# The Patterns of Changing Mathematical Representations in Solving Mathematical Problems Among Junior High School Students

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***Abstract*. The ability to represent mathematics is one of eight mathematical skills that must be acquired by students in Indonesia. The representation being shown remarks students' basic ability to develop concepts and to think mathematically. Changes in mathematical representations are needed to understand mathematical problems. Mathematical problems that are naturally abstract can be changed into other forms that are more concrete and making them easier to understand. The purpose of the current study was to analyze and to describe the process of changes from one specific representation to other mathematical representations. The subjects of this study were the seventh graders who had completed an assignment sheet. Tests and interviews were conducted to collect the necessary data. The former was given to reveal the process of changing mathematical representation, while the latter was to complete and clarify the data. The data analysis technique included reducing data, describing data, and drawing conclusions. The current study has indicated that the process of changing practiced by the students encompassed; verbal to visual representations through symbolic intermediaries, verbal to symbolic representations without intermediary representation, symbolic to verbal representations without intermediary representation, symbolic to visual representations without intermediary representation, visual to verbal representations *without* intermediary representation, and visual to symbolic representations through intermediary representation of verbal representations.**
***Keywords*: *representation, verbal, visual, symbolic, and intermediary***

## INTRODUCTION

Representation is one of the psychological concepts used to investigate students' ways of thinking and to indicate the intended mechanisms [1], [2] . Mathematical representation refers to any expressions of ideas or thoughts under particular conditions demonstrated by students in numerous forms and serves as a means of inventing solutions to mathematical problems [3]. Mathematical representation ability is one of eight major mathematical skills which Indonesian students are required to acquire, referring to the Regulations of Ministry of Education and Culture No. 58 of 2014 concerning on 2013 Curriculum of Mathematics in junior high school level, denoting that one of primary achievements in Mathematics is to present concepts into several forms of mathematical representations.

Portraying diverse representations is a fundamental skill that students need to acquire to formulate concepts and to think mathematically. Therefore, students require training to formulate representations on their own so that they are equipped with good and flexible skills, and understanding of concepts for problem-solving [4]. The use of mathematical representations to explore mathematical ideas for solving a problem allows students to make some changes on particular forms of representations to other possible forms, called representation change [5]. Changes in mathematical representations are indispensable to understand mathematical problems, especially the abstract ones, particularly due to the fact that they can be elaborated into more concrete forms to raise the sense of comprehensibility. Besides, students need to understand Mathematics by manipulating tangible objects and representing them based on their understanding.

The diversity of representations in a concept must be explored to enrich students' knowledge and experiences since mathematical representation, in general, contains expressions of students' thoughts in search of solutions to problems they face [6]. A problem can be represented through visual, verbal, and mathematical equations or expressions [7]. Moreover, mathematical representation is defined as a process of mental development acquired by an individual, which is revealed and visualized in numerous mathematical models, namely verbal, visual, and mathematical equations, or even the combination of all [8].

A number of difficulties are encountered by students when they attempt to define the existing variable and to answer an essay question since they need to convert the question into a mathematical model [9]. In this case, the possible mathematical representations to use are verbal and symbolic representations of the mathematical equation. In addition, it allows students to employ various forms

of representations and to change the representations into other possible forms. The problem in the current study was related to a one-variable linear equation. This study was aimed at analyzing and describing the process of 1) changing verbal representations to visual representations and mathematical equations, 2) changing mathematical equations to visual and verbal representations, and 3) changing visual representations to mathematical equations and verbal representations.

## METHOD

The current study employed a qualitative approach to reveal how the students carried out the processes of changing mathematical representations. It was also necessary to generate written and spoken information from the students through observation and interview. Accordingly, descriptive design was applied by the researcher to describe the process of changing from (1) verbal to visual representations and vice versa; (2) verbal representations to mathematical equations and vice versa; and (3) visual representations to mathematical equations and vice versa.

The subjects were six students of the seventh grade who answered a question sheet given. Specifically, the subjects were those equipped with good communication skills to explain what they had written during the interview session. The study was conducted at SMP Nasional Malang, Indonesia. The data were taken from the students and collected directly in the forms of the answers written on the sheet, which were related to the process of changing mathematical representations in solving one-variable linear equations. The results of the interview delimited the clarifications of the students' written answers.

Data collection instruments were tests and interviews. The test was completed by the students individually. The test items intended to reveal the process of changing mathematical representations were designed in three forms, namely essay, visual, and mathematical equation. The test of changing mathematical representation is a kind of test designed by involving changes in various representations. Meanwhile, the interview was carried out to collect the data of spoken communication regarding the process of changing mathematical representations aiming for the clarification of the students' written answers. To avoid the loss of information, the researcher asked for permission to use a voice recorder. Before the interview was started, the researcher briefly and clearly explained the background and topic of the study.

In general, the primary instrument of this study was the researcher guided by a worksheet. Additionally, the researcher served to perform several procedures, such as collecting, analyzing, interpreting, and reporting the data. The worksheet given to the students was in the form of an essay test containing mathematical problems (verbal, visual, and mathematical equation representations). The problems given were to trigger the students to perform changing mathematical representations adapted from a mathematics textbook for the seventh grade about one-variable linear equations.

Qualitative analysis technique was applied to analyze the data, which consisted of three main procedures performed at the same time: data reduction, data presentation, and conclusion. Data reduction refers to a process of simplification through selection, focus, and abstraction on raw data to be converted into meaningful data. Further, the data were presented narratively according to the results of the test and interview after reduction. The conclusion was drawn by comparing the results of the students' works to those of the interview to sum up the processes of the students' changing mathematical representations.

## RESULT & DISCUSSION

The current study was aimed at describing and exploring the processes of the students' changing mathematical representations. The data were collected by using a test about one-variable linear equations. There were six subjects recruited and considered having good communication skills. The data were processed by describing all answers of the subjects collected from both the written test and interview.

Target: The students are able to state any problems expressed in mathematical equations into verbal and visual representations.

### 1) The Result from Student 1 (S1)
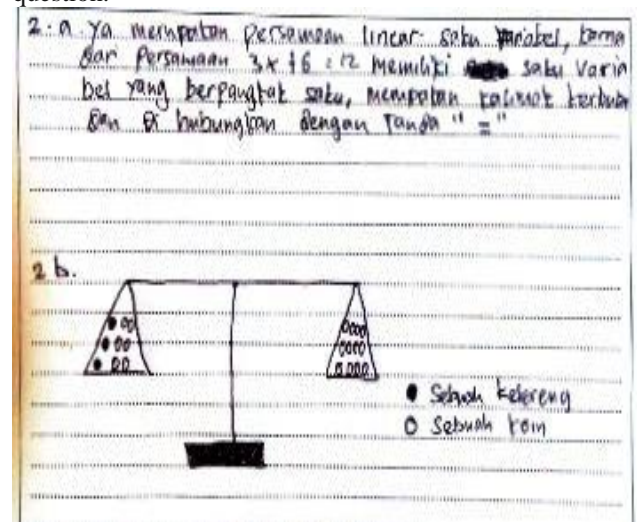The following is the answer of S1 to the given question.



Figure 1. S1's answer

The student could represent the problem by using the concept of one-variable linear equations in a precise and complete verbal form. Besides, the student drew a figure of balance, in which the left side consisted of three marbles and six coins, while the right one contained twelve coins. Then, the researcher interviewed S1 to acquire further information regarding the question as denoted in the following excerpt from the interview:

P   :   Referring to the equation of **3x + 6 = 12**, which one is the variable?
S1  :   The **x**, Sir.
P   :   Did you find variables other than **x**?
S1  :   No, Sir. That's only **x**.
P   :   For question 2b, could you please tell me about this figure of balance?
S1  :   So, there are 3 marbles and 6 coins on the left, and 12 coins are on the right.

P   :   Why did you put the marbles and coins on the left?
S1  :   Look at the equation, Sir. We have "**3x + 6**" on the left, so I put 3 marbles there to represent **3x** and added it with 6 coins, Sir.

P   :   Ah, I see. So, the **3x** represents 3 marbles, doesn't it?
S1  :   Alright, Sir.
P   :   Then, how could it be balanced?
S1  :   1 marble is equal to 2 coins.

Based on the result of the interview, S1 mentioned that the symbol "**x**" in the equation **3x + 6 = 12** was a variable, and the student also could see and state straightforwardly that variable **x** served as the marble by way of sketching a figure of balance, using marble and coin. To sum up, 1 marble was equal to 2 coins. Therefore, **x = 2**.

**2)   The Result from Student 2 (S2)**
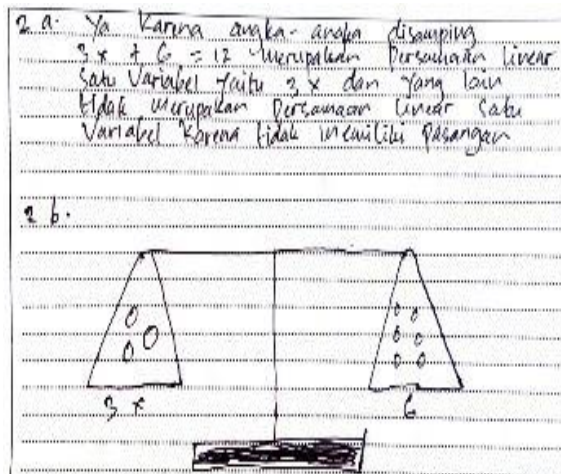The following is the answer made by S2 to the given question.



Figure 2. S2's answer

The student represented the problem into a verbal form by assuming that **3x** denoted one-variable linear equation since it appeared in a pair, while 6 and 12 did not represent the problem since both of which were written barely, not paired with any elements. Further, S2 drew a figure of balance in which three balls were put on the left side, representing **3x** and 6 balls on the right side numerically written as 6. The researcher conducted an interview with S2 in order to obtain further information related to the S2's answer as utterly shown in the following excerpt from the interview:

P   :   Of the equation **3x + 6 = 12**, which one is the variable?
S2  :   It's the **x**, Sir.
P   :   Could you please explain why you stated "the others are not categorized into the one-variable linear equation"?
S2  :   The others not belonging to the one-variable linear equation are 6 and 12, Sir. That's because both of them are not paired with any element. Meanwhile, **3x** is.
P   :   What do you mean by "not paired with any element?"
S2  :   They do not have any **x** variable, Sir.
P   :   For the question 2b, tell me briefly about the figure of balance you've made.
S2  :   The left side with three big balls refers to **3x**, while the right side has six small balls, Sir.
P   :   Referring to the equation **3x + 6 = 12**, where is the **12** located?
S2  :   <<in silence>>
P   :   Are you sure the figure of balance you've created is correct?
S2  :   Actually, I'm still lost, Sir.
P   :   What makes you confused?
S2  :   : Defining the compositions of the balance, Sir.

Based on the results of the interview, S2 was not able to understand the concept of one-variable linear equations well. As a consequence, S2 was mistaken to represent it verbally and found it challenging to imagine objects to put in the sense of the equation **3x + 6 = 12**.

**3)   The Result from Student 3 (S3)**
The following is the answer made by S3 to question number 2.
The student could represent the problem through a one-variable linear equation concept verbally and correctly, yet incomplete. S3 drew a figure of balance, with the left side containing three empty circles and three balls with an **x** symbol and the right side with 12 empty balls. Further, an interview was conducted with S3 to obtain further information related to the S3's answer to question

number 2 as presented in the following interview excerpt:

P : Look at the equation **3x + 6 = 12**. Which one do you think is the variable?
S3 : The **x**, Sir.
P : Do you think "**3x + 6 = 12**" belongs to an open or a close sentence?
S3 : That's an open sentence, Sir.
P : For the answer to question 2b, could you please explain more about this figure?
S3 : On the left, there are three balls with **x** sign, and three other balls are empty; thus, there are 6 in total.
   Meanwhile, I put 12 balls on the right side.
P : Why did you make the balls different? Three are with **x** sign, and three others are empty?
S3 : It represents the left side of the equation, **3x + 6**.
P : Which one is for **3x**? and which one is 6? Show me.
S3 : 3 balls are with an **x** sign.
P : Please tell me the meaning of "There are 6 in total".
S3 : To make it balanced with the right side, the **x** should be equal to two empty balls.

The interview excerpt indicates that S3 could identify the value of **x** and could express that it belonged to an open sentence. In addition, S3 could convey the representation correctly by sketching a figure of balance.
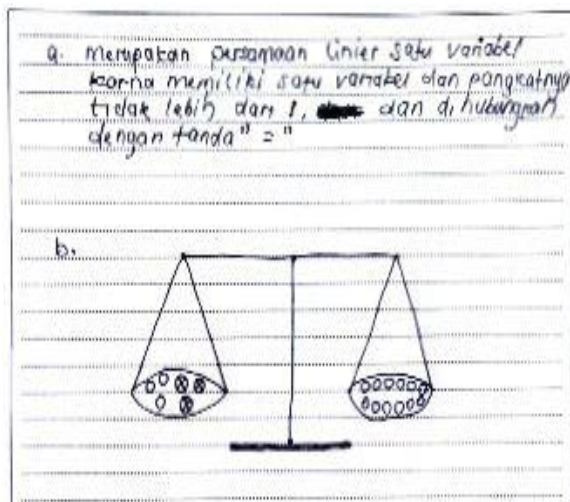


Figure 3. S3's answer

### 4) The Result from Student 4 (S4)
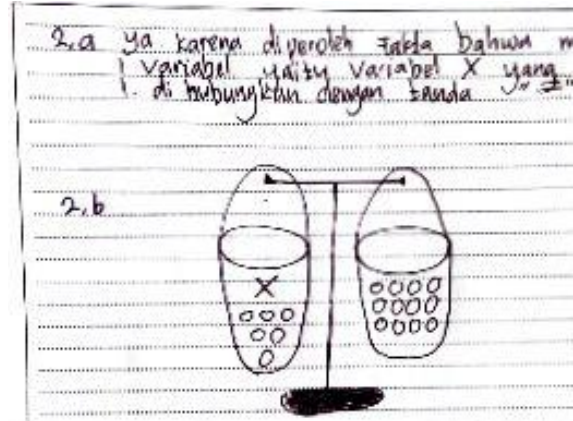The following is the answer made by S4 to the question given.



Figure 4. S4's answer

The student could represent the problem by applying the concept of one-variable linear equation in a verbal form correctly, yet incomplete. S4, in addition, created a figure of balance with **x** symbol and 6 balls put on the left, and 12 balls on the right. The following is the excerpt from the interview between the researcher and S4:

P : From the equation **3x + 6 = 12**, which one is a variable to the power of one?
S4 : The **x**, Sir.
P : Do you think "**3x + 6 = 12**" is an open or a close sentence?
S4 : That's an open sentence, Sir.
P : Referring to the question 2b, please tell me about the balance you've made.
S4 : : So, there are **x** variable and 6 balls on the left, and 12 balls on the right, Sir
P : What does the variable **x** refer to, according to the question?
S4 : It refers to the **3x**, Sir.
P : Look. There's written **3x**, but why did you only write an **x**?
S4 : My bad. That's mistaken, isn't it, Sir?

Based on the interview, it is shown that S4 could express the **x** symbol and that the equation belonged to an open sentence. Nonetheless, S4 was still in trouble to sketch a balance correctly.

### 5) The Result from Student 5 (S5)
The following is the answer made by S5 to the question given.

The student represented the problem by pronouncing that **3x** was the variable, while **6** was the constant. Next, S5 drew a figure of balance, with 3 apples and 6 strawberries put on the left side, and 6 apples and 6 strawberries on the right side. Then, the researcher interviewed S5 to obtain necessary information related to the answer to question 2 as shown in the following excerpt from the interview:

P : Regarding the equation **3x + 6 = 12**, which one is the variable?

S5   :   *The **x**, Sir.*

P   :   *On the question 2a, do you think **3x** is the variable?*

S5   :   *Sorry, I made a wrong answer; **x** is the only variable.*

P   :   *So, why does the equation **3x + 6 = 12** belong to a one-variable linear equation?*

S5   :   *That's because **x** is the variable, while 6 serves as the constant.*

P   :   *Do you think "**3x + 6 = 12**" is an open sentence?*

S5   :   *<<in silence>>*

P   :   *Look. What is "**3x + 6 = 12**" connected with?*

S5   :   *With "=", Sir*

P   :   *For the answer of 2b, please tell me briefly about this balance illustration.*

S5   :   *Here, we have 3 apples and 6 strawberries on the left side, and 6 apples and 6 strawberries on the right side. So, in total, there are 12.*

P   :   *Look at the equation here, **3x + 6 = 12**. What do apples and strawberries represent?*

S5   :   *'3 apples' represent the **3x**, '6 strawberries' represent the **6**, and '6 apples and 6 strawberries' on the right' represent the **12**, Sir.*

P   :   *So, which one does the **x** represent? Apple? Or strawberry?*

S5   :   *The apple, Sir. We have **3x** to indicate '3 apples', don't we?*

P   :   *Please look at it again, you also have 6 apples here. Does it also represent the **x** variable?*
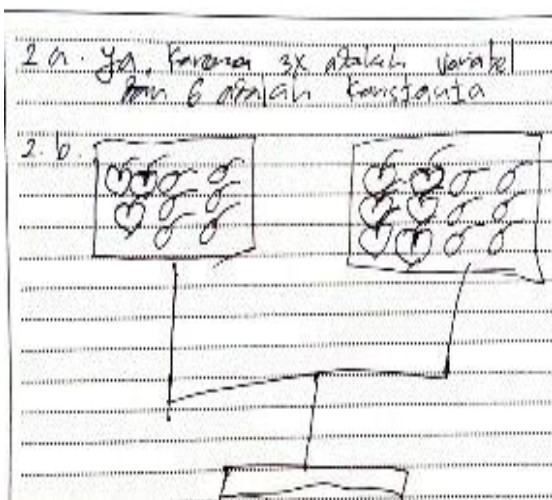
S5   :   *<<in silence>>.*



Figure 5. S5's answer

From the interview, it can be concluded that S5 only gave a reason by mentioning the elements of algebra operation, namely variable and constant.

However, S5 was aware of the **x** variable connected with an "=" symbol. Meanwhile, regarding the question 2b, S5 was mistaken in understanding the essence of balance between the left and the right sides. In sum, S5 was still perplexed to apprehend the meaning of **x** on the left side of the equation.

**6) The Result from Student 6 (S6)**

The following is the answer written by S6 to the question given.
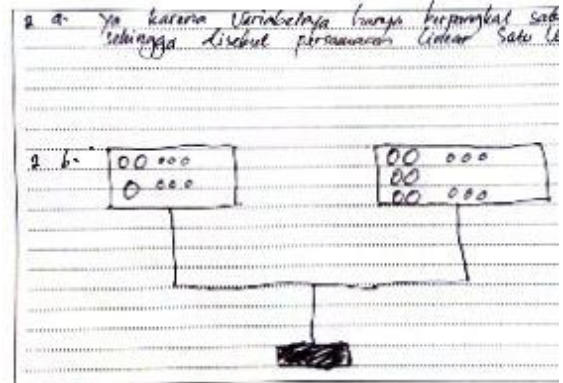


Figure 6. S6's answer

The student was able to represent the problem employing one-variable linear equation concept in a verbal form correctly, yet incomplete. S6 sketched a figure of balance, with 3 big balls and 6 small balls given on the left, and 6 big balls and 6 small balls on the right. Then, the researcher conducted an interview with S6 in search of further information about S6's answer to the given question, as presented in the following excerpt of the interview.

P   :   *Look at the equation, **3x + 6 = 12**. Which one is the variable?*

S6   :   *The **x**, Sir.*

P   :   *Is it because the variable is only to the power of 1?*

S6   :   *<<in silence>>*

P   :   *What do you think the equation of **3x + 6 = 12** is connected with?*

S6   :   *"=", Sir*

P   :   *For the answer of 2b, could you please explain more about this balance?*

S6   :   *So, we have 3 big balls and 6 small balls on the left, and 12 balls are on the right.*

P   :   *Then, which one does **x** refer to? The big or small ones?*

S6   :   *The big one, Sir.*

P   :   *Do you think the big balls are also supposed to be on the right side?*

S6   :   *Yes, Sir. But all the balls are supposed to be small.*

P   :   *So, how many small balls do you need to make a big ball?*

S6   :   *Two, Sir.*

Based on the interview, it is clear that S6 could indicate the $x$ variable in the equation $3x + 6 = 12$, to the power of 1, so that it belonged to the one-variable linear equation category and was connected by a relation symbol of "=". In addition, S6 was correct to understand the left side of the balance, yet remained incorrect to represent the balls on the right side.

The processes of changing mathematical representations found in the current study are shown in the following Table 1.

Based on Table 1, it is evident that changing mathematical representations from verbal to visual representation could be performed using intermediary representation, i.e., the symbolic representation, as shown by S1, S2, S3, S4, and S5. However, it was found that S1, somehow, did not require the intermediary representation. Further, changing mathematical representations from verbal to mathematical model could be performed using

S1, S2, S3, S4, and S6. In addition, changing the mathematical model to visual representation could be performed through symbolic representation, as shown by S2, S3, and S4.

However, under similar conditions, there were some of the subjects who preferred not to use any intermediary representation, namely S1, S5, and S6.

Some students required intermediary representation in the process of changing mathematical representation from visual to verbal representation, namely S1 and S3. Besides, the rest of the students, i.e., S2, S4, S5, and S6, did it without any intermediary representation. Changing mathematical representation from visual representation to the mathematical model could be performed in three ways: through symbolic representation as shown by S2; through verbal representation (as an intermediary representation) as committed by S4, S5 and S6; and without any intermediary representation as performed by S1 and

**Table 1.** Process of changing mathematical representations shown by the subjects

| Problems | Changing Processes Found | Subjects |
|---|---|---|
| 1a. | Verbal ⟶ Symbolic ⟶ Visual | S1, S2, S3, S4, S5 |
| | Verbal ⟶ Visual | S6 |
| 1b. | Verbal ⟶ Symbolic ⟶ Visual ⟶ Mathematical model | S3, S6 |
| | Verbal ⟶ Symbolic ⟶ Mathematical model | S1, S2, S4, S6 |
| 2a. | Mathematical model ⟶ Verbal | S1, S2, S3, S4, S6 |
| | Mathematical model ⟶ Symbolic ⟶ Verbal | S5 |
| 2b. | Mathematical model ⟶ Visual | S1, S5, S6 |
| | Mathematical model ⟶ Symbolic ⟶ Visual | S2, S3, S4 |
| 3a. | Visual ⟶ Verbal | S2, S4, S5, S6 |
| | Visual ⟶ Mathematical model ⟶ Verbal | S1, S3 |
| 3b. | Visual ⟶ Mathematical model | S1, S3 |
| | Visual ⟶ Symbolic ⟶ Mathematical model | S2 |
| | Visual ⟶ Verbal ⟶ Mathematical model | S4, S5, S6 |

intermediary representations, both symbolic and visual representations, as shown by S3 and S6. On the other hand, only symbolic representation was shown by S1, S2, S4, and S5 to answer the question.

Furthermore, it was found that changing the mathematical model to verbal representation could be performed through intermediary representation, i.e., the symbolic representation, as practiced by S5. Nevertheless, with the same case, some subjects did not require any intermediary representation, namely

S3.

The process of changing mathematical representations performed by the students in solving the problems encompassed the changes from (a) verbal to visual representation; (b) verbal representation to mathematical model; (c) mathematical model to verbal representation; (d) mathematical model to visual representation; (e) visual to verbal representation; and (f) the visual representation to a mathematical model through the

intermediaries. It indicated that the processes of changing mathematical representation from A to B might need representation C, D, and so forth. In other words, the processes do not occur instantly [5]. It is parallel with the process of changing problems presented in verbal representations to graphics performed by the students needs some intermediaries, such as symbolic, scheme, and numeric representations [10].

## CONCLUSION

The students did a change from verbal to visual representations using an intermediary, the symbolic representation. Meanwhile, verbal representations were changed to mathematical models without any intermediary. Similarly, a mathematical model was changed to verbal representation without any presence of an intermediary. The students made a change from a mathematical model to visual representation without any help of an intermediary. To change from visual to verbal representation, the students did not require any intermediary. At last, verbal representation was employed as an intermediary by the students to change from visual to a mathematical model.

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