Simulation of New Predicting Yarn Characteristics on Open-End (OE) Yarn Based on Torus Coordinate

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ABSTRACT
In this paper, we have made a new dimensional Euclidean space on torus coordinate to explain yarn characteristics based on fibre movement in the process of rotor spinning machine. In this study, a simulation model of fibre movement for predicting yarn characteristics on OE spun yarn based on torus coordinate in three dimensional Euclidean metric has been derived and formulated comprehensively. Based on this research, we have found that the yarn properties of OE spun yarn can be influenced by the movement of fibre. By understanding the fibre movement inside of yarn, the relationship among yarn strength, yarn diameter and yarn twist was established and validated by experimentally and by theoretically.
Keywords: *twist, yarn strength, fibre movement, torus coordinate, textile.*

1. INTRODUCTION

Open End spinning or Rotor spinning has established itself as a commercially possible technology with much higher productivity than conventional ring spinning for coarse and medium yarn counts. However, to get the optimum benefits from this technology, the parameters process has to be analyzed and determined. In textile science, during the spinning process, fibres are fed into the OE rotor machine continuously to form a yarn .spinning is a process to produce fibre or filament to form a yarn used in the textile industry. The study of fibre movement and its influence of characteristics of yarn have been studied by many researchers both theoretically and experimentally[1-22]. Some researchers developed models for modelling fibre migration based on some coordinate and they reported that the so called helical model or cylindrical model of yarn structure may be considered as a simple model of yarn structure[1-7]. The foundations of structural analysis and mechanics of fibres and yarn have been analyzed by all researchers on the assumption of axial uniformity of fibres in cylindrical coordinate or helical models of yarn structure [5-7] However it has always been obvious that a cylindrical model is theoretically deficient in continuous filament and also staple fibre yarns. According to Putra et.al [1-3]The helical models offer only approximate predictions and require some limitations factors. The fibre movement of yarn can be analyzed by classical mechanics in a certain coordinate to show the real movement of fibre inside yarn, especially fibre movement laid inside rotor spinning machine which is moved in torus coordinate as it is shown in Figure 1 [1-3]. Putra, et al. [1] formulated the relationship of angle of twist to diameter of yarn on torus coordinate, whereas the relationship among yarn strength, twist multiplier and yarn twist wasn't formulated and established. Putra, et al.[2] formulated the relation of twist and yarn count on certain coordinate and also in solenoid coordinate, whereas the relationship of yarn strength to yarn twist wasn't derived. According to Lawrence[7], the properties of spun yarn can be explained and determined by the fibre movement and yarn structure. Backer, Hearle and Grosberg [5] presented the formulation of structural mechanics of fibres, yarns and fabrics based on theoretically and experimentally to predict some properties of yarns used in textile. According to many researchers [1-14], variation of yarn parameters such as yarn strength, diameter, twist, yarn count number etc. is ignorable or unavoidable, especially for staple yarn. Variation of yarn properties can cause problems both during the production or after production.[8] According to Backer, Hearle and Grosberg [5], the strength of yarn is influenced by the rate of twist and the relation is explained as: the higher the strength of yarn, the lower is the rate of twist and vice versa. Furter and Meier[6] reported that the properties of yarn is influenced by the yarn movement and the result is explained as: the higher the twist, the lower is the hairiness and also the higher the twist, the higher the yarn count. Some researchers developed a model based on helical or cylindrical models of yarn structure for predicting yarn properties, but it has been obvious that a cylindrical model is theoretically deficient in continuous filament and staple fibre yarn. Rohlena[11] and Lawrence [7] explained that fibre migration and yarn movement can influence the rate of twist and the strength of yarn . According to Trommer [12], Lawrence[7] and Backer, Hearle and

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Grosberg [5] twist is defined as the ratio of rotor angular velocity to yarn delivery speed (linear velocity) or it can be meant as the amount of turned yarn per length of yarn. According to Trommer [12], Lawrence[7], Putra, et al.[3] and Backer, Hearle and Grosberg [5] the relation of twist and rotor angular speed can be formulated using the following equation:

$$T = \frac{n_r}{v_d} \quad \ldots(1)$$

where$T$ is the rate of twist in unit(1/m);n_r, rotor angular velocity in unit (rpm); v_d, yarn delivery speed in unit (m/min). Putra et.al [2], Lawrence[7] and Backer, Hearle and Grosberg [5] determined and formulated the relation of yarn twist, twist multiplier and yarn count number, as shown below:

$$T = \frac{\tan\theta}{\pi d} \quad \ldots(2)$$

where T is rate of twist; θ, angle of twist in unit (degree); d, diameter of yarn in unit (mm).Trommer [12], Backer, Hearle and Grosberg [5] and Putra,et al.[1-3] formulated the relation among twist multiplier(α_m) twist, T, and yarn count number, N_m as:

$$T = a_m\sqrt{N_m} \quad \ldots(3)$$

According to Musa K & Ayse[8] O, Penava Z & Oreskovic[9], Prendžova [10], the relationship of yarn strength is proportional to diameter of yarn and the relationship of yarn strength is inversely proportional to rate of yarn twist. Numerous researchers have investigated the relationships between yarn diameter, twist and yarn strength.[1-23] Putra et al. [1] investigated and formulated the relationships between yarn diameter and angle of yarn twist. It is stated that there is a strong relationship between diameter, angle of twist and twist, as shown below:

$$T = \frac{\tan\alpha}{\pi d_{yarn}} \quad \ldots(4)$$

$$T = \left(\frac{\frac{v_d}{n_r} + 0.02\sqrt{tex}}{\frac{a^2}{2}\pi tex}\right)\sqrt{\left(0.02\sqrt{tex}\right)^2 M_S\left(\frac{v_d}{n_r} + 0.02\sqrt{tex}\right)^2 - 1} \quad \ldots(5)$$

where α is angle of twist; tex, yarn count number; M_S, a constant value. Putra, et al.[1,2] derived the Eq (5) using calculus tensor based on classical mechanics on torus coordinate. According to Putra et al.[1] , the diameter of yarn is influenced by the angle of twist and the rate of twist, as mentioned in Eq (4) and Eq (5). According to many researchers [23-28] mathematical models have their basis in applied physics and it can be used to determine the motion of matter. Applied physics can be used to model and to provide a better understanding of the complex relationships of the different parameters that determine the movement of a certain matter.

The influence of fibre movement on the properties of yarn, such as diameter of yarn, angle of twist, strength, yarn delivery speed, and also yarn twist has been discussed and determined in this study considering the fibre-yarn movement on torus coordinate. A relationship between the process parameters and the yarn structure is important for the resulting of yarn properties such as yarn strength, yarn twist, yarn diameter and yarn count number. To understand the mechanics of yarn produced by such new systems, a more detailed understanding of the fibre movement inside yarn is first desirable.

2. MATERIAL AND METHODS

2.1. Development of fibre movement on torus coordinate

It is usually necessary to rewrite the vector equations in terms of suitable coordinate before the final solution can be obtained. In Cartesian coordinate the position of a point $P(x,y,z)$ is determined by the intersection of three mutually perpendicular plane, x,y,z. when x,y,z are the three quantities which form $x = x(r,u,v)$; $y = y(r,u,v)$ and $z = z(r,u,v)$ with inverses, $r = r(x,y,z)$; $u = u(x,y,z)$ and $v = v(x,y,z)$. The quantities (r,u,v) are the curvilinear coordinate of a point $P(x,y,z)$.

Fibre moving on torus coordinate can be derived using a mathematical method. By knowing the coordinate system, the fibre equation inside yarn can be analyzed. In particular, a fibre with length (dS) moves inside a yarn, whose length is du during a time (dt) and length of yarn cross section (dv). A yarn is assumed to be formed as torus coordinate, whose radius of yarn (r) and the length of gap (b), are shown in Figure 1.

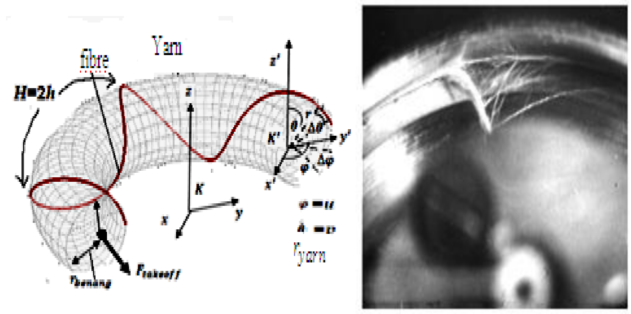


Figure 1. Model of OE yarn on torus coordinate and fibre formation on Open-End Spinning[7]

A coordinate system, usually to be encountered in addition to the Cartesian, is of the curvilinear type, such as torus. Consider the three quantities of C_m(m=1,2,3) which relate to the rectangular coordinates and the three transformed quantities (torus coordinate) of $\widetilde{C_\mu}$ (μ =1,2,3) related to C_m, as:

$$S = (x, y, z) = (r, u, v) \qquad \ldots(6)$$

$$(x, y, z) = ((b + r\cos v)\cos u, (b + r\cos v)\sin u, r\sin v). \qquad \ldots(7)$$

Equation (7) can be used to get the fibre movement using geodesic equation , the square of line element on torus coordinate($\boldsymbol{dS^2}$) is given below

$$dS^2 = ((b + r\cos v)^2 du^2 + r^2 dv^2 + dr^2). \qquad \ldots(8)$$

The metric element ($\boldsymbol{g_{mn}}$) can be written as shown below

$$g_{mn} = \begin{pmatrix} (b + r\cos v)^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad \ldots(9)$$

The components of a unit vector can be written in matrix format, as shown below

$$\begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{r} \end{pmatrix} = \begin{pmatrix} -\sin u & \cos u & 0 \\ -\sin v\cos u & -\sin v\sin u & \cos v \\ \cos v\cos u & \cos v\sin u & \sin v \end{pmatrix} \begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix}. \qquad \ldots(10)$$

The acceleration of fibre on each axis ($\boldsymbol{a^u}, \boldsymbol{a^v}, \boldsymbol{a^r}$) on torus coordinate can be written as (after some calculations):

$$\frac{d^2x^1}{dt^2} + \Gamma^1_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = \frac{d^2u}{dt^2} - \frac{2r\sin v}{b+r\cos v}\frac{du}{dt}\frac{dv}{dt} + \frac{2\cos v}{b+r\cos v}\frac{du}{dt}\frac{dr}{dt} = a^u, \qquad \ldots(11)$$

$$\frac{d^2x^2}{dt^2} + \Gamma^2_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = \frac{d^2v}{dt^2} + \frac{1}{r}\sin v(b + r\cos v)\frac{du}{dt}\frac{du}{dt} + \frac{2}{r}\frac{dv}{dt}\frac{dr}{dt} = a^v, \qquad \ldots(12)$$

$$\frac{d^2x^3}{dt^2} + \Gamma^3_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = \frac{d^2r}{dt^2} - \cos v\,(b + r\cos v)\frac{du}{dt}\frac{du}{dt} - r\frac{dv}{dt}\frac{dv}{dt} = a^r. \qquad \ldots(13)$$

The general acceleration ($\boldsymbol{a^i}$)for a certain mass of yarn ($\boldsymbol{m}$), including the stress tensor ($\boldsymbol{\sigma^{ij}}$) and the external force ($\boldsymbol{F^i}$), can be written as shown below:

$$\sigma^{ij}{}_{,j} + F^i = ma^i \qquad \ldots(14)$$

to get the general solution of fibre migration influenced by the stress tensor as traction tensor ($\boldsymbol{t^r}, \boldsymbol{t^u}, \boldsymbol{t^v}$) caused by the external force, therefore we are able to substitute Eq. (11), Eq. (12) and Eq. (13) to Eq. (14) and after some calculations, it can be written as below:

$$\frac{\partial \boldsymbol{t^r}}{\partial r} + \frac{1}{b+r\cos v}\frac{\partial \boldsymbol{t^u}}{\partial u} + \frac{1}{r}\frac{\partial \boldsymbol{t^v}}{\partial v} + \frac{\cos v\boldsymbol{t^r} - \sin v\boldsymbol{t^v}}{b+r\cos v} + \frac{\boldsymbol{t^r}}{r} + \bar{F} = m\bar{a}. \quad \ldots(15)$$

Eq.(15) can be derived as

$$\frac{\partial\sigma^{rr}}{\partial r}\hat{r} + \frac{\partial\sigma^{rr}}{\partial r}\frac{\partial\sigma^{ru}}{\partial r}\hat{u} + \frac{\partial\sigma^{rv}}{\partial r}\hat{v} + \frac{1}{b+r\cos v}\left(\frac{\partial\sigma^{ur}}{\partial u}\hat{r} + \frac{\partial\sigma^{uu}}{\partial u}\hat{u} + \frac{\partial\sigma^{uv}}{\partial u}\hat{v}\right) + \frac{1}{r}\left(\frac{\partial\sigma^{vr}}{\partial v}\hat{r} + \frac{\partial\sigma^{vu}}{\partial v}\hat{u} + \frac{\partial\sigma^{vv}}{\partial v}\hat{v}\right) + \frac{\cos v(\sigma^{rr}\hat{r}+\sigma^{ru}\hat{u}+\sigma^{rv}\hat{v}) - \sin v(\sigma^{vr}\hat{r}+\sigma^{vu}\hat{u}+\sigma^{vv}\hat{v})}{b+r\cos v} + \frac{(\sigma^{rr}\hat{r}+\sigma^{ru}\hat{u}+\sigma^{rv}\hat{v})}{r} + \bar{F} = m\bar{a}, \quad \ldots(16)$$

If we take $\hat{\boldsymbol{r}}$axis and we analyze the fibre migration inside yarn influenced by the external force occurred by the spinning tension in rotor($\boldsymbol{F_o}$) and internal force (caused by deformation force of fibre) and also substitute Eq. (13) to Eq.(16), therefore we get Eq.(17)

$$\frac{\partial\sigma^{rr}}{\partial r} + \frac{1}{b+r\cos v}\left(\frac{\partial\sigma^{ur}}{\partial u}\right) + \frac{1}{r}\left(\frac{\partial\sigma^{vr}}{\partial v}\right) + \frac{\sigma^{rr}\cos v - \sigma^{vr}\sin v}{b+r\cos v} + \frac{\sigma^{rr}}{r} - F_o = m\left(\frac{d^2r}{dt^2} - \cos v\,(b + r\cos v)\dot{u}^2 - r\dot{v}^2\right), \quad \ldots(17)$$

If $\boldsymbol{v}$ is kept constant and the speed of rotor$\dot{\boldsymbol{u}} \approx \dot{\boldsymbol{v}}$ and there's no fibre migration on $\hat{\boldsymbol{r}}$ axis, $\frac{\boldsymbol{d^2r}}{\boldsymbol{dt^2}} = \boldsymbol{0}$, hence it gives

$$\frac{\partial\sigma^{rr}}{\partial r} + \frac{1}{b+r\cos v}\left(\frac{\partial\sigma^{ur}}{\partial u}\right) + \frac{1}{r}\left(\frac{\partial\sigma^{vr}}{\partial v}\right) + \frac{\sigma^{rr}\cos v - \sigma^{vr}\sin v}{b+r\cos v} + \frac{\sigma^{rr}}{r} - F_o = -m\cos v(b + r\cos v)\dot{u}^2 - mr\dot{v}^2 \approx -m\dot{u}^2(\cos v(b + r\cos v) + r), \quad \ldots(18)$$

In case $r = a \ll b$ and $\dot{u} \approx \dot{v}$ or $n_r \approx n_d$, hence

$$\frac{\partial\sigma^{rr}}{\partial r} + \frac{1}{b+r\cos v}\left(\frac{\partial\sigma^{ur}}{\partial u}\right) + \frac{1}{r}\left(\frac{\partial\sigma^{vr}}{\partial v}\right) + \frac{\sigma^{rr}\cos v - \sigma^{vr}\sin v}{b+r\cos v} + \frac{\sigma^{rr}}{r} - F_o = -m\cos v(b + r\cos v)\dot{u}^2, \quad \ldots(19)$$

For a case, if angle of twist,$\boldsymbol{\theta}$, is kept constant, $\boldsymbol{\sigma^{ii}}$ **has a constant**value and$\boldsymbol{r = a \ll b = R_{rotor}}$, $\boldsymbol{\sigma^{ij} = 0}$, we can find the following equation

$$F_o - \sigma^{rr}\left(\frac{\cos v}{b+r\cos v} + \frac{1}{r}\right) = m\sin\theta(b + r\sin\theta)\dot{u}^2, \quad \ldots(20)$$

Now by Eq.(20), we have

$$F_o - \frac{\sigma^{rr}}{r}\left(\frac{2r\cos v+b}{b+r\cos v}\right) = F_o - \frac{\sigma^{rr}}{r} \approx m\sin\theta b\dot{u}^2. \quad \ldots(21)$$

$$m(R_{rotor}\sin^2\theta)\dot{u}^2 = F_o - \frac{\sigma^{rr}}{r}, \quad \ldots(22)$$

After some calculations, we have

$$\tan\theta = \frac{\sqrt{F_o}}{n_r\pi d_{rotor}}\sqrt{N_m} = const\sqrt{N_m}. \quad \ldots(23)$$

Eq.(23) can be written as

$$\frac{\tan\theta}{const} = \sqrt{N_m}, \quad \ldots(24)$$

According to Trommer [12], Backer et al. [5] and Putra et al.[1-3] , the relation between angle of twist($\boldsymbol{\theta}$)and yarn diameter ($\boldsymbol{d_{yarn}}$) can be related by Eq. (2) and Eq. (24) as shown below:

$$T = \alpha_m\sqrt{N_m} = \alpha_m\frac{\tan\theta}{const}, \quad \ldots(25.a)$$

$$\alpha_m\frac{\tan\theta}{const} = \frac{\tan\theta}{\pi d_{yarn}}, \quad \ldots(25.b)$$

$$d_{yarn} = \frac{const}{\tan\theta}. \quad \ldots(25.c)$$

to derive the yarn strength equation, we can expand from Eq.(19), as written below

$$\left(\frac{\partial\boldsymbol{\sigma^{rr}}}{\partial\boldsymbol{r}} + \frac{\boldsymbol{1}}{\boldsymbol{b+r\cos v}}\left(\frac{\partial\boldsymbol{\sigma^{ur}}}{\partial\boldsymbol{u}}\right) + \frac{\boldsymbol{1}}{\boldsymbol{r}}\left(\frac{\partial\boldsymbol{\sigma^{vr}}}{\partial\boldsymbol{v}}\right) + \frac{\boldsymbol{\sigma^{rr}\cos v - \sigma^{vr}\sin v}}{\boldsymbol{b+r\cos v}} + \frac{\boldsymbol{\sigma^{rr}}}{\boldsymbol{r}}\right) - \boldsymbol{F_o} = \boldsymbol{m}\left(\frac{\boldsymbol{d^2r}}{\boldsymbol{dt^2}} - \boldsymbol{\cos v\,(b + r\cos v)\dot{u}^2 - r\dot{v}^2}\right), \quad \ldots(26)$$

$$\boldsymbol{F_{yarn} - F_o = m\left(\frac{d^2r}{dt^2} - \cos v\,(b + r\cos v)\dot{u}^2 - r\dot{v}^2\right)}, \quad \ldots(27)$$

If fibre angular speed ,$\dot{\boldsymbol{v}}$, is kept constant and the angular speed of yarn in rotor$\dot{\boldsymbol{u}} \approx \dot{\boldsymbol{v}}$ and there's no fibre migration on $\hat{\boldsymbol{r}}$ axis, $\frac{\boldsymbol{d^2r}}{\boldsymbol{dt^2}} = \boldsymbol{0}$, hence it gives

$$F_{yarn} - F_o = m(-r\dot{v}^2). \quad \ldots(28)$$

$$m(R_r\sin^2\theta)\dot{v}^2 \approx F_o - F_{yarn}. \quad \ldots(29)$$

We calculate and derive from Eq.(29) and we have

$$\sin\theta \approx \sqrt{-\frac{F_{yarn}}{mR_r l_{fibre} \dot{v}^2} l_{fibre} + \frac{F_O}{mR_r l_{fibre} \dot{v}^2} l_{fibre}} \quad \ldots(30)$$

$$\tan\theta \approx \sin\theta = \sqrt{-\frac{F_{yarn}}{ml^2 \dot{v}^2} l + \frac{F_O}{ml^2 \dot{v}^2} l}, \quad \ldots(31)$$

Using the definition of twist ,$\boldsymbol{T}$, as a function of twist angle and also fibre count ($\boldsymbol{T_t = l/m}$) and Spinning tension occurred by the rotor as $\boldsymbol{F_o = F_{rotor}}$, we have

$$(2\pi r)^2 T^2 = \frac{F_{rotor}}{ml^2 \dot{v}^2} l - \frac{F_{yarn}}{ml^2 \dot{v}^2} l \quad \ldots(32)$$

$$\frac{F_{yarn}}{T_{tf} l^2 \dot{v}^2} = \frac{F_{rotor}}{T_{tf} l^2 \dot{v}^2} - (2\pi r)^2 T^2 \quad \ldots(33)$$

$$\frac{F_{yarn}}{(T_t) l^2 \dot{v}^2} = \frac{F_{rotor}}{(T_t) l^2 \dot{v}^2} - (2\pi r)^2 T^2 \quad \ldots(34)$$

to derive the relationship of rotor angular speed,$\boldsymbol{\dot{v}}$,and the yarn count number, N_m, we can expand from Eq.(25.a) and Eq.(34), as written below

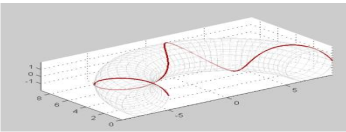
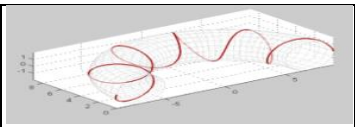
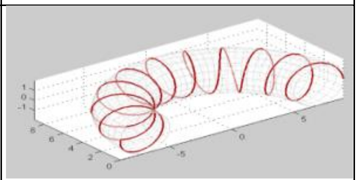
$$\left[\frac{F_{rotor}}{(T_t)l^2} - \frac{F_{yarn}}{(T_t)l^2}\right] = \dot{v}^2 (\pi d)^2 {\alpha_m}^2 N_m \quad \ldots(35)$$

$$R_{rotor} - R_{yarn} = l^2 \dot{v}^2 (\pi d)^2 {\alpha_m}^2 N_m \quad \ldots(36)$$

$$\dot{v} = \sqrt{\frac{R_{rotor} - R_{yarn}}{l^2 \pi^2 \dot{v}^2 {\alpha_m}^2 N_m}} = \frac{1}{\pi d l \alpha_m} \sqrt{\frac{R_{rotor} - R_{yarn}}{N_m}} \quad \ldots(37)$$

Using Eq.(37) we can establish the relationship of rotor angular speed and yarn count number. Eq.(25.a) explains that the higher the yarn count number, the higher is the yarn twist. The prediction of fibre movement-related by the yarn twist has been simulated and is shown in Table 1. The simulation of fibre movement inside the OE yarn on torus coordinate was carried out using Eq.(26), Eq.(29) and the computer program.

Table 1. Simulation of Fibre Movement Inside Yarn.

Influence of angular velocity on fibre inside a yarn $\dot{v}$ =10 $T_{simulation} = 3$	
Influence of angular velocity on fibre inside a yarn $\dot{v}$ =20 $T_{simulation} = 5$	
Influence of angular velocity on fibre inside a yarn n_{yarn} =$\dot{v}$ =40 $T_{simulation} =$ 10	

According to the simulation in Table 1, it can be shown that the higher the yarn angular speed, the higher is the yarn twist. Using Eq.(25.a) we can also get the relationship between yarn angular speed and yarn count number from yarn twist. It can be shown The relationship between fibre movement inside yarn and yarn strength on torus coordinate after some calculations using Eq.(32) and Eq.(33), we get

$$R_{yarn} = R_o(1 - {d_{yarn}}^2 T^2) = \frac{F_O}{T_t} N \left(1 - {d_{yarn}}^2 \left(\frac{n_r}{v_d}\right)^2\right) = N_m F_O N \left(1 - {d_{yarn}}^2 \left(\frac{n_r}{v_d}\right)^2\right) = N_m a \left(1 - \left(\frac{b}{v_d}\right)^2\right). \quad \ldots(38)$$

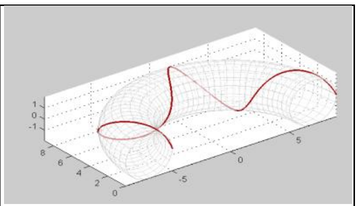
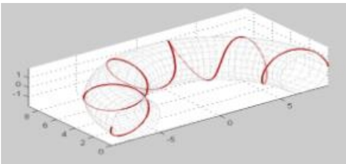
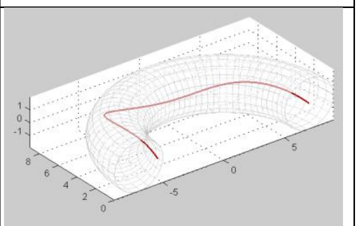
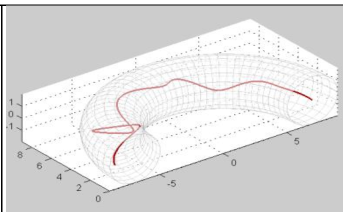
$\boldsymbol{R_{yarn}}$ is defined as the tenacity of yarn in unit [cN/tex];$\boldsymbol{R_o}$, tenacity take off;N, number of fibre inside yarn; $\boldsymbol{d_{yarn}}$, diameter of yarn; T, rate of twist of yarn and $\boldsymbol{a}, \boldsymbol{b}$ are constant number influenced by yarn properties. Using Eq.(38) we can predict the relationship between yarn tenacity and yarn twist. Eq.(35) explains that the higher the twist, the lower is the strength of the yarn. According to the experimental data in industry by the following data (Table 2):

Table 2. Experimental result

***rotor speed* (rpm)**	***tenacity of yarn* (cN/tex)**	a_m	***Strength* (cN)**	***Yarn count Ne***	***Speed of deliery yarn* v_d (m/min)**
72000	15.82	136	212	40	70.7
72000	15.44	136	170	50	63.0
72000	14.85	136	139	60	57.4

and using Eq.(17) we can establish the relationship of rotor angular speed and fibre migration through $\hat{\boldsymbol{r}}$ axis. Eq.(17) explains that If $\boldsymbol{v}$ isn't kept constant and there's occurred fibre migration on $\hat{\boldsymbol{r}}$ axis, hence $\frac{d^2r}{dt^2} \neq \mathbf{0}$. The prediction of fibre movement related by the yarn twist when $\frac{d^2r}{dt^2} = \mathbf{0}$ and $\frac{d^2r}{dt^2} \neq \mathbf{0}$ can be simulated and shown in Table 3. The simulation of fibre movement inside the yarn on torus coordinate on $\hat{\boldsymbol{r}}$ axis was carried out using Eq.(17), Eq.(18) and the computer program.

Table 3. Simulation of fibre movement inside yarn looked on r axis.

Influence of angular velocity on fibre inside a yarn $\dot{v}$ =10 *in case* $\frac{d^2r}{dt^2} = \mathbf{0}$	
Influence of angular velocity on fibre inside a yarn $\dot{v}$ =20 *in case* $\frac{d^2r}{dt^2} = \mathbf{0}$	
Influence of angular velocity on fibre inside a yarn n_{yarn} =$\dot{v}$ =10 *in case* $\frac{d^2r}{dt^2} \neq \mathbf{0}$	
Influence of angular velocity on fibre inside a yarn n_{yarn} =$\dot{v}$ =20 *in case* $\frac{d^2r}{dt^2} \neq \mathbf{0}$	

By using Table 2 and Eq. (35) we can show the relation of tenacity (cN/tex) and twist (tpm) as Figure 2

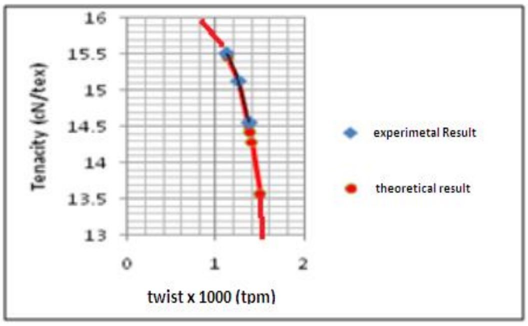


Figure 2. Relationship of tenacity and twist both theoretically and experimentally

using multivariate regression we can also show the relationship of yarn speed and yarn count number in unit ($\boldsymbol{N_e}$ = 0,59$\boldsymbol{N_m}$) to yarn tenacity in unit (cN/tex) . For some yarns with a certain yarn count number (40,50 and 60 Ne) and yarn speed $\mathbf{70,7\ yd/min}$, 63 $\mathbf{yd/min}$ dan 57,4 $\mathbf{yd/min}$ as well as yarn tenacity 15.82 cN/tex, 15.44 cN/tex and 14.85 cN/tex, we can also make a model using non linear multivariate regression as written below

$$\sum_{i=1}^{n} \hat{y}_i = a_o + a_1 \sum x_{i1} + a_2 \sum x_{i2} + \cdots + a_k \sum x_{ik}$$
$$\hat{y}_1 = a_o + a_1 x_{11} + a_2 x_{12} + +a_3 x_{13} + \cdots + +a_k x_{1k}$$
$$\hat{y}_2 = a_o + a_1 x_{21} + a_2 x_{22} + +a_3 x_{23} + \cdots + +a_k x_{2k}$$
$$\hat{y}_n = a_o + a_1 x_{n1} + a_2 x_{n2} + +a_3 x_{n3} + \cdots + +a_k x_{nk} \qquad ...(39)$$

$$\begin{pmatrix}\hat{y}_1\\ \vdots \\ \hat{y}_n\end{pmatrix} = \begin{pmatrix}1 & \dots & x_{1k}\\ 1 & \ddots & \vdots \\ 1 & \dots & x_{nk}\end{pmatrix}\begin{pmatrix}a_0\\ \vdots \\ a_k\end{pmatrix} \quad ...(40)$$

$$\hat{y}_i = x_{ik}a_k \quad ...(41)$$

$$\hat{\boldsymbol{y}} = \boldsymbol{Xa} \quad ...(42)$$

The residuals of experimental data and the prediction of fibres movement inside yarn, called an error ϵ, have a magnitude as

$$\sum_{i=1}^{n}(y_i - \hat{y}_i) = \epsilon \quad ...(43)$$

Sum of squared of the residuals in Eq.(43) can be written in matrix form as proposed below

$$\epsilon^T\epsilon = L \quad ...(44)$$

We can do to minimize Eq.(44) to find a as written below

$$L = \epsilon^T\epsilon = \sum (y_i - \hat{y}_i)^T (y_i - \hat{y}_i) \quad ...(45)$$

$$L = \epsilon^T\epsilon = (y - Xa)^T(y - Xa) = (y^T - a^TX^T)(y - Xa) = y^Ty - y^TXa - a^TX^Ty + a^TX^TXa$$
$$= y^Ty - y^TXa - (y^TXa)^T + a^TX^TXa \quad ...(46)$$

$$\frac{dL}{da} = \frac{d}{da}(y^Ty - y^TXa - (y^TXa)^T + a^TX^TXa) = -2y^TX + 2a^TX^TX = 0 \quad ...(47)$$

$$2y^TX = 2a^TX^TX \quad ...(48)$$

$$a^TX^TX = y^TX \text{with} a^T = (X^TX)^{-1}y^TX$$

$$(a^TX^TX)^T = (y^TX)^T \quad ...(49)$$

$$(X^TX)a = X^Ty \quad ...(50)$$

$$a = (X^TX)^{-1}X^Ty \quad ...(51)$$

and

$$\hat{y} = Xa = X(X^TX)^{-1}X^Ty \quad ...(52)$$

error $\boldsymbol{\epsilon}$

$$\epsilon = y - \hat{y} = (1 - X(X^TX)^{-1}X^T)y = (1 - H)y = \bar{H}y \quad ...(53)$$

Using Eq. (53) and after some calculations, we get

$$T_w = a_oN_e{}^{a_1}v_f{}^{a_2} \quad …(54)$$

$$\ln T_w = \ln a_o + a_1 lnN_e + a_1 lnv_f \quad …(55)$$

$$Y = A_o + A_1X_1 + A_2X_2 \quad …(56)$$

$$\begin{pmatrix}\hat{y}_1\\ \hat{y}_2\\ \hat{y}_3\end{pmatrix} = \begin{pmatrix}1 & \ln(40{,}1) & \ln(70.7)\\ 1 & \ln(50{,}1) & \ln(63{,}0)\\ 1 & \ln(60{,}1) & ln(57{,}4)\end{pmatrix}\begin{pmatrix}a_0\\ a_1\\ a_2\end{pmatrix} \quad …(57)$$

Using Eq.(57) and$a_o = e^{Ao}, a_1 = A_1, a_2 = A_2$

$$T_w = 8150\ v_f{}^{-0{,}913}N_e{}^{-0{,}638} \quad …(58)$$

The results of model and experimental data can be revealed in Table 4 using Eq.(58):

Table 4. Comparison of the model and experimental data

Yarn count number Ne (hank/lbs)	***Delivery speed Vd* (yd/min)**	***Tenacity R* experiment (cN/tex)**	***Tenacity R* model (cN/tex)**
40	70,7	15.82	15.86
50	63	15.44	15.28
60	57,4	14.85	14.91

The relationship of yarn speed and yarn count number to yarn tenacity can be seen in Figure 3

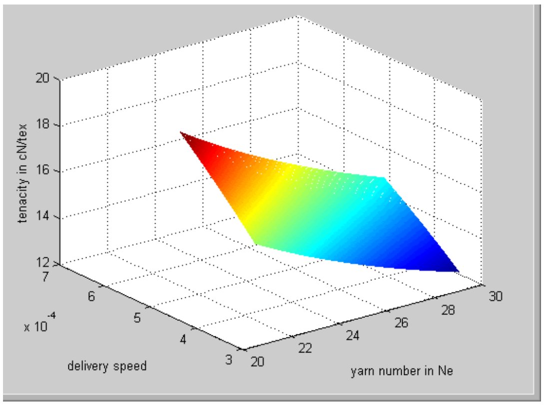


Figure 3. Interaction effect of delivery speed and yarn count number on yarn tenacity.

3. RESULTS AND DISCUSSION

The prediction of fibre movement inside OE yarn has been derived using torus coordinate occurred on rotor open end spinning machine. In this new theory, the relationship among yarn strength per yarn count number(R_{yarn}), number of fibre inside yarn(N), twist(T) and diameter of yarn(d_{yarn}) has been established. The model in this prediction has shown the relationship of those parameters as strength of yarn. Based on the theoretical consideration, it can be found that the higher the twist(T), the higher is the yarn count number as shown in Eq. (25). From Eq. (25), it can be assumed that the higher the yarn diameter, the lower is the twist, for the same yarn count number. In this research, the relationship between twist and yarn diameter is established. According to Rohlena [11]The relationship between yarn strength with yarn diameter is related to the movement of fibre inside the yarn. The more fibre inside yarn, the higher is the strength of yarn. According to Lawrence.[7], the strength of yarn per tex is influenced by the rate of twist and the relation is shown as: the lower the twist, the higher is the strength of yarn per tex and vice versa. According to Backer,*et al*.[5], the strength of yarn per tex is influenced by the rate of twist. Based on this research, the relation of yarn strength to twist has been predicted as Eq.(35). Based on the theoretical consideration Eq.(35) and also the data from experimental results, it can be assumed that the relation of twist to yarn properties (i.e. strength of yarn) can be formulated by using following equation:

$$\frac{F_{yarn}}{T_t} = \frac{F_o}{T_t} N\left(1 - d_{yarn}{}^2 T^2\right). \qquad \ldots(59)$$

From Eq. (59) it can be assumed that the higher the twist of yarn, the lower is the yarn strength (F_{yarn}). Based on Eq.(59) we can conclude that for a constant yarn count number (T_t) in unit tex and yarn diameter (d_{yarn}) in unit mm, hence the relationship of yarn strength is inversely proportional to yarn twist. In this study, we developed a simulation for modelling fibre migration based on torus coordinate on yarn structure and we may be considered a real model of yarn structure inside rotor formulated by torus coordinate .However it has always been obvious that a torus model is theoretically better than helical model to explain fibre movement in continuous filament and also staple fibre yarns. According to Trommer [12], Lawrence[7] , Putra, et al.[3] and Backer, et al.[5] the relation of twist and yarn speed (v_d) can be formulated using Eq.(1).According to Trommer[12]the higher the yarn speed, the lower is the twist as well as the higher the yarn number in metric, the higher is the twist. The relationship of yarn twist as a function of yarn speed is inversely proportional to yarn tenacity as shown in Fig. 3. In this research we have found that the lower yarn speed, the lower is yarn tenacity as shown in Eq.(38)

4. CONCLUSION

In this study, the relationship between yarn strength, yarn count number and yarn twist was investigated theoretically and experimentally. As stated by many researchers[1-13] yarn strength decreases where yarn twist increases for a constant yarn diameter and yarn count number. In this study, we have found the new equations as written in Eq.38 and Eq.58 to predict yarn strength as a function of twist in torus coordinate for a case where number of fibre inside yarn and diameter of yarn are kept constant. In this research we have found that the lower yarn speed, the lower is yarn tenacity as shown in Eq.(38). By understanding the fibre movement inside of yarn, the relationship among yarn strength, yarn count number, yarn diameter and yarn twist was established and validated by experimental. However it has been obvious that torus coordinate is theoretically better to explain fibre movement in continuous filament and also staple fibre yarns. than helical models in cylindrical coordinate which have been developed by numerous researchers.

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