

Modeling Investment Trends: A Logarithmic-Modified Markov Chain Approach

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ABSTRACT

The study aimed at stabilizing the changing variance using the logarithmic transformation to achieve a significant proportion of stability and a faster rate of convergence of the steady state transition probability in Markov chains. The traditional Markov chain and logarithmic-modified Markov chain were considered. On exploring the yearly data on the stock prices from 2015 to 2018 as obtained from the Nigerian Stock Exchange, it was found that the steady state of logarithmic-modified Markov chain converged faster than the traditional Markov chain with efficiency in tracking the correct cycles where the stock movements are trending irrespective of which cycle it starts at time zero with differences in probability values by 1.1%, 0.7%, -0.41% and -1.37% for accumulation, markup, distribution and mark-down cycles, respectively. Thus, it could be deduced that the logarithmic modification enhances the ability of the Markov chain to tract the variation of the steady state probabilities faster than the traditional counterpart.

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1. INTRODUCTION

In Statistics, trends are steady and methodical fluctuations in the variance of a process over a long period of time and their underlying features are embedded with useful information that are viable for investment decisions [1–4]. The uncertainty or probabilistic nature of stock investment has made trend analysis a potential and promising tool for investors, who seek capital appreciation, hence, the quest for an appropriate model that could offer the best prediction.

The motivation for this study is drawn from the fact that investment trends are stochastic and follow a Markov process which deals with probability of any particular future behavior of a process with an exactly known current state which cannot be altered by additional knowledge concerning its past behavior [5]. According to [6], Markov chains are fundamental stochastic processes that have many diverse applications and it represents any dynamical system whose new state is simply a function of the last state and an auxiliary random on the past and present only through the present state.

The striking reason for continuous application of Markov chains in investment trend analysis is the existence of a limiting probability distribution which is independent of initial states, and this is associated with the convergence in probability of finding the Markov chain in a particular state irrespective of which state the chain began at time zero.

Prior studies such as [7–10] applied Markov chain model approach to analyze and forecast stock trends. On the other hand, given the fact that Markov chain models are appropriate in tracking long-run behavior of trends but failed in tracking short-term behavior prompted the studies of [11–15] to provide the missing link.

However, no previous studies have attempted to take into consideration the effect of changing variance while analyzing investment trends through Markov chains. It is evident in the literature that stocks trends are characterized by changing variance [16–23]. The consequential effect of neglecting changing variance is that it takes longer time to achieve convergence of the probabilities of Markov chain. For instance, in the study of [9], it takes about 2080 steps (about 8 trading years) to attain near convergence.

Therefore, this study seeks to bridge this gap by stabilizing the changing variance through logarithmic transformation to achieve good stability and quick convergence. This is advantageous in that, it reduces the number of n -step computation and increases the precision of the estimates of the probabilities, thus enhances investors to make quick and stable decision.

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The remaining part of the paper is organized as follows: Section 2 treats materials and methods. Results and discussion are presented in Section 3 while the conclusion is handled in Section 4.

2. MATERIALS AND METHODS

2.1. Markov Process

A Markov chain $\{X_t\}$ is a stochastic process with the property that, given the value of X_t , the values of X_s for $s > t$ are not influenced by the values of X_u for $u < t$. A discrete-time Markov chain is a Markov process whose state space is a finite or countable set, and whose (time) index set is $T = (0, 1, 2, \dots)$.

Mathematically, the Markov property is that

$$\Pr \{X_{n+1} = j | X_0 = i_0, \dots, X_{n-1} = i_{n-1}, X_n = i\} = \Pr \{X_{n+1} = j | X_n = i\}, \tag{1}$$

for all time points n and all states $i_0, \dots, i_{n-1}, i, j$ [6,5].

2.2. Transition Probability Matrices of a Markov Chain

The probability of X_{n+1} being in state j given that X_n is in state i is called the one-step transition probability and is denoted by $P_{ij}^{n, n+1}$. That is,

$$P_{ij}^{n, n+1} = \Pr \{X_{n+1} = j | X_n = i\}. \tag{2}$$

The notation emphasizes that in general, the transition probabilities are functions not only of the initial and final states, but also of the time of transition as well. When the one-step transition probabilities are independent of the time variable n , we say that the Markov chains have stationary transition probabilities. Given the Markov chains that have stationary transition probabilities, then, $P_{ij}^{n, n+1} = P_{ij}$ is independent of n , and P_{ij} is the conditional probability that the state value undergoes a transition from i to j in one trial. Thus P_{ij} is arranged in a matrix, in the infinite square array

$$P = \begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} & \cdots \\ P_{10} & P_{11} & P_{12} & P_{13} & \cdots \\ P_{20} & P_{21} & P_{22} & P_{23} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{i0} & P_{i1} & P_{i2} & P_{i3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

where $P = \|P_{ij}\|$ is referred to as the Markov matrix or transition probability matrix of the process. The i th row of P , for $i = 0, 1, \dots$, is the probability distribution of the values of X_{n+1} under the condition that $X_n = i$. If the number of states is finite, then P is a finite square matrix whose order (the number of rows) is equal to the number of states. Clearly, the quantities P_{ij} satisfy the conditions

$$P_{ij} \geq 0 \text{ for } i, j = 0, 1, 2, \dots, \tag{3}$$

$$\sum_{j=0}^{\infty} P_{ij} = 1 \text{ for } i = 0, 1, 2, \dots \tag{4}$$

A Markov process is completely defined once its transition probability matrix and initial state X_0 (or, more generally, the probability distribution of X_0) are specified [5,6].

2.3. Regular Transition Probability Matrices

Suppose that a transition probability matrix $P = \|P_{ij}\|$ on a finite number of states labeled $0, 1, \dots, n$ has the property that when raised to some power n , the matrix P^n has all of its elements strictly positive. Such a transition probability matrix, or the corresponding Markov chain is called regular. The most important fact concerning a regular Markov chain is the existence of limiting probability distribution $\pi = (\pi_0, \pi_1, \dots, \pi_n)$, where $\pi_j > 0$ for $j = 0, 1, \dots, n$ and $\sum_j \pi_j = 1$, and this distribution is independent of the initial state. Formally, for a regular transition probability matrix $P = \|P_{ij}\|$ we have the convergence

$$\lim_{n \rightarrow \infty} P_{ij}^{(n)} = \pi_j > 0 \text{ for } j = 0, 1, \dots, n. \tag{5}$$

Or, in terms of the Markov chain $\{X_n\}$

$$\lim_{n \rightarrow \infty} Pr\{X_n = j | X_0 = i\} = \pi_j > 0 \text{ for } j = 0, 1, \dots, n. \quad (6)$$

This convergence means that, in the long run ($n \rightarrow \infty$), the probability of finding the Markov chain in state j is approximately π_j , no matter in which state the chain began at time zero [5].

2.4. Proposed Transformation

In traditional Markov chain process, the problem of estimating transition probabilities is based on the observations on the Markov chain at time $t = \{0, 1, \dots, T\}$ using

$$n_{ij} = \sum_{t=0}^T n_{ij}(t), \quad (7)$$

where $n_{ij}(t)$ is the number of chain transitions from the state i to the state j at time t .

$$n_i = \sum_{j=1}^{k_i} n_{ij}, \quad (8)$$

where n_i is the total number of chain transitions from the state i over the time T [24,25]. Hence, the estimates of transition probabilities from the state i to state j are obtained as follows:

$$\hat{P}_{ij} = \frac{n_{ij}}{n_i}, \quad i = 1, \dots, k; j = 1, \dots, k_i. \quad (9)$$

Traditional Markov chain is so defined to enhance convergence to a certain probability distribution of interest. However, to achieve quick convergence rate of the steady state probabilities, this study considers natural logarithmic transformation of the observations on the Markov chain $t = \{0, 1, \dots, T\}$ which is reflected in the following sample statistics:

$$n'_{ij} = \sum_{t=0}^T \ln(n_{ij})(t), \quad (10)$$

where $n'_{ij}(t)$ is the number of logarithmic chain transitions from the state i to the state j at time t .

$$n'_i = \sum_{j=1}^{k_i} \ln(n_{ij}), \quad (11)$$

where n'_i is the total number of logarithmic chain transitions from the state i over the time T .

$$\hat{P}'_{ij} = \frac{n'_{ij}}{n'_i}, \quad i = 1, \dots, k; j = 1, \dots, k_i, \quad (12)$$

where \hat{P}'_{ij} are the estimates of logarithmic transition probabilities from the state i to state j . This new approach distinguishes itself from the traditional Markov chain approach by incorporating the natural logarithm for the purpose of accounting for changing variance. Stabilizing the changing variance in turn aids quicker convergence of transition probabilities of the Markov chain leading to significant improvements in the estimates of the steady state probabilities.

3. RESULTS AND DISCUSSION

3.1. Data

The data on stock prices from 2015 to 2018 were obtained from Nigerian Stock Exchange annual report 2018 and are presented in Table 1 while the logarithmic-transformed stock prices are presented in Table 2. The data are grouped into four phases according to stock market cycles: accumulation phase (where the stocks are oversold); mark-up phase (where the demand for the stocks surpasses the supply); distribution phase (where the stocks are overbought) and mark-down phase (where the supply for the stocks overwhelms the demand).

The plots of data in Tables 1 and 2 are shown in Figures 1 and 2, respectively. It is obvious that the direction of movement of the stock prices according to stock cycles changes due to variance stabilization through natural logarithm as shown in Figure 2 compared to the series in Table 1.

Table 1 Stock prices (“000,000,000”) from 2015 to 2018 according to stock cycle.

	Accumulation	Mark-up	Distribution	Mark-down	Row Total
Accumulation	478	501	487	421	1887
Mark-up	524	523	536	540	2123
Distribution	605	521	526	507	2159
Mark-down	527	536	430	425	1918
Grand Total					8087

Table 2 Natural logarithmic-transformed stock prices (“000,000,000”) from 2015 to 2018 according to stock cycle.

	Accumulation	Mark-up	Distribution	Mark-down	Row Total
Accumulation	6.1696	6.2166	6.1883	6.0426	24.6171
Mark-up	6.2615	6.2596	6.2841	6.2916	25.0968
Distribution	6.4052	6.2558	6.2653	6.2285	25.1548
Mark-down	6.2672	6.2841	6.0638	6.0521	24.6672
Grand Total					99.5359

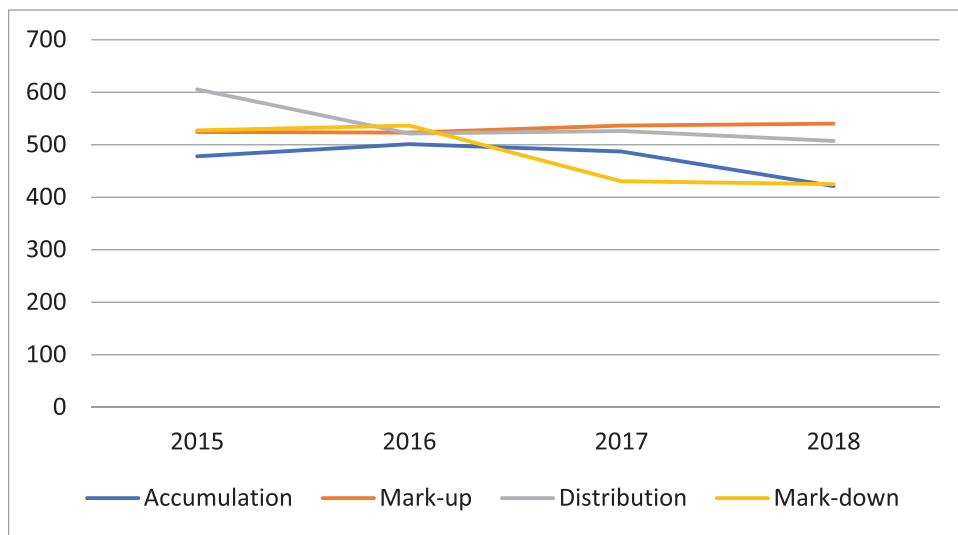


Figure 1 | Graph stock prices according to stock cycle.

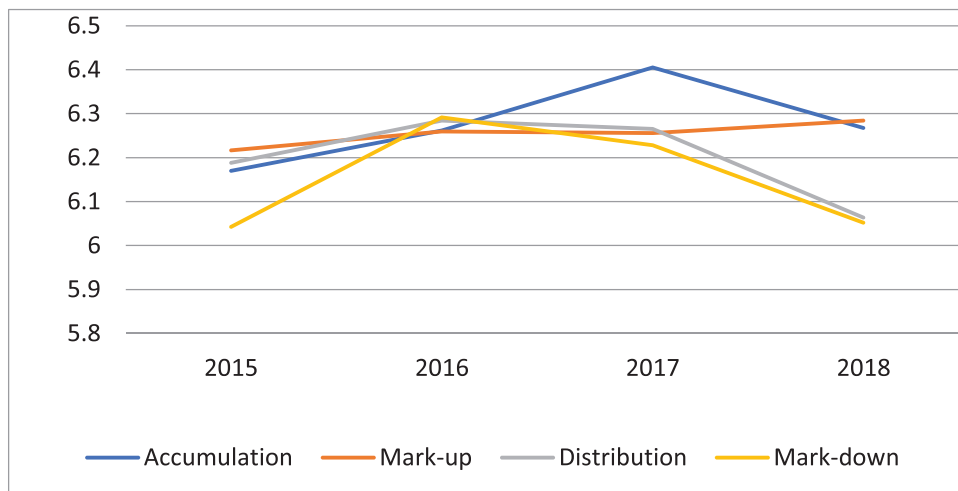


Figure 2 | Graph of natural logarithmic-transformed stock prices according to stock cycle.

3.2. Building Transition Probability Matrix of the Stock Prices

The transition probability matrix of the stock cycle using the information in Table 1 are obtained as

		Accumulation	Mark-up	Distribution	Mark-down
$P_{Stock\ Cycle} =$	Accumulation	0.2533	0.2655	0.2581	0.2231
	Mark-up	0.2468	0.2463	0.2525	0.2544
	Distribution	0.2802	0.2413	0.2436	0.2348
	Mark-down	0.2748	0.2795	0.2242	0.2216

3.3. Convergence of Steady State Probability of the Stock Prices

The steady state probabilities are the constant probabilities to be attained in the states in the future. The interest is to ascertain the number of steps it takes $P_{StockCycle}$ to attain a steady state.

		Accumulation	Mark-up	Distribution	Mark-down
$P_{Stock\ Cycle}^2 =$	Accumulation	0.2633	0.2573	0.2453	0.2341
	Mark-up	0.2640	0.2582	0.2444	0.2334
	Distribution	0.2633	0.2583	0.2452	0.2331
	Mark-down	0.2623	0.2578	0.2458	0.2341

		Accumulation	Mark-up	Distribution	Mark-down
$P_{Stock\ Cycle}^3 =$	Accumulation	0.2633	0.2579	0.2452	0.2337
	Mark-up	0.2632	0.2579	0.2452	0.2337
	Distribution	0.2632	0.2579	0.2452	0.2337
	Mark-down	0.2633	0.2579	0.2451	0.2337

		Accumulation	Mark-up	Distribution	Mark-down
$P_{Stock\ Cycle}^4 =$	Accumulation	0.2632	0.2579	0.2452	0.2337
	Mark-up	0.2632	0.2579	0.2452	0.2337
	Distribution	0.2632	0.2579	0.2452	0.2337
	Mark-down	0.2632	0.2579	0.2452	0.2337

Therefore, the $P_{StockCycle}$ is found to converge to a steady state at the fourth power.

3.4. Convergence of Steady State Probability of the Natural Logarithmic-Transformed Stock Prices

The focus is to determine how fast the logarithmic-transformed transition probability matrix, $P_{Ln(StockCycle)}$ converges to the steady state.

		Accumulation	Mark-up	Distribution	Mark-down
$P_{Ln(Stock\ Cycle)}$ =	Accumulation	0.2506	0.2525	0.2514	0.2455
	Mark-up	0.2495	0.2494	0.2504	0.2507
	Distribution	0.2546	0.2487	0.2491	0.2476
	Mark-down	0.2541	0.2548	0.2458	0.2454

		Accumulation	Mark-up	Distribution	Mark-down
$P_{Ln(Stock\ Cycle)}^2$ =	Accumulation	0.2522	0.2513	0.2492	0.2473
	Mark-up	0.2522	0.2514	0.2492	0.2473
	Distribution	0.2522	0.2514	0.2492	0.2473
	Mark-down	0.2522	0.2513	0.2492	0.2473

		Accumulation	Mark-up	Distribution	Mark-down
$P_{Ln(Stock\ Cycle)}^3$ =	Accumulation	0.2522	0.2513	0.2492	0.2473
	Mark-up	0.2522	0.2513	0.2492	0.2473
	Distribution	0.2522	0.2513	0.2492	0.2473
	Mark-down	0.2522	0.2513	0.2492	0.2473

It could be found that $P_{Ln(Stock\ Cycle)}$ converges to the third power.

3.5. Percentage Difference between the Steady State Transition Probability Matrix of Stock Prices and Natural Logarithm of Stock Prices

As indicated in Table 3, when the variance (variation in the prices) is not accounted for, at the accumulation cycle, the stocks have the probability of 0.2632, that is, about 26.32% of being oversold. At the mark-up cycle, the probability of the demand for the stocks surpassing the supply is 0.2579 which is about 25.79%. At the distribution cycle, the probability of the stock being overbought is 0.2452 which is about 24.52%, and at the mark-down cycle, the probability of the supply of stocks overwhelming the demand is 0.2337 (23.37%). On the other hand, when the stabilization of variance is considered, the accumulation cycle has the probability of 0.2522 (25.22%) of the stocks being oversold. At the mark-up cycle, the probability that the demand for stocks is higher than supply is 0.2513 (25.13%). At distribution cycle, the probability of the stocks being overbought is 0.2492 (24.92%), and at the mark-down cycle, the supply is greater than demand with the probability of 0.2473 (24.73%). Comparing the two approaches, it is shown that, when the logarithmic transformation is not considered, the probabilities that the stocks would remain in the accumulation and mark-up cycles are hyped by 1.1 % and 0.7%, respectively while the probabilities that the stocks remain in distribution and mark-down are played down by 0.41% and 1.36%, respectively.

Going forward, the logarithmic-modified transition probability matrix converged at third power, a step faster than that of the ordinary transition probability matrix which converged at the fourth power. In addition, the true direction of the stock cycles appeared to be misleading when the changing variance is not accounted for. The findings of this study are in agreement with previous studies of [7–15], and that Markov chain approach is sufficient for modeling stock/investment trends but proud of its novelty by application of logarithmic transformation of the transition matrix and provided the needed improvement to the study of [9]. The implication is that, a quicker convergence with higher precision estimates of the steady state probabilities is achieved when the variance is stabilized.

4. CONCLUSION

Concisely, the findings of the study showed that the logarithmic-modified approach to Markov chain modeling of the investment trends results in a faster convergence rate of the steady state transition probability with robust probabilities than the ordinary approach. Thus the aim of this study is sufficed by these findings. The weakness of the study is its inability to assess the expected number of stocks in the long run using logarithmic-modified approach and it is therefore recommended to be undertaken in our future studies.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest regarding the publication of this paper.

Table 3 | Comparison of the steady state transition probability matrix of stock prices and natural logarithm of stock prices.

Stock Cycle	Steady State Transition Probability Matrix of Stock Prices	Steady State Transition Probability Matrix of Natural Logarithm of Stock Prices	Percentage Differences (%)
Accumulation	0.2632	0.2522	1.1
Mark-up	0.2579	0.2513	0.7
Distribution	0.2452	0.2492	-0.41
Mark-down	0.2337	0.2473	-1.36

AUTHORS' CONTRIBUTIONS

MIU designed the study and wrote the first draft. JAU managed the literature search, manuscript writing and data collection. EAA managed the analysis. All authors read and approved the final manuscript.

DATA AVAILABILITY STATEMENT

The authors confirm that the data supporting the findings of the study are available within the article.

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