

# Conceptual Errors of Students in Solving Mathematics Problems on the Topic of Function

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## ABSTRACT

The purposes of this study were to describe the students' conceptual errors in solving mathematics problems on the topic of functions and the cause of errors. This research is categorized into qualitative research with a descriptive-explorative approach. Diagnostic tests and interviews are the data collection techniques of this study. The research subjects are four selected students out of the total twenty eight students, who were evenly divided into two categories, namely, high-ability and low-ability. The results of the survey are that in low ability students, conceptual errors were found to be systemic from problem-solving besides algebraic operational errors and understanding of the instructions for solving problems. In high-ability students, there are errors in the operation of addition and errors in concluding. Solutions that can be applied to these problems are building an understanding of the concept of function operations, group learning, and intensive practice questions. Meanwhile, high ability students need to improve a more conducive learning environment.

**Keywords:** *Conceptual errors, solving mathematics problems, function topic*

## 1. INTRODUCTION

Mathematics is basic knowledge as a tool to support success in understanding other fields of study and means of scientific thinking to develop the ability to think logically, systematically, and critically. [1]. Learning as one of organizing student activities requires the teacher's creativity. The role of a teacher is not merely a source of information but also the direct and as a facilitator to facilitate the learning process to be more meaningful. Furthermore, teaching is an attempt to provide stimulus, guidance, and encouragement to students to study. Therefore, the teacher should give a particular condition to enable the student's participation and produce responses to certain situations.[2], [3].

In learning mathematic, teachers may have difficulty with the emergence of various student errors. The mistakes of students were systematic, consistent, incidental, and deviations from the rightness [4]. The references about the mathematical learning revealed that many types of errors made by students in solving problems. The errors are classified into concept and procedure errors. The types of concept errors include incorrectly using formulas, misinterpreting concepts, mistyping formulas, and not writing equations to answer questions.

On the other hand, the types of procedural errors made by students include seven types. Firstly, the student was not able to perform procedures to draw a line. Secondly, they did not understand the purpose of the problem. The third error indicated that the students incorrectly substitute variable values into formulas. In fourth problem was the errors due to less skill in using algebraic ideas. The fifth errors were the students done the arithmetic operations incorrectly. Other conditions showed by the errors because students did not continue the work process. The last type of errors is concluding without a valid reason [5].

Specifically, conceptual errors are usually caused by the learning environment. Theoretical knowledge requires students' ability to describe the relationships between categories, facts, and the nature of the problem. Students must be able to provide information about the relationship between variables mathematically. Conceptual errors appear in the inability of students to describe the relationship between variables and cause errors in solving mathematical problems. Besides, sometimes there are errors in determining theorems or formulas to answer a question. As a result, there is an error in using theories or formulas. Besides, students also did not write the theorem.

This research describes the problem of low mathematics learning outcomes at one of the high schools in Indonesia. The students' mathematics learning achievement in many senior high schools in Indonesia is still low. From the interview conducted by researchers, several mathematics teachers in MAN 2 Parepare met the problem of conceptual mistakes in solving mathematical problems. So it is necessary to identify the types of factors that cause errors and find alternative solutions to overcome these problems.

One of the solutions to overcome students' mistakes in solving mathematical problems is applying constructivism learning based on mathematics teaching standards, according to NCTM (National Council of Teacher of Mathematics)[6]. These standards using four components, namely, task, discourse, environment, and analysis. Knowledge aspects that are important in learning mathematics are understanding, the experience of students, and intellectual abilities. Mathematics teachers are required to develop students' abilities in describing arguments and problem-solving in mathematics problems. The cognitive domain in mathematics learning requires a systematic description of the procedure using the right concepts. The development of student reasoning is significant for mathematical communication skills.

This study emphasizes the topic of function, which aimed at uncovering the types of conceptual errors, the causes of errors, and overcome students' conceptual errors in solving mathematical problems.

## 2. RESEARCH METHODS

This research is categorized into qualitative research with a descriptive-exploratory approach. A descriptive method is used to describe the facts about students' conceptual errors in solving mathematical problems. The approach is intended to explore the data qualitatively through interviews. The subjects of this study were twenty eleventh-grade students who study in MAN 2 Parepare, a senior high school in South Sulawesi Province, Indonesia. The informant keys were chosen intentionally or purposively such that there were high, medium, and low ability students. The low ability categories by the bottom 25% of the students. At the same time, the high ability was obtained by the top 25% of the students. Meanwhile, medium-capable students are students who are located between the top and bottom students.

Like qualitative research, the researcher is the main instrument. The other supporting instruments were the diagnostic test and the interview guide used to reveal the cause of the error based on the results. In collecting data, four students were selected consisting of two students in the top category and in the low group who made the most mistakes. They were then interviewed to apply data

triangulation to validate data and obtain information on the causes of conceptual errors

Furthermore, data analysis techniques used Miles and Huberman Model. The first step is data reduction that including collecting the diagnostic test results and then make the students scoring. Furthermore, the analysis continued to determine the types of errors made based on the results of the diagnostic tests. The types of errors are conceptual errors in solving mathematical problems, recording the ranking of the students. Besides that, the data about the types of errors and the data of factors causing students to make mistakes in solving mathematical problems related to functions are the basis of interview guidelines which can be analyzed to draw conclusions and verification.

## 3. RESULT AND DISCUSSION

The description of error types is obtained by identifying the students' answers to math questions that are given. The result of error identification, namely:

### 3.1. Error in substituting values to variables

The function  $f(x) = 3x - 4$  is not substituted to  $X$  value in functioning  $(x) = 2x^2 + 4x - 5$  in the mapping  $(g \circ f)$ .

### 3.2. The Irregularity steps in completing a task

Inconsistent in operating the algebraic form of the function  $f(x) = 3x - 4$ .

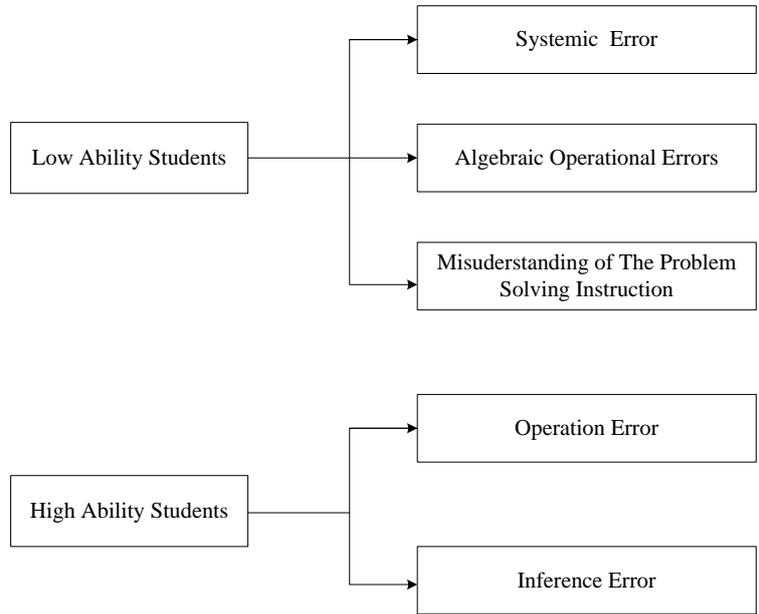
### 3.3. Concept mapping error

- The students do not understand the concept of  $(g \circ f)(x) = 4x^2 + 12x$  so they cannot map the values of  $g(x)$  and  $(g \circ f)(x)$  to get  $f(x)$ .
- Error in mapping function  $g$  which is the pair of points  $(1,6)$  can be continued its mapping to a function  $f$  that is the pair of marks  $(6, -2)$  to obtain the results of the mapping  $(f \circ g)$  at the point  $(-2)$ .

### 3.4. Math operation error

- Operating error in the quadratic function  $4x^2 + 12x = f(x) - 9$  becomes  $f(x) - 2 = -4x^2 + 12x - 9$  which should be answered to be  $f(x) - 2 = 4x^2 + 12x + 9$ .
- Error operating square root of the function  $(f(x)) - 2 = (f(x)) - 2 = \sqrt{4x^2 + 12x + 9}$ .
- Equation solving error  $x = \frac{y-12}{3}$  into  $y = \frac{12}{3}x$ , which the students should answer  $x = \frac{1}{3}y - 4$ .
- Students solve the equation  $x = \frac{y-12}{3}$  into  $y - 4$ , while the answer should be  $x = \frac{1}{3}y - 4$ .

Based on the identification, the researcher conducted interviews with two groups of students. The results of the interview are summarized in Figure 1.



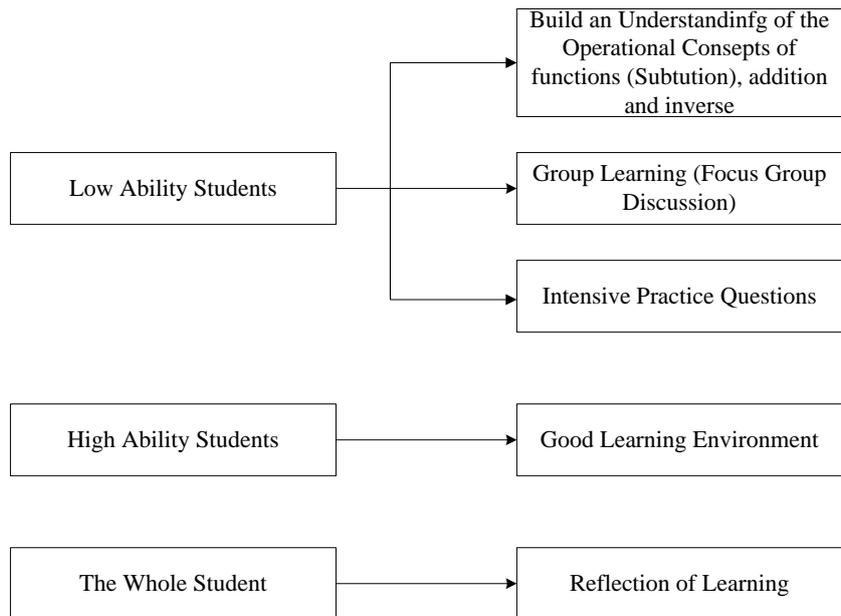
**Figure 1** The cause of conceptual errors

The searching results were confirmed with standard NCTM recommendations. Recommendations that can be applied to low ability students are to build an intensive understanding of the concept of addition, substitution, and inversion functions. Providing great practice questions and proper learning patterns are the task of group learning and focus group discussions.

The high ability students can apply an excellent learning environment improvement method. In this

research, students have understood the concept well. However, the uncomfortable room conditions and the noisy classroom atmosphere made it difficult for students to concentrate.

Furthermore, for all students, reflection activity on conceptual errors that occurs becomes a medium for increasing student enthusiasm and mastery. The description of the solution to the problem can be seen in Figure 2.



**Figure 2** A solution to the conceptual error problem

The description above becomes a reference for teachers to improve learning achievement students in mathematic, especially on the subject of functions. Therefore, mathematics teachers must create a learning environment that fosters the development of students' mathematical abilities. Efforts that can be made by the teacher are to provide and manage the time needed to explore important mathematical ideas. Besides, teachers can also use space and objects to facilitate students in learning mathematics. The existence of contests that encourage the development of students' math skills and proficiency is also a stimulant for students to find practical ways of mastering mathematical concepts.

On the other hand, students must also develop themselves independently and collaborate with other students to understand mathematics. With self-confidence, students will dare to take intellectual risks by asking questions and formulating possible answers [7]. A good learning environment for student development is the presence of mutual respect between teachers and students. The teacher respects students' original ideas and assesses the reasons for thinking that refer to solving math problems. The material presented by the teacher about the value of student ideas affects students' tendencies in the classroom. Students take risks in proposing their expected answers, strategies, and solutions in an environment where teachers value students' ideas. Teachers also need to help students to respect and be interested in other people's opinions.

#### **4. CONCLUSION**

In low ability students, conceptual errors were found to be systemic from problem-solving besides algebraic operational errors and understanding of the instructions for solving problems. In high-ability students, there are errors in the operation of addition and errors in concluding. Solutions that can be applied are building an understanding of the concept of function operations, group learning, and intensive practice questions. Meanwhile, high ability students need to improve a more conducive learning environment.

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