



Reliability Analysis of Weighted- *k*-out-of- *n*: G System Consisting of Two Different Types of Nonidentical Components Each with its Own Positive Integer-Valued Weight

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ABSTRACT

This paper introduces a special case of weighted- *k*-out-of- *n*:G system formed from two types of nonidentical components with different weights. This system consists of *n* nonidentical components each with its own positive integer-valued weight which are categorized into two groups with respect to their duties and services. In fact, we have a system consisting *n* components such that n_1 of them each with its own weight ω_i and reliability p_{1i} and n_2 of them each with its own weight ω_i^* and reliability p_{2i} . If the total weights of the functioning components exceeds a prespecified threshold *k*, the system is supposed to work. The reliability of system is obtained based on the total weight of all working components in both group. The survival function and mean time to failure are presented. Also, the component importance of this system are studied.

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1. INTRODUCTION

In most real-life systems, the total contribution of the components plays an important role and must be above a specified performance level. In many situations, the components contribute differently to the capacity of the system. The systems with weighted components with unequal weights are introduced by Wu and Chen [1] to deal with this situation which has been studied in the literature. A system including n components with their different positive integer weights is known as weighted-k-out-of-n:G system when it works if and only if the total weight of working components is above a given threshold k.

Chen and Yang [2] extended the one-stage weighted- k-out-of- n model to the two-stage weighted- k-out-of- n model with components in common. Samaniego and Shaked [3] presented a review on weighted- k-out-of- n systems. Li and Zuo [4] provided two models of multistate weighted- k-out-of- n system models and presented the recursive algorithms for their reliability evaluation. Navarro *et al.* [5] extended the signature-based representations of the reliability functions of coherent systems to systems with heterogeneous components. Eryilmaz [6] introduced a k-out-of- n system with random weights for components and investigated the reliability properties of the system. Eryilmaz and Bozbulut [7] studied a multi-state weighted- k-out-of- n:G system model in a dynamic setup and yielded an algorithmic approach for its dynamic reliability analysis. In order to measure the components importance in k-out-of- n system with random weights, Rahmani *et al.* [8] defined the weighted importance (WI) measure that depends only on the distribution of component weights and also, Meshkat and Mahmoudi [9] generalized this measure for two component i and j and the relation of these measures is investigated with Birnbaum reliability importance measure.

Eryilmaz and Sarikaya [10] studied the special case of weighted- *k*-out-of- *n*: G system containing two types of components, each group having different weights and reliabilities such that one group have the common weight ω and reliability p_1 , while the other have common weight ω^* and reliability p_2 . They also obtained the nonrecursive equations for the system reliability, survival function and mean time to failure (MTTF). Recently, Eryilmaz [11] introduced the (k_1, k_2, \dots, k_m) -out-of- *n* system including n_i components of type *i* for $i = 1, \dots, m$ and $n = \sum_{i=1}^m n_i$. The corresponding system is assumed to work if at least k_1 components of type 1, k_2 components of type 2, ..., k_m components of type *m* function. Its reliability and the setup of weighted- (k_1, k_2, \dots, k_m) -out-of- *n* system is also defined and studied.

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In this study, we consider a general setup of weighted- *k*-out-of- *n*:G system in which one group consists of n_1 components each with its own positive integer-valued weight ω_i and reliability p_{1i} , while the other group consists of n_2 components each with its own positive integer-valued weight ω_i^* and reliability p_{2i} , $(n = n_1 + n_2)$. This system operates if the total weight of all working components is at least *k*. Therefore, its reliability is obtained based on the total weight of all working components in both group. This setup of weighted- *k*-out-of- *n*:G system containing two types of nonidentical components might be useful in practice. As an illustration, assume that a production line (system) is constituted by *n* devices (components) in two groups with n_1 and n_2 devices ($n = n_1 + n_2$), respectively, which each group has their own duties and services and different capacities (weights). For an instance, a production line in tile factory consists of two groups of devices, one group including some devices to combine the materials and bake the tiles and the other group including some devices to implement the layout and color on the tiles and glaze them. Finally to have a complete tile, some devices in the line should be operate such that a certain capacity of total capacities is provided. Therefore, we have a system consisting *n* components such that n_1 of them each with its own weight ω_i and reliability p_{1i} and n_2 of them each with its own weight ω_i^* and reliability p_{2i} . To operate the system, a certain capacity of components denoted by threshold *k*, should be provided.

The remainder of the paper is arranged as follows: in Section 2, for mentioned weighted-*k*-out-of-*n*:G system, the description of system and its reliability evaluation is provided. The survival function and MTTF of this system are studied in Section 3. In Section 4, the Birnbaum reliability importance is investigated. Finally, the concluding remarks are given in Section 5.

2. THE SYSTEM MODEL

In this section, first some notations and the description of modelling main assumptions are provided for this general setup of weighted*k*-out-of- *n*:G system.

2.1. Notations

- *n* Number of components in a system
- X_i State of *i*-th component of the system:
 - $X_i = 1$ if component is functioning; $X_i = 0$ if it is failed
- *k* Minimum total weight(capacity) of all working components to operate system
- *C* The set of all components
- *C*₁ First group of components
- *C*₂ Second group of components
- n_1 Number of components in C_1
- n_2 Number of components in C_2
- ω_i Weight of *i*-th component in C_1
- ω_i^* Weight of *i*-th component in C_2
- p_{1i} Reliability of *i*-th component in C_1
- p_{2i} Reliability of *i*-th component in C_2

2.2. System Description

Consider a system including *n* independent and nonidentical components which are placed in two groups with respect to their duties and services. Each component has its own positive integer-valued weight. Let the state of *i*-th component X_i be an independent binary random variable such that $p_{1i} = P(X_i = 1)$ if $i \in C_1$ with corresponding weight ω_i and $p_{2i} = P(X_i = 1)$ if $i \in C_2$ with corresponding weight ω_i^* . According to the above representation, clearly $C = C_1 \cup C_2$, $C_1 \cap C_2 = \emptyset$, $|C_1| = n_1$ and $|C_2| = n_2$. Assume that $\phi(X)$ denotes the structure function of the system where $X = (X_1, \dots, X_n)$ is the state vector of components. As mentioned, in this general setup of weighted- *k*-out-of- *n*:G system, if the total weight of all functioning components is above predefined threshold *k*, the system is supposed to work. So, the structure function of this system is defined by

$$\phi(\mathbf{X}) = \begin{cases} 1 & \text{if } \sum_{i \in C_1} \omega_i X_i + \sum_{i \in C_2} \omega_i^* X_i \ge k, \\ 0 & o.w. \end{cases}$$

Evidently, the reliability of system defined as probability that system is working, can be given by the probability

$$R = P(\phi(X) = 1) = P\left(\sum_{i \in C_1} \omega_i X_i + \sum_{i \in C_2} \omega_i^* X_i \ge k\right).$$
(1)

2.3. Reliability Evaluation

Now, taking the independent and nonidentical components into account with the reliability of *i*-th component p_{1i} if $i \in C_1$ with corresponding weight ω_i^* , the reliability of system defined by (1) can be formulated as follows:

$$R = \sum_{s=0}^{n_1} \sum_{u=0}^{n_2} \sum_{\substack{E_s \\ (E_s, E_u) \in \mathbb{E}}} \sum_{m=1}^{s} p_{1l_m} \prod_{m=s+1}^{n_1} (1-p_{1l_m}) \prod_{m=1}^{u} p_{2l_m} \prod_{m=u+1}^{n_2} (1-p_{2l_m}),$$
(2)

where $s = \sum_{i \in C_1} X_i$, $u = \sum_{i \in C_2} X_i$, summation $E_s(E_u)$ extends over all combinations $l_1, \dots, l_s(l_1, \dots, l_u)$ of $\{1, \dots, n_1\}(\{1, \dots, n_2\})$ and

$$\mathbb{E} = \left\{ (E_s, E_u) : \sum_{m=1}^s \omega_{l_m} X_{l_m} + \sum_{m=1}^u \omega_{l_m}^* X_{l_m} \ge k \right\}$$

In the following, illustrative results are presented to observe the value of *R* for a weighted- k-out-of-5:G system with respect to the different values of *k*, n_1 and the component weights and reliabilities.

Example 2.1.

Consider a weighted- *k*-out-of- 5:G system with two especial case $n_1 = 2$ and $n_1 = 3$. First case $n_1 = 2$, suppose that $\omega_1 = 3$, $\omega_2 = 1$, $\omega_1^* = 1$, $\omega_2^* = 2$ and $\omega_3^* = 3$ with corresponding component reliabilities as

Then for k = 7,

$$R = [p_{11}(1 - p_{12})(p_{21}(1 - p_{22})p_{23} + (1 - p_{21})p_{22}p_{23} + p_{21}p_{22}p_{23}) + (1 - p_{11})p_{12}p_{21}p_{22}p_{23}]$$

+
$$[p_{11}p_{12}((1 - p_{21})(1 - p_{22})p_{23} + p_{21}p_{22}(1 - p_{23}) + p_{21}(1 - p_{22})p_{23} + (1 - p_{21})p_{22}p_{23} + p_{21}p_{22}p_{23})]$$

and for k = 9,

$$R = [p_{11}(1 - p_{12})p_{21}p_{22}p_{23})] + [p_{11}p_{12}((1 - p_{21})p_{22}p_{23} + p_{21}p_{22}p_{23})]$$

Now case $n_1 = 3$, let $\omega_1 = 3$, $\omega_2 = 1$, $\omega_3 = 2$, $\omega_1^* = 1$ and $\omega_2^* = 2$ with corresponding component reliabilities as

The calculation of *R* for k = 7 and k = 9 is similar.

In Table 1, the reliability of the weighted- k-out-of- 5:G system is computed with respect to the different values of k, n_1 and the weight of components. As observed, the system reliability is sensitive to the values of n_1 which determines the number of components in each group and also, the values of threshold k plays a significant role in system reliability.

	,	0	,
n	n_1	k	R
5	2	7	0.9893
		9	0.8910
	3	7	0.9210
		9	0.6697

 Table 1
 Reliability of the weighted-k-out-of-5:G system.

Proposition 2.1. As an especial case, consider a weighted- k-out-of- n:G system including independent and identical components in each of two groups with the reliability of i-th component p_1 if $i \in C_1$ and p_2 if $i \in C_2$, but with different corresponding weight ω_i and ω_i^* respectively in each group. Then, the reliability given by (2) can be rewritten as

$$R = \sum_{s=0}^{n_1} \sum_{u=0}^{n_2} \sum_{\substack{E_s \\ (E_s, E_u) \in \mathbb{E}}} p_1^s (1-p_1)^{n_1-s} p_2^u (1-p_2)^{n_2-u},$$

where the notations are mentioned for Equation (2).

Remark 2.1. If all the weights ω_i and ω_i^* are respectively equal to ω and ω^* with identical components in each of two groups, the especial case of weighted- *k*-out-of- *n*:G system containing two types of components presented by Eryilmaz and Sarikaya [10] is concluded.

3. SURVIVAL ANALYSIS

Let T_1, \dots, T_n represent the lifetimes of *n* independent and nonidentical components. As representative of the performance of the weighted*k*-out-of-*n*:G system, the total weight of the system at time $t(\geq 0)$ can be described by

$$W_n(t) = \sum_{i \in C_1} \omega_i I(T_i > t) + \sum_{i \in C_2} \omega_i^* I(T_i > t),$$

where $I(T_i > t)$ is indicator function (which is 1 if $T_i > t$ and 0 if $T_i \le t$). Suppose that the survival function of *i*-th component is $\overline{F}_i(t) = P(T_i > t), \forall i \in C_1 \text{ and } \overline{G}_i(t) = P(T_i > t), \forall i \in C_2$, then the mean weight of the system at time *t* is obtained as

$$E[W_n(t)] = \sum_{i \in C_1} \omega_i P(T_i > t) + \sum_{i \in C_2} \omega_i^* P(T_i > t)$$
$$= \sum_{i=1}^{n_1} \omega_i \overline{F}_i(t) + \sum_{i=1}^{n_2} \omega_i^* \overline{G}_i(t).$$

Now, considering T as the lifetime of the system, then it is defined as

$$T = \inf\{t : W_n(t) < k\}$$

So, by substituting p_{1l_m} and p_{2l_m} in Equation (2) by $\overline{F}_{l_m}(t)$ and $\overline{G}_{l_m}(t)$, respectively, the survival function defined as probability that system is working at time *t* for weighted-*k*-out-of-*n*:G system with two type nonidentical components can be established as follows:

$$S(t) = P(T > t) = \sum_{s=0}^{n_1} \sum_{u=0}^{n_2} \sum_{\substack{E_s \\ (E_s, E_u) \in \mathbb{E}}} \sum_{m=1}^s \overline{F}_{l_m}(t) \prod_{m=s+1}^{n_1} F_{l_m}(t) \prod_{m=1}^u \overline{G}_{l_m}(t) \prod_{m=u+1}^{n_2} G_{l_m}(t),$$
(3)

where the notations are mentioned for Equation (2) in previous section.

Proposition 3.1. For the especial case mentioned in Proposition 2.1, substituting $\overline{F}(t)$ and $\overline{G}(t)$, respectively, the survival function (3) at time t can be rewritten as

$$S(t) = \sum_{s=0}^{n_1} \sum_{u=0}^{n_2} \sum_{\substack{E_s \\ (E_s, E_u) \in \mathbb{E}}} \overline{F}^s(t) F^{n_1 - s}(t) \overline{G}^u(t) G^{n_2 - u}(t),$$

where the notations are mentioned for Equation (2) in previous section.

As one of the most important reliability characteristics, the MTTF of a system is defined as expected value of the lifetime of the system

$$MTTF = E(T) = \int_0^\infty P(T > t) dt.$$

Hence, the MTTF of our proposed system is computed by

$$MTTF = \sum_{s=0}^{n_1} \sum_{u=0}^{n_2} \sum_{\substack{E_s \\ (E_s, E_u) \in \mathbb{E}}} \int_0^\infty \prod_{m=1}^s \overline{F}_{l_m}(t) \prod_{m=s+1}^{n_1} F_{l_m}(t) \prod_{m=1}^u \overline{G}_{l_m}(t) \prod_{m=u+1}^{n_2} G_{l_m}(t) dt.$$
(4)

Example 3.1.

Considering the weighted- *k*-out-of- 5:G system mentioned in Example 2.1, let the survival function of *i*-th component for each classes is $\overline{F}_i(t) = e^{-\lambda_i t}$ and $\overline{G}_i(t) = e^{-\mu_i t}$, respectively. For first case $n_1 = 2$, suppose that $\omega_1 = 3, \omega_2 = 1, \omega_1^* = 1, \omega_2^* = 2$ and $\omega_3^* = 3$ with corresponding component survival function $\lambda = (0.2, 0.3)$ and $\mu = (0.6, 0.1, 0.4)$. For second case $n_2 = 3$, suppose that $\omega_1 = 3, \omega_2 = 1, \omega_3^* = 1$ and $\omega_2^* = 2$ with corresponding component survival function $\lambda = (0.2, 0.3)$ and $\mu = (0.2, 0.6, 0.3)$ and $\mu = (0.1, 0.4)$.

From Figure 1, the graphs of survival function with respect to different values of k indicate that survival of system depends on the combination of the survival function and the weight of components in each of classes. Obviously, survival function is sensitive to the values of n_1 which determines the number of components in each group and also, the values of threshold k plays a significant role in system reliability.

As well, MTTF is

$$MTTF = \sum_{s=0}^{n_1} \sum_{u=0}^{n_2} \sum_{E_s} \sum_{E_u} \int_0^{\infty} \prod_{m=1}^s e^{-\lambda_{l_m} t} \prod_{m=s+1}^{n_1} (1 - e^{-\lambda_{l_m} t}) \prod_{m=1}^u e^{-\mu_{l_m} t} \prod_{m=u+1}^{n_2} (1 - e^{-\mu_{l_m} t}) dt,$$
$$= \sum_{s=0}^{n_1} \sum_{u=0}^{n_2} \sum_{E_s} \sum_{E_u} \int_0^{\infty} e^{-t (\sum_{m=1}^s \lambda_{l_m} + \sum_{m=1}^u \mu_{l_m})} \prod_{m=s+1}^{n_1} (1 - e^{-\lambda_{l_m} t}) \prod_{m=u+1}^{n_2} (1 - e^{-\mu_{l_m} t}) dt.$$

In Table 2, the MTTF of the weighted- k-out-of- 5:G system is computed with respect to the different values of k, n_1 and the weight of components. As observed, MTTF is sensitive to the values of n_1 which determines the number of components in each group and also, the values of threshold k plays a significant role in system reliability.

4. COMPONENT IMPORTANCE

Considering the important role of ranking the components according to their importance measure in system reliability, the Birnbaum reliability importance is obtained in a weighted- *k*-out-of- *n*:G system with two type nonidentical components. Birnbaum [12] introduced the importance of the *i*-th component in a coherent system. If the event S shows that the system works, then Birnbaum reliability importance of the *i*-th component is defined by

$$I_i = P(S|X_i = 1) - P(S|X_i = 0).$$



Figure 1 The survival function with respect to different values of (right side).

Table 2Mean time to failure (MTTF) of the weighted-k-out-of-5:G system.

п	<i>n</i> ₁	k	MTTF
5	2	7	1.8597
		9	1.1442
	3	7	1.4637
		9	0.6250

In this setup of weighted- *k*-out-of- *n*:G system, the Birnbaum reliability importance of the *i*-th component is rewritten as follows: For $i \in C_1$,

$$P(S|X_i = 1) = \sum_{s=1}^{n_1} \sum_{u=0}^{n_2} \sum_{\substack{E_{s \setminus i} \ E_u}} \sum_{k_u} \prod_{m=1}^{s-1} p_{1l_m} \prod_{m=s+1}^{n_1} (1 - p_{1l_m}) \prod_{m=1}^{u} p_{2l_m} \prod_{m=u+1}^{n_2} (1 - p_{2l_m}),$$
(5)

and

$$P(S|X_i = 0) = \sum_{s=0}^{n_1-1} \sum_{u=0}^{n_2} \sum_{\substack{E_{s\backslash i} \\ E_{s\backslash i}, E_u \rangle \in \mathbb{E}}} \sum_{m=1}^{s} p_{1l_m} \prod_{m=s+1}^{n_1-1} (1-p_{1l_m}) \prod_{m=1}^{u} p_{2l_m} \prod_{m=u+1}^{n_2} (1-p_{2l_m}),$$
(6)

where summation $E_{s\setminus i}$ extends over all combinations l_1, \dots, l_s of $\{1, \dots, i-1, i+1, \dots, n_1\}$, denoted by $\{1, \dots, n_1\}\setminus\{i\}$. For $i \in C_2$,

$$P(S|X_i = 1) = \sum_{s=0}^{n_1} \sum_{u=1}^{n_2} \sum_{E_s} \sum_{E_{u\setminus i}} \prod_{m=1}^{s} p_{1l_m} \prod_{m=s+1}^{n_1} (1 - p_{1l_m}) \prod_{m=1}^{u-1} p_{2l_m} \prod_{m=u+1}^{n_2} (1 - p_{2l_m}),$$
(7)

and

$$P(S|X_i = 0) = \sum_{s=0}^{n_1} \sum_{u=0}^{n_2-1} \sum_{E_s} \sum_{E_{u\setminus i}} \prod_{m=1}^{s} p_{1l_m} \prod_{m=s+1}^{n_1} (1-p_{1l_m}) \prod_{m=1}^{u} p_{2l_m} \prod_{m=u+1}^{n_2-1} (1-p_{2l_m}), \tag{8}$$

where summation $E_{u \land i}$ extends over all combinations l_1, \dots, l_u of $\{1, \dots, n_2\} \backslash \{i\}$. Let $I_1(I_2)$ denote the importance of the components in $C_1(C_2)$ that equals the difference between Equations (5) and (6) (Equations (7) and (8)). Next example presents the value of Birnbaum reliability importance for a weighted-*k*-out-of- 5:G system with respect to the values of *k*, n_1 , and the component weights and reliabilities.

Example 4.1.

Consider a weighted- *k*-out-of- 5:G system with the especial case $n_1 = 3$ and k = 5. Suppose that $\omega_1 = 3$, $\omega_2 = 1$, $\omega_3 = 2$, $\omega_1^* = 1$ and $\omega_2^* = 2$ with corresponding component reliabilities $p_1 = (p_{11}, p_{12}, p_{13})$ and $p_2 = (p_{21}, p_{22})$.

For instance, if $i = 3 \in C_1$,

$$P(S|X_i = 1) = [(1 - p_{11})(1 - p_{12})p_{21}p_{22}] + [p_{11}(1 - p_{12})((1 - p_{21})(1 - p_{22}) + p_{21}(1 - p_{22}) + (1 - p_{21})p_{22} + p_{21}p_{22})] + [(1 - p_{11})p_{12}((1 - p_{21})p_{22} + p_{21}p_{22})] + [p_{11}p_{12}((1 - p_{21})(1 - p_{22}) + p_{21}(1 - p_{22}) + (1 - p_{21})p_{22} + p_{21}p_{22})],$$

and

$$P(S|X_i = 0) = [p_{11}(1 - p_{12})((1 - p_{21})p_{22} + p_{21}p_{22})] + [p_{11}p_{12}(p_{21}(1 - p_{22}) + (1 - p_{21})p_{22} + p_{21}p_{22})].$$

In Table 3, the Birnbaum reliability importance of the weighted- k-out-of- 5:G system is computed with respect to the different vectors of reliability of the components in both group C_1 and C_2 . As observed, the value of I_1 and I_2 depends on the combination of the weight and the reliability of the components.

5. CONCLUSION

In many situations, the components contribute differently to the capacity of the system which the weighted-k-out-of-n:G system are used to deal with. A system including n components with their different positive integer weights that it works if and only if the total weight of working components is above a given threshold k. In most of the studies in the literature, the weighted-k-out-of-n:G systems have been studied with identical components.

In this paper, we introduce a special case of weighted- k-out-of- n:G system formed from two types of nonidentical components with different weights in which one group consists of n_1 components each with its own positive integer-valued weight ω_i and reliability p_{1i} ,

<i>P</i> ₁	<i>P</i> ₂	i	ω_i	ω_i^*	I_1	I ₂
(0.95, 0.97, 0.85)	(0.90, 0.95)	1	3	1	0.1940	0.0102
		2	1	2	0.0105	0.0605
		3	2	_	0.0534	_
(0.80, 0.90, 0.95)	(0.90, 0.85)	1	3	1	0.1992	0.0472
		2	1	2	0.0516	0.1957
		3	2	_	0.1911	_
(0.80, 0.90, 0.85)	(0.95, 0.97)	1	3	1	0.1790	0.0243
		2	1	2	0.0117	0.1866
		3	2	_	0.1965	_
(0.70, 0.75, 0.65)	(0.85, 0.65)	1	3	1	0.5490	0.1472
		2	1	2	0.0919	0.2765
		3	2	_	0.2765	_

Table 3 Birnbaum reliability importance of the weighted-*k*-out-of-5:G system for $n_1 = 3$ and k = 5 with respect to reliability of the components.

while the other group consists of n_2 components each with its own positive integer-valued weight ω_j^* and reliability p_{2j} , $(n = n_1 + n_2)$. The components are categorized into two groups with respect to their duties and services with different capacity. If the total weights of the functioning components exceeds a prespecified threshold k, the system is supposed to work. The reliability and component importance of this system are studied. The survival function and MTTF are presented. This setup of weighted- k-out-of- n:G system containing two types of components might be useful in practice. As result, the system reliability, survival function, MTTF and component importance of this system are sensitive to the values of n_1 which determines the number of components in each group and also, the values of threshold k plays a significant role in system reliability.

CONFLICTS OF INTEREST

There is no conflict of interest in this article. Mrs Meshkat is the phd student under supervision of Prof. Eisa Mahmoudi and this work is related to her thesis.

AUTHORS' CONTRIBUTIONS

RahmatSadat Meshkat wrote the initial draft of the paper and did the analysis. Eisa Mahmoudi helped in analyzing the finding and supervised overall work.

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