal{A}_{pq}^a}{\sum_{a=1}^l \mathcal{A}_{pq}^a}}, \prod_{a=1}^l (\rho_{pq}^a)^{\frac{\mathcal{A}_{pq}^a}{\sum_{a=1}^l \mathcal{A}_{pq}^a}} \right),$ or by utilizing SFPWG operator $M_{pq} = (\aleph_{pq}, \hbar_{pq}, \rho_{pq}) = \text{SFPWG}(M_{pq}^{(1)}, M_{pq}^{(2)}, \dots, M_{pq}^{(l)}) = \left(\prod_{a=1}^l (\aleph_{pq}^a)^{\frac{\mathcal{A}_{pq}^a}{\sum_{a=1}^l \mathcal{A}_{pq}^a}}, \prod_{a=1}^l (\hbar_{pq}^a)^{\frac{\mathcal{A}_{pq}^a}{\sum_{a=1}^l \mathcal{A}_{pq}^a}}, \sqrt{1 - \prod_{a=1}^l (1 - (\rho_{pq}^a)^2)^{\frac{\mathcal{A}_{pq}^a}{\sum_{a=1}^l \mathcal{A}_{pq}^a}}} \right).$
Step 4.	Find the values of \mathcal{A}_{pq} ($p = 1, 2, \dots, m$)($q = 1, 2, \dots, n$) as follows $\mathcal{A}_{pq} = \prod_s^{q-1} S^*(M_{ps}), (p = 1, 2, \dots, m)(q = 1, 2, \dots, n), \text{ such that } \mathcal{A}_{ps} = 1.$

(Continued)

Table 1 | Algorithm. (Continued)

Algorithm	Steps to solve MCGDM problem by using SFPW aggregation operators
Step 5.	For each alternative \mathfrak{Y}_p , aggregate the SFN M_{pq} by using the presented SFPWA or SFPWG operator. $M_p = (\mathfrak{S}_p, \mathfrak{h}_p, \rho_p) = \text{SFPWA}(M_{p1}, M_{p2}, \dots, M_{pn}) = \left(\sqrt{1 - \prod_{q=1}^n (1 - (\mathfrak{S}_{pq})^2)^{\frac{\mathcal{A}_{pq}}{\sum_{q=1}^n \mathcal{A}_{pq}}}}, \prod_{q=1}^n (\mathfrak{h}_{pq})^{\frac{\mathcal{A}_{pq}}{\sum_{q=1}^n \mathcal{A}_{pq}}}, \prod_{q=1}^n (\rho_{pq})^{\frac{\mathcal{A}_{pq}}{\sum_{q=1}^n \mathcal{A}_{pq}}} \right),$ or by utilizing SFPWG operator $M_p = (\mathfrak{S}_p, \mathfrak{h}_p, \rho_p) = \text{SFPWG}(M_{p1}, M_{p2}, \dots, M_{pn}) = \left(\prod_{q=1}^n (\mathfrak{S}_{pq})^{\frac{\mathcal{A}_{pq}}{\sum_{q=1}^n \mathcal{A}_{pq}}}, \prod_{q=1}^n (\mathfrak{h}_{pq})^{\frac{\mathcal{A}_{pq}}{\sum_{q=1}^n \mathcal{A}_{pq}}}, \sqrt{1 - \prod_{q=1}^n (1 - (\rho_{pq})^2)^{\frac{\mathcal{A}_{pq}}{\sum_{q=1}^n \mathcal{A}_{pq}}} \right).$
Step 6.	Calculate the score values by using Equation (3).
Step 7.	Select the alternative having highest score value.

Table 2 | Spherical fuzzy decision matrix (SFDM) $\mathbb{D}^1 = (E_{pq}^1)_{4 \times 4}$.

	\mathcal{L}_1	\mathcal{L}_2	\mathcal{L}_3	\mathcal{L}_4
\mathfrak{Y}_1	(0.7,0.3,0.5)	(0.9,0.2,0.3)	(0.5,0.4,0.3)	(0.1,0.6,0.4)
\mathfrak{Y}_2	(0.4,0.6,0.1)	(0.6,0.4,0.3)	(0.6,0.6,0.3)	(0.8,0.5,0.3)
\mathfrak{Y}_3	(0.7,0.5,0.4)	(0.5,0.4,0.2)	(0.8,0.2,0.2)	(0.7,0.5,0.2)
\mathfrak{Y}_4	(0.8,0.2,0.1)	(0.6,0.5,0.5)	(0.9,0.2,0.1)	(0.9,0.1,0.1)

Table 3 | Spherical fuzzy decision matrix (SFDM) $\mathbb{D}^2 = (E_{pq}^2)_{4 \times 4}$.

	\mathcal{L}_1	\mathcal{L}_2	\mathcal{L}_3	\mathcal{L}_4
\mathfrak{Y}_1	(0.5,0.3,0.3)	(0.8,0.1,0.2)	(0.6,0.5,0.3)	(0.3,0.5,0.4)
\mathfrak{Y}_2	(0.3,0.6,0.1)	(0.6,0.4,0.2)	(0.6,0.4,0.3)	(0.8,0.4,0.3)
\mathfrak{Y}_3	(0.6,0.4,0.4)	(0.5,0.3,0.2)	(0.9,0.2,0.1)	(0.7,0.5,0.1)
\mathfrak{Y}_4	(0.8,0.2,0.1)	(0.7,0.5,0.4)	(0.8,0.3,0.2)	(0.8,0.2,0.1)

Table 4 | Spherical fuzzy decision matrix (SFDM) $\mathbb{D}^3 = (E_{pq}^3)_{4 \times 4}$.

	\mathcal{L}_1	\mathcal{L}_2	\mathcal{L}_3	\mathcal{L}_4
\mathfrak{Y}_1	(0.6,0.3,0.2)	(0.8,0.2,0.3)	(0.5,0.5,0.2)	(0.2,0.5,0.3)
\mathfrak{Y}_2	(0.4,0.5,0.2)	(0.7,0.3,0.2)	(0.6,0.4,0.4)	(0.8,0.4,0.4)
\mathfrak{Y}_3	(0.8,0.3,0.4)	(0.5,0.4,0.4)	(0.8,0.2,0.3)	(0.7,0.5,0.3)
\mathfrak{Y}_4	(0.8,0.3,0.2)	(0.7,0.5,0.5)	(0.9,0.3,0.1)	(0.8,0.2,0.2)

$$\mathcal{A}_{mn}^{(2)} = \begin{bmatrix} 0.7167 & 0.8933 & 0.6667 & 0.4967 \\ 0.5967 & 0.7033 & 0.6367 & 0.7667 \\ 0.6933 & 0.6833 & 0.8533 & 0.7333 \\ 0.8633 & 0.6200 & 0.9200 & 0.9300 \end{bmatrix},$$

$$\mathcal{A}_{mn}^{(3)} = \begin{bmatrix} 0.4945 & 0.7712 & 0.4489 & 0.2781 \\ 0.3421 & 0.5064 & 0.4478 & 0.6108 \\ 0.4714 & 0.4829 & 0.7850 & 0.5451 \\ 0.7453 & 0.4298 & 0.7698 & 0.8029 \end{bmatrix}.$$

Step 3. Apply the SFPWA operator to combine all the matrices into a combine (aggregated) single matrix (Table 5).

Step 4. Calculate the values of \mathcal{A}_{pq} ($p = 1, 2, \dots, m$), ($q = 1, 2, \dots, n$).

$$\mathcal{A}_{mn} = \begin{bmatrix} 1 & 0.7129 & 0.6233 & 0.4172 \\ 1 & 0.5962 & 0.4361 & 0.2939 \\ 1 & 0.7194 & 0.4945 & 0.3999 \\ 1 & 0.8582 & 0.5612 & 0.5015 \end{bmatrix}.$$

Step 5. Apply the SFPWA operator to combine the values E_{pq} ($p = 1, 2, 3, 4$) of \mathbb{D} in Table 5 to get E_p .

$$\begin{aligned} \mathfrak{Y}_1 &= (0.6695, 0.3327, 0.3101), & \mathfrak{Y}_2 &= (0.5769, 0.4846, 0.1902), \\ \mathfrak{Y}_3 &= (0.6999, 0.4276, 0.2616), & \mathfrak{Y}_4 &= (0.7968, 0.2891, 0.1897). \end{aligned}$$

Step 6. Calculate the score values of all SFNs obtained in Step 5.

$$\begin{aligned} S^*(\mathfrak{Y}_1) &= \frac{1}{3} (2 + (0.6695)^2 - (0.3327)^2 - (0.3101)^2) = 0.7471, \\ S^*(\mathfrak{Y}_2) &= \frac{1}{3} (2 + (0.5769)^2 - (0.4846)^2 - (0.1902)^2) = 0.6873, \\ S^*(\mathfrak{Y}_3) &= \frac{1}{3} (2 + (0.6999)^2 - (0.4276)^2 - (0.2616)^2) = 0.7462, \\ S^*(\mathfrak{Y}_4) &= \frac{1}{3} (2 + (0.7968)^2 - (0.2891)^2 - (0.1897)^2) = 0.8384. \end{aligned}$$

Therefore,

$$S^*(\mathfrak{Y}_4) > S^*(\mathfrak{Y}_1) > S^*(\mathfrak{Y}_3) > S^*(\mathfrak{Y}_2).$$

Step 7. The best alternative is \mathfrak{Y}_4 .

Now, we solve the problem by utilizing SFPWG operator.

Step 1*. This Step is same as that of Step 1.

Step 2*. This Step is same as that of Step 2.

Step 3*. Apply the SFPWG operator to combine all the matrices into an aggregated single matrix (Table 6).

Step 4*. Calculate the values of \mathcal{A}_{pq} ($p = 1, 2, \dots, m$), ($q = 1, 2, \dots, n$).

$$\mathcal{A}_{mn} = \begin{bmatrix} 1 & 0.6896 & 0.5977 & 0.3982 \\ 1 & 0.5937 & 0.4320 & 0.2906 \\ 1 & 0.7124 & 0.4864 & 0.3883 \\ 1 & 0.8569 & 0.5563 & 0.4933 \end{bmatrix}.$$

Step 5*. Apply the SFPWG operator to combine the values E_{pq} ($p = 1, 2, 3, 4$) of \mathbb{D} in Table 6 to get E_p .

$$\begin{aligned} \mathfrak{Y}_1 &= (0.5202, 0.3326, 0.3432), & \mathfrak{Y}_2 &= (0.5071, 0.4848, 0.3923), \\ \mathfrak{Y}_3 &= (0.6358, 0.4273, 0.3118), & \mathfrak{Y}_4 &= (0.7691, 0.2742, 0.2909). \end{aligned}$$

Table 5 | Aggregated spherical fuzzy decision matrix (SFDM) $\mathbb{D} = (E_{pq})_{4 \times 4}$.

	\mathcal{L}_1	\mathcal{L}_2	\mathcal{L}_3	\mathcal{L}_4
\mathfrak{Y}_1	(0.6329,0.3779,0.3452)	(0.8466,0.1585,0.2619)	(0.5353,0.4499,0.2753)	(0.1942,0.5541,0.3824)
\mathfrak{Y}_2	(0.3729,0.5810,0.1130)	(0.6264,0.3745,0.2403)	(0.6000,0.4859,0.3191)	(0.8000,0.4394,0.3230)
\mathfrak{Y}_3	(0.7011,0.4164,0.3999)	(0.5000,0.3656,0.2334)	(0.8409,0.5000,0.1803)	(0.7000,0.5000,0.1763)
\mathfrak{Y}_4	(0.8000,0.2245,0.1219)	(0.6559,0.5000,0.4674)	(0.8738,0.2580,0.1267)	(0.8456,0.1552,0.1226)

Table 6 | Aggregated spherical fuzzy decision matrix (SFDM) $\mathbb{D} = (E_{pq})_{4 \times 4}$.

	\mathcal{L}_1	\mathcal{L}_2	\mathcal{L}_3	\mathcal{L}_4
\mathfrak{Y}_1	(0.6064,0.3779,0.3952)	(0.8361,0.1585,0.2712)	(0.5296,0.4499,0.2821)	(0.1516,0.5541,0.3865)
\mathfrak{Y}_2	(0.3661,0.5810,0.1239)	(0.6216,0.3745,0.2509)	(0.6000,0.4859,0.3248)	(0.8000,0.4394,0.3294)
\mathfrak{Y}_3	(0.6859,0.4164,0.4000)	(0.5000,0.3653,0.2611)	(0.8311,0.5000,0.2138)	(0.7000,0.5000,0.2068)
\mathfrak{Y}_4	(0.8000,0.2245,0.1366)	(0.6493,0.5000,0.4731)	(0.8645,0.2580,0.1427)	(0.8352,0.1552,0.1375)

Step 6*. Calculate the score values of all SFNs obtained in Step 5*.

$$\begin{aligned} S^*(\mathfrak{Y}_1) &= \frac{1}{3} \left(2 + (0.5202)^2 - (0.3326)^2 - (0.3432)^2 \right) = 0.6807, \\ S^*(\mathfrak{Y}_2) &= \frac{1}{3} \left(2 + (0.5071)^2 - (0.4848)^2 - (0.3923)^2 \right) = 0.6227, \\ S^*(\mathfrak{Y}_3) &= \frac{1}{3} \left(2 + (0.6358)^2 - (0.4273)^2 - (0.3118)^2 \right) = 0.7081, \\ S^*(\mathfrak{Y}_4) &= \frac{1}{3} \left(2 + (0.7691)^2 - (0.2742)^2 - (0.2909)^2 \right) = 0.8106. \end{aligned}$$

Therefore,

$$S^*(\mathfrak{Y}_4) > S^*(\mathfrak{Y}_3) > S^*(\mathfrak{Y}_1) > S^*(\mathfrak{Y}_2).$$

Step 7*. The best alternative is \mathfrak{Y}_4 .

Table 7 represents the score values and ranking order of alternatives.

The graphical representation of the score values of alternatives using presented operators is displayed in Figure 1.

5. COMPARATIVE ANALYSIS

In this section, we provide a comparison of our presented approach with existing methods to check the authenticity and reliability of our proposed operators. Now, we solve the above numerical example using spherical fuzzy weighted averaging (SFWA) operator [25] and spherical fuzzy weighted geometric (SFWG) operator [25]. In order to make comparison, we assign the weight vector $W = (0.4, 0.3, 0.2, 0.1)^T$ to the criteria.

The results of the alternatives using existing operators are illustrated as follows:

- When we apply the SFWA operator [25], we obtain the following aggregated values of the alternatives:

$$\begin{aligned} \mathfrak{Y}_1 &= (0.6720, 0.3054, 0.3283), & \mathfrak{Y}_2 &= (0.5061, 0.5134, 0.2104), \\ \mathfrak{Y}_3 &= (0.6670, 0.3916, 0.3437), & \mathfrak{Y}_4 &= (0.7601, 0.3293, 0.2203). \end{aligned}$$

Table 7 | Final scores and ranking of alternatives.

Operators	$S^*(\mathfrak{Y}_1)$	$S^*(\mathfrak{Y}_2)$	$S^*(\mathfrak{Y}_3)$	$S^*(\mathfrak{Y}_4)$	Final Ranking
SFPWA	0.7471	0.6873	0.7462	0.8384	$S^*(\mathfrak{Y}_4) > S^*(\mathfrak{Y}_1) > S^*(\mathfrak{Y}_3) > S^*(\mathfrak{Y}_2)$
SFPWG	0.6807	0.6227	0.7081	0.8106	$S^*(\mathfrak{Y}_4) > S^*(\mathfrak{Y}_3) > S^*(\mathfrak{Y}_1) > S^*(\mathfrak{Y}_2)$

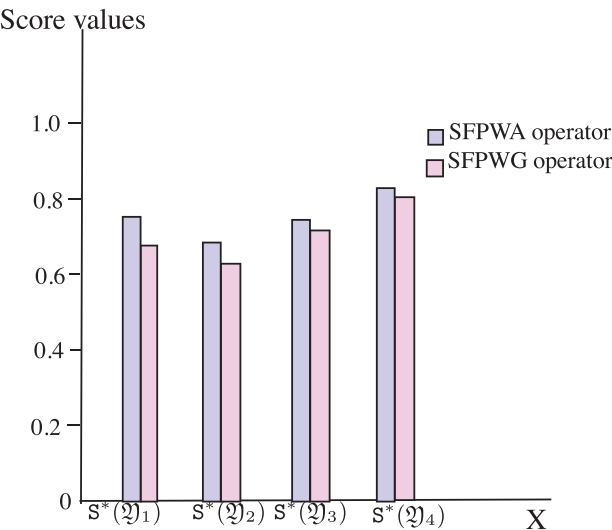


Figure 1 | Graphical representation using proposed operators.

Using Equation (1) on above aggregated values, we get the following score values:

$$S(\mathfrak{Y}_1) = 0.6794, \quad S(\mathfrak{Y}_2) = 0.5941, \quad S(\mathfrak{Y}_3) = 0.6439, \quad S(\mathfrak{Y}_4) = 0.7368.$$

The ranking order of the alternatives on the basis of score values is $S(\mathfrak{Y}_4) > S(\mathfrak{Y}_1) > S(\mathfrak{Y}_3) > S(\mathfrak{Y}_2)$, which shows that \mathfrak{Y}_4 is the best alternative among all.

- On applying the SFWG operator [25], we obtain the following aggregated values of the alternatives:

$$\begin{aligned} \mathfrak{Y}_1 &= (0.5105, 0.3054, 0.3112), & \mathfrak{Y}_2 &= (0.4490, 0.5134, 0.2068), \\ \mathfrak{Y}_3 &= (0.5999, 0.3916, 0.3282), & \mathfrak{Y}_4 &= (0.7328, 0.3293, 0.2777). \end{aligned}$$

Using Equation (1) on above aggregated values, we get the following score values:

$$S(\mathfrak{Y}_1) = 0.6313, \quad S(\mathfrak{Y}_2) = 0.5763, \quad S(\mathfrak{Y}_3) = 0.6267, \quad S(\mathfrak{Y}_4) = 0.7086.$$

The ranking order of the alternatives on the basis of score values is $S(\mathfrak{Y}_4) > S(\mathfrak{Y}_1) > S(\mathfrak{Y}_3) > S(\mathfrak{Y}_2)$, which depicts that \mathfrak{Y}_4 is the best alternative among all.

Table 8 represents the aggregated results obtained by utilizing the existing operators.

Table 9 displays the score values and ranking order of alternatives.

Table 10 represents the final ranking order of alternatives by applying the existing and proposed operators.

It can be seen that the results calculated in this section by using existing operators are similar to our results. It can also be observed from Table 10 that the optimal (best) alternative by using existing and proposed operators is \mathfrak{Y}_4 . Thus, the Table 10 depicts that the results of [25] are consistent with our SFPWA and SFPWG approach, which exhibits the validity and authenticity of the method. The graphical representation of the score values of alternatives using SFWA and SFWG operators is displayed in Figure 2.

6. CONCLUSIONS

Spherical fuzzy model is an indispensable tool to model the unclear information, with the condition $0 \leq \mathfrak{N}^2 + \mathfrak{h}^2 + \rho^2 \leq 1$, and an efficient tool to deal the information, when there occurs a neutral or abstinence kind of opinion. This dominating feature of spherical fuzzy model makes it more effective to represent the relevant information of an alternative. Inspired by the structure and properties of SFSs, we have presented some AOs under the spherical fuzzy environment. In this paper, we have explored the concept of prioritized weighted averaging operators within spherical fuzzy frame work. Two AOs, namely, SFPWA operator and SFPWG operator are introduced. In DM problems, it might be possible that the criteria and decision makers are at different priority levels. The assumption of the same priority levels for the

Table 8 | Aggregated values using existing operator.

Operators	\mathfrak{Y}_1	\mathfrak{Y}_2	\mathfrak{Y}_3	\mathfrak{Y}_4
SFWA operator [25]	(0.6720,0.3054,0.3283)	(0.5061,0.5134,0.2104)	(0.6670,0.3916,0.3437)	(0.7601,0.3293,0.2203)
SFWG operator [25]	(0.5105,0.3054,0.3112)	(0.4490,0.5134,0.2068)	(0.5999,0.3916,0.3282)	(0.7328,0.3293,0.2777)

SFWA, spherical fuzzy weighted averaging; SFWG, spherical fuzzy weighted geometric.

Table 9 | Final scores and ranking using existing operators.

Operators	$S(\mathfrak{Y}_1)$	$S(\mathfrak{Y}_2)$	$S(\mathfrak{Y}_3)$	$S(\mathfrak{Y}_4)$	Final Ranking
SFWA [25]	0.6794	0.5941	0.6439	0.7368	$S(\mathfrak{Y}_4) > S(\mathfrak{Y}_1) > S(\mathfrak{Y}_3) > S(\mathfrak{Y}_2)$
SFWG [25]	0.6313	0.5763	0.6267	0.7086	$S(\mathfrak{Y}_4) > S(\mathfrak{Y}_1) > S(\mathfrak{Y}_3) > S(\mathfrak{Y}_2)$

SFWA, spherical fuzzy weighted averaging; SFWG, spherical fuzzy weighted geometric.

Table 10 | Final ranking.

Operators	Final Ranking
SFWA operator [25]	$S(\mathfrak{Y}_4) > S(\mathfrak{Y}_1) > S(\mathfrak{Y}_3) > S(\mathfrak{Y}_2)$
SFWG operator [25]	$S(\mathfrak{Y}_4) > S(\mathfrak{Y}_1) > S(\mathfrak{Y}_3) > S(\mathfrak{Y}_2)$
Our proposed SFPWA operator	$S^*(\mathfrak{Y}_4) > S^*(\mathfrak{Y}_1) > S^*(\mathfrak{Y}_3) > S^*(\mathfrak{Y}_2)$
Our proposed SFPWG operator	$S^*(\mathfrak{Y}_4) > S^*(\mathfrak{Y}_3) > S^*(\mathfrak{Y}_1) > S^*(\mathfrak{Y}_2)$

SFWA, spherical fuzzy weighted averaging; SFWG, spherical fuzzy weighted geometric; SFPWA, spherical fuzzy prioritized weighted averaging; SFPWG, operator and spherical fuzzy prioritized weighted geometric.

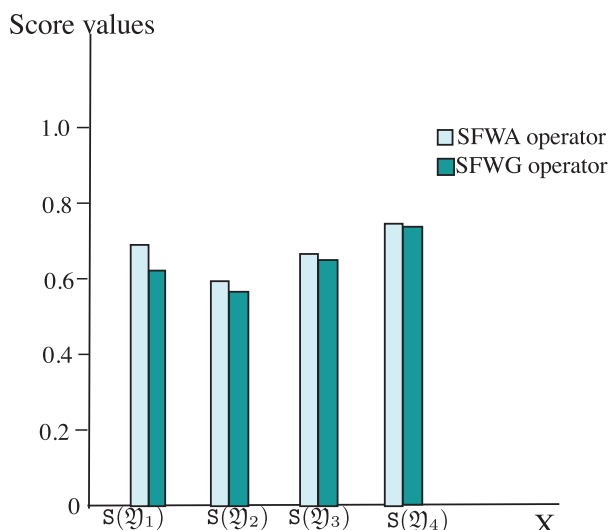


Figure 2 | Graphical representation using existing operators.

criteria and decision makers may not be feasible in all situations. Therefore, the spherical fuzzy prioritized weighted AOs have the ability to tackle the information having prioritization among the data. This salient feature of spherical fuzzy prioritized weighted AOs makes it superior, as it can handle the prioritized information within the spherical fuzzy circumstances.

To rank the alternatives, we have developed a new score function. Apart from this, we have investigated several desirable properties including, idempotency, monotonicity and boundedness of the aforesaid operators with their proofs. By utilizing the presented AOs, we have successfully developed an MCGDM method. Furthermore, we have given an algorithm of the MCGDM method and provided the solution of a numerical example to elaborate the applicability as well as authenticity of proposed operators. We have also delivered a comparison of our newly proposed method with already established methods. Hence, it is concluded that the presented work provides a convenient and a more reasonable platform to address the MCGDM problems.

We are planning to extend our work to MCGDM techniques, including ELECTRE-II, III, IV, PROMETHEE-I, II methods in complex spherical fuzzy environment.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

AUTHOR CONTRIBUTIONS

Investigation, Muhammad Akram, Samirah Alsulami, Ayesha Khan and Faruk Karaaslan; Writing original draft, Muham-mad Akram and Ayesha Khan; Writing review and editing, Faruk Karaaslan and Samirah Alsulami.

REFERENCES

- [1] L.A. Zadeh, Fuzzy sets, *Inf. Control.* 8 (1965), 338–353.
- [2] K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets Syst.* 20 (1986), 87–96.
- [3] R.R. Yager, Pythagorean fuzzy subsets, in *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, IEEE, Edmonton, Canada, 2013, pp. 57–61.
- [4] B.C. Cuong, Picture fuzzy sets-first results, Part 1, in *Seminar Neuro-Fuzzy Systems with Applications*, Institute of Mathematics, Vietnam Academy of Science and Technology, Hanoi, Vietnam, 2013.
- [5] B.C. Cuong, Picture fuzzy sets-first results, Part 2, in *Seminar Neuro-Fuzzy Systems with Applications*, Institute of Mathematics, Vietnam Academy of Science and Technology, Hanoi, Vietnam, 2013.
- [6] F.K. Gündođdu, C. Kahraman, Spherical fuzzy sets and spherical fuzzy TOPSIS method, *J. Intell. Fuzzy Syst.* 36 (2019), 337–352.
- [7] F.K. Gündođdu, C. Kahraman, Spherical fuzzy sets and decision making applications, in: C. Kahraman, S. Cebi, S. Cevik Onar, B. Oztaysi, A. Tolga, I. Sari (Eds.), *Intelligent and Fuzzy Techniques in Big Data Analytics and Decision Making*, INFUS 2019, Advances in Intelligent Systems and Computing, vol. 1029, Springer, Cham, Switzerland, 2020.

- [8] F.K. Gündogdu, C. Kahraman, A novel spherical fuzzy analytic hierarchy process and its renewable energy application, *Soft Comput.* 24 (2020), 4607–4621.
- [9] C. Kahraman, F.K. Gündogdu, S.C. Onar, B. Oztaysi, Hospital location selection using spherical fuzzy TOPSIS, in 2019 Conference of the International Fuzzy Systems Association and the European Society for Fuzzy Logic and Technology (EUSFLAT 2019), Atlantis Press, 2019.
- [10] M. Akram, D. Saleem, T. Al-Hawary, Spherical fuzzy graphs with application to decision-making, *Math. Comput. Appl.* (2020), 25, 1–8.
- [11] M. Akram, Decision making method based on spherical fuzzy graphs, in: C. Kahraman, I. Otay (Eds.), *Spherical Fuzzy Sets Book*, Studies in Fuzziness and Soft Computing, Springer, Cham, Switzerland, 2020.
- [12] Z. Xu, Intuitionistic fuzzy aggregation operators, *IEEE Trans. Fuzzy Syst.* 15 (2007), 1179–1187.
- [13] Z. Xu, R.R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, *Int. J. Gen. Syst.* 35 (2006), 417–433.
- [14] D.F. Li, Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets, *Expert Syst. Appl.* 37 (2010), 8673–8678.
- [15] S. Zeng, W. Sua, Intuitionistic fuzzy ordered weighted distance operator, *Knowl. Based Syst.* 24 (2011), 1224–1232.
- [16] R.R. Yager, Pythagorean membership grades in multicriteria decision making, *IEEE Trans. Fuzzy Syst.* 22 (2013), 958–965.
- [17] X. Peng, H. Yuan, Fundamental properties of Pythagorean fuzzy aggregation operators, *Fundam. Inform.* 147 (2016), 415–446.
- [18] W.F. Liu, J. Chang, X. He, Generalized pythagorean fuzzy aggregation operators and applications in decision making, *Control Decis.* 31 (2016), 2280–2286.
- [19] M. Akram, A. Khan, Complex pythagorean dombi fuzzy graphs for decision making, *Granul. Comput.* (2020).
- [20] M. Akram, A. Khan, A.B. Saeid, Complex pythagorean dombi fuzzy operators using aggregation operators and their decision-making, *Expert Syst.* (2020).
- [21] M. Akram, G. Shahzadi, A hybrid decision-making model under q-rung orthopair fuzzy Yager aggregation operators, *Granul. Comput.* (2020).
- [22] Z.S. Chen, C. Yu, K.S. Chin, L. Martínez, An enhanced ordered weighted averaging operators generation algorithm with applications for multicriteria decision making, *Appl. Math. Modell.* 71 (2019), 467–490.
- [23] G. Shahzadi, M. Akram, A.N. Al-Kenani, Decision-making approach under Pythagorean fuzzy Yager weighted operators, *Mathematics.* 8 (2020), 70.
- [24] Y. Yang, Z.S. Chen, Y.H. Chen, K.S. Chin, Interval-valued Pythagorean fuzzy Frank power aggregation operators based on an isomorphic Frank dual triple, *Int. J. Comput. Intell. Syst.* 11 (2018), 1091–1110.
- [25] S. Ashraf, S. Abdullah, Spherical aggregation operators and their application in multi-attribute group decision-making, *Int. J. Intell. Syst.* 34 (2019), 493–523.
- [26] S. Ashraf, S. Abdullah, T. Mahmood, Spherical fuzzy Dombi aggregation operators and their application in group decision making problems, *J. Amb. Intell. Hum. Comput.* 11 (2020), 2731–2749.
- [27] M. Akram, A. Habib, J.C.R. Alcantud, An optimization study based on Dijkstra algorithm for a network with trapezoidal picture fuzzy numbers, *Neural Comput. Appl.* (2020).
- [28] H. Garg, M. Munir, K. Ullah, T. Mahmood, N. Jan, Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators, *Symmetry.* 10 (2018), 670.
- [29] P.A. Perveen, F.J.J. Sunil, K.V. Babitha, H. Garg, Spherical fuzzy soft sets and its applications in decision-making problems, *J. Intell. Fuzzy Syst.* 37 (2019), 8237–8250.
- [30] K. Ullah, H. Garg, T. Mahmood, N. Jan, Z. Ali, Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making, *Soft Comput.* 24 (2020), 1647–1659.
- [31] K. Ullah, T. Mahmood, H. Garg, Evaluation of the performance of search and rescue robots using T-spherical fuzzy hamacher aggregation operators, *Int. J. Fuzzy Syst.* 22 (2020), 570–582.
- [32] S. Zeng, H. Garg, M. Munir, T. Mahmood, A. Hussain, A multi-attribute decision making process with immediate probabilistic interactive averaging aggregation operators of T-spherical fuzzy sets and its application in the selection of solar cells, *Energies.* 12 (2019), 4436.
- [33] P. Liu, G. Shahzadi, M. Akram, Specific types of q-rung picture fuzzy Yager aggregation operators for decision-making, *Int. J. Comput. Intell. Syst.* 13 (2020), 1072–1091.
- [34] X. Ma, M. Akram, K. Zahid, J.C.R. Alcantud, Group decision-making framework using complex Pythagorean fuzzy information, *Neural Comput. Appl.* (2020).
- [35] D. Yu, Intuitionistic fuzzy prioritized operators and their application in multi-criteria group decision making, *Technol. Econ. Develop. Econ.* 19 (2013), 1–21.
- [36] D.J. Yu, Multi-criteria decision making based on generalized prioritized aggregation operators under intuitionistic fuzzy environment, *Int. J. Fuzzy Syst.* 15 Mar(2013), 147–54.
- [37] R. Arora, H. Garg, Group decision-making method based on prioritized linguistic intuitionistic fuzzy aggregation operators and its fundamental properties, *Comput. Appl. Math.* 38 (2019), 36.
- [38] H. Garg, Multi-criteria decision-making method based on prioritized Muirhead mean aggregation operator under neutrosophic set environment, *Symmetry.* 10 (2018), 280.
- [39] R. Arora, H. Garg, Prioritized averaging/geometric aggregation operators under the intuitionistic fuzzy soft set environment, *Sci. Iran.* 25 (2018), 466–482.
- [40] H. Gao, Pythagorean fuzzy Hamacher prioritized aggregation operators in multiple attribute decision making, *J. Intell. Fuzzy Syst.* 35 (2018), 2229–2245.
- [41] M.S.A. Khan, S. Abdullah, A. Ali, F. Amin, Pythagorean fuzzy prioritized aggregation operators and their application to multi-attribute group decision making, *Granul. Comput.* 4 (2019), 249–263.

- [42] P. Liu, M. Akram, A. Sattar, Extensions of prioritized weighted aggregation operators for decision-making under complex q-rung orthopair fuzzy information, *J. Intell. Fuzzy Syst.* (2020).
- [43] R. Verma, B. Sharma, Intuitionistic fuzzy Einstein prioritized weighted average operators and their application to multiple attribute group decision making, *Appl. Math. Inf. Sci.* 9 (2015), 3095.
- [44] R.R. Yager, Prioritized aggregation operators, *Int. J. Approx. Reason.* 48 (2008), 263–274.