Advances in Intelligent Systems Research, volume 174
Proceedings of the 8th Scientific Conference on Information Technologies for Intelligent Decision Making Support (ITIDS 2020)

Application of Noise-Resistant Modular Codes to Increase the Fault Tolerance of Infocommunication Systems with OFDMD

Igor Kalmykov
Institute of Information Technologies and Telecommunications
North Caucasus Federal University
Stavropol, Russia
kia762@yandex.ru

Maksim Kalmykov
Institute of Information Technologies and Telecommunications
North Caucasus Federal University
Stavropol, Russia
kia762@yandex.ru

Ivan Efremenkov
Institute of Information Technologies and Telecommunications
North Caucasus Federal University
Stavropol, Russia
blood384@mail.ru

Igor Provornov*
Institute of Information Technologies and Telecommunications
North Caucasus Federal University
Stavropol, Russia
igorprovornov@yandex.ru

***Abstract*—The use of methods of intellectual decision support in modern infocommunication systems OFDM allows to ensure high reliability of data transmission due to their adaptation to various destructive influences. To make an optimal decision that allows to effectively counteract such impacts, it is necessary to conduct a multi-scale analysis of the parameters of the communication channel using a discrete wavelet transform. The purpose of research is to improve the accuracy and reliability of the implementation of discrete wavelet transform in infotelecommunication systems OFDM. To achieve this aim it is proposed to use nonpositional modular codes, in particular code of residue number system. The paper presents the implementation of discrete wavelet transform based on Haar discrete wavelet transform by code of residue number system (RNS). It is shown that the use of the modular RNS code does not only improve the accuracy in the OFDM-DWT, but helps to detect and correct errors that occur in the process of transformation. Thus, the application of new modular technologies in infotelecommunication systems OFDM allows not only to increase the computational accuracy, but also to ensure the correct result due to parallelization at the level of operation and processing of low-bit data OFDM.**

Keywords——orthogonal frequency division multiplexing discrete wavelet transform of signals, residue number system, correction of errors, positional characteristics

I. Introduction

Currently, infotelecommunication systems widely used the technology of orthogonal frequency division multiplexing OFDM (Orthogonal Frequency Division Multiplexing) [1 - 3]. This allows providing a high-speed transmission of multimedia data in wireless communication systems. In addition, OFDM technology has a number of advantages: high spectral efficiency, efficient operation in multipath signal conditions; high resistance to narrowband interference [1]. However, the use of Fast Fourier transform (FFT) in OFDM systems leads to drawbacks, among which there are two computational tracts, as well as accumulation of rounding errors.

It is possible to increase the efficiency of performing discrete wavelet transform of signals through the use of non-positional modular codes. It is known that the use of such codes allows to perform algorithms of digital signal processing in real time with high accuracy [1, 4, 5]. It is connected to the fact that basic operations of digital signal processing are operations of addition, subtraction and multiplication that can be effectively realized with use of modular integer arithmetic. These particular operations provide a basis of algorithms for the discrete wavelet transform of signals. Besides, modular codes are able to detect and correct errors that occur during the transmission of information due to interference in the communication channel. The ability of such codes to form decisions about the correctness of the implementation of the message transmission process and the need for their correction at reception allow us to refer the issues discussed in the article to the field of intellectual decision support. Therefore implementation of discrete wavelet transform (DWT) of signals with use of modular codes is the important task.

II. Research objective

Currently the wavelet transform has found wide application due to the fact that the Fourier transform and its fast algorithms are inefficient to provide analysis of non-stationary signals, which are localized in some interval of time [6, 7, 8, 9, 10, 11, 12]. This is because the information about the signal in the time domain is lost when performing the discrete Fourier transform (DFT) and fast Fourier transform (FFT). Therefore, to gain a true picture of the signal analysis it is necessary to make this procedure both in time domain and in frequency domain.

In order to increase the accuracy of the discrete wavelet transform [7] it was proposed to use algebraic structures of a finite field. In this case DWT was carried out on modulo p which is the characteristic of Galois field of GF (p). Execution of integer DWT provides maximum accuracy of the large-scale analysis of signals. However, increase in digit capacity of input signal parameters leads to sharp increase of modulo p. It affects negatively the instrumental expenses that cause lowering of error safety of a special processor (SP) of DWT. It is possible to solve the problem of simultaneous increase in accuracy of the large-scale analysis of a signal and strengthening the failures resistance which can arise in SP DWT operations by means of redundant modular codes. Therefore the purpose of our research is the increase in accuracy and reliability of the discrete wavelet transform of signals by means of adjusting codes of residue number system.

III. MATERIAL AND METHODS OF RESEARCH

It is known that the use of modular codes allows you to speed up and improve accuracy of DSP implementation due to the transition from the one-dimensional signal processing to multidimensional signal processing, using the isomorphism generated by the Chinese Remainder Theorem (CRT) [RNS]. The use of algebraic systems data allows to perform in parallel modular operations using low-bit integer residues. Currently the residue number system (RNS) has the largest application in this field. The main fields of usage of the modular codes are given in [1, 5, 13, 14].

In the residue number system integer A is represented as a set of residues , where ; i = 1,2, ..., n, obtained by dividing it by pairwise prime modules pi [1, 4].. The main advantages of RNS are the high speed and accuracy of modular operations, such as addition, subtraction and multiplication [2, 3]. However, these operations are widely used in large-scale analysis, according to

$$\begin{gathered} W_a(0,0) = \sum_{b=0}^{N-1} x(b)\psi_{00}(b) \\ W_d(m,j) = \sum_{b=0}^{N-1} x(b)\psi_{mj}(b) \end{gathered}, \tag{1}$$

where X = [x (0), x (1), ..., x (N-1)] - input vector; $\psi_{00}(b), \psi_{mj}(b)$ - skaling-function of discrete wavelet transform; Wa (0,0) and Wd (m, j) - approximate and detail sequence.

In case of large-scale analysis of signals in the last Galois field expression (1) is transformed into

$$\begin{gathered} W_a(0,0) = \left(\sum_{b=0}^{N-1} \left| x(b) \right|_p^+ \left| \psi_{00}(b) \right|_p^+ \right) \bmod p \\ W_d(m,j) = \left(\sum_{b=0}^{N-1} \left| x(b) \right|_p^+ \left| \psi_{mj}(b) \right|_p^+ \right) \bmod p \end{gathered}. \tag{2}$$

The use of residue number system code allows us to transform the calculation of one-dimensional DWT modulo p into a large-scale multi-dimensional analysis. In this case, we obtain an approximate sequence of RNS code residues

$$\begin{cases} w_1(0,0) = \left(\sum_{b=0}^{N-1} \left| x(b) \right|_{p_1}^+ \left| \psi_{00}(b) \right|_{p_1}^+ \right) \bmod p_1 \\ \vdots \\ w_k(0,0) = \left(\sum_{b=0}^{N-1} \left| x(b) \right|_{p_k}^+ \left| \psi_{00}(b) \right|_{p_k}^+ \right) \bmod p_k \end{cases}, \tag{3}$$

where $w_i(0,0) \equiv W_a(0,0) \bmod p_i$; i = 1, 2, ..., k.

At the same time detail sequence will look like

$$\begin{cases} w_1(m,j) = \left(\sum_{b=0}^{N-1} \left| x(b) \right|_{p_1}^+ \left| \psi_{mj}(b) \right|_{p_1}^+ \right) \bmod p_1 \\ \vdots \\ w_k(m,j) = \left(\sum_{b=0}^{N-1} \left| x(b) \right|_{p_k}^+ \left| \psi_{mj}(b) \right|_{p_k}^+ \right) \bmod p_k \end{cases}, \tag{4}$$

where $w_i(m,j) = W_d(m,j) \bmod p_i; i = 1,2,...,k$.

However, the transition to parallel computing leads to increased circuit costs, which affects negatively the reliability of DWT devices. A promising way to resolve this contradiction is to give processors the fault tolerance. The use of modular code allows you to solve this problem at lower circuit costs compared to the classical method of fault isolation "2 of 3".

For fail correction using modular codes r number of redundant bases $p_{k+1},..., p_{k+r}$ is used, while

$$p_1 < ... < p_{k-1} < p_k < p_{k+1} < ... < p_{k+r}, \tag{5}$$

where k - the number of working bases.

Introduction of redundant bases causes a full range of code

$$P^* = \prod_{i=1}^{k+r} p_i = P \prod_{i=k+1}^{k+r} p_i , \tag{6}$$

where $P = \prod_{i=1}^{k} p_i$ - the working range of RNS code.

RNS code $A = (\alpha_1, \alpha_2, ..., \alpha_{k+r})$ deemed to be permitted if it belongs to the working range of P, i.e.

$$A = (\alpha_1, \alpha_2, ..., \alpha_{k+r}) < P = \prod_{i=1}^{k} p_i \tag{7}$$

If an error occurs at the i-base RNS code, the incorrect code is:

$$\begin{gathered} \tilde{A} = (a_1, ..., a_{i-1}, \tilde{a}_i, a_{i+1}, ..., a_{k+r}) = \\ = (a_1, ..., a_{i-1}, a_i + \Delta a_i, a_{i+1}, ..., a_{k+r}) \end{gathered}, \tag{8}$$

where $\tilde{\alpha}_i$ - distorted residue of RNS code; Δa_i - the depth of error for the i-base.

According to CRT converting to a positional number system (PNS) we have

$$\tilde{A} = \sum_{\substack{j=1 \\ j \neq i}}^{k+r} (a_j B_j + (a_i + \Delta a_i) B_i) \bmod P^* . \quad (9)$$

Simplifying the equation

$$\tilde{A} = \sum_{\substack{j=1 \\ j \neq i}}^{k+r} (a_j B_j + (a_i + \Delta a_i) B_i) \bmod P^* = A + \Delta a_i B_i \quad (10)$$

Analysis of equation (10) indicates that an error $\tilde{A} > P$.

Since modular codes are non-positional codes, the positional characteristics are used to correct errors in these codes. They show the location of an error combination of modular code with respect to the working range of the system. In [15, 16, 17] the algorithm and the circuit realization calculation interval number, the physical meaning of which is defined as $L = [A/P]$. If RNS code has no error, i.e. $A < P$, the value of interval number is zero, i.e. L = 0. If an error occurs in RNS code, the positional characteristic $L \neq 0$. In [18] it is proposed to use as the positional characteristics the leading coefficients of the general polyadic system. In [19, 20] it is proposed for the error correction to apply algorithm for calculating the RNS trace code. However, the algorithms mentioned above require substantial circuit costs and are time-consuming. It is possible to reduce the costs by use of the developed algorithm for spreading the system of RNS code bases.

To calculate residues for control bases we use following interval of RNS code

$$L = \left[\frac{A}{P}\right] = \left[\frac{\sum_{i=1}^{k+r} \alpha_i B_i \bmod P^*}{P}\right] = \left[\frac{\sum_{i=1}^{k+r} \alpha_i \left(K_i P + B_i^*\right) \bmod P^*}{P}\right], \quad (11)$$

where $B_i = K_i P + B_i^*$ - orthogonal basis of the I-base RNS; $B_i \equiv 1 \bmod p_i$; B_i^* - orthogonal basis RNS without reference bases.

It is known that the values in interval L change from zero when there is no error in RNS code, to a value $\tilde{P} - 1 = \prod_{i=k+1}^{k+r} p_i - 1$. So the expression (11) can be written as

$$L = \left(\sum_{i=1}^{k+r} \alpha_i K_i + \left(\frac{\sum_{j=1}^{k} \alpha_j B_j^*}{P} \right) \right) \bmod \tilde{P} = \left(\sum_{i=1}^{k+r} \alpha_i K_i + R^* \right) \bmod \tilde{P}, \quad (12)$$

where R* is the rank of RNS control system without reference bases.

If an error in RNS code is absent, L = 0. Substitute this value in (12) and determine the value of the reference bases $(\alpha^*_{k+1}, ..., \alpha^*_{k+r})$ on the values of working residues $(\alpha_1, ..., \alpha_k)$. Then we have

$$\begin{cases} -\alpha^*_{k+1} K_{k+1} = \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \bmod p_{k+1} \\ \vdots \\ -\alpha^*_{k+r} K_{k+r} = \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \bmod p_{k+r} \end{cases} \quad (13)$$

To avoid the negative values of residues lets transform (13) to

$$\begin{cases} \alpha^*_{k+1} = \left(p_{k+1} - M_{k+1} \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \bmod p_{k+1} \\ \vdots \\ \alpha^*_{k+r} = \left(p_{k+r} - M_{k+r} \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \bmod p_{k+r} \end{cases}, \quad (14)$$

where $M_{k+j} = \left(K_{k+j}\right)^{-1} \bmod p_{k+j}$; j = 1, 2, ..., r.

To detect and correct the error in RNS code using the developed algorithm aimed at expanding the bases we must find the error syndrome

$$\begin{cases} S_{k+1} = \left| \alpha_{k+1} - \alpha^*_{k+1} \right|^+_{p_{k+1}} = \\ = \left(\alpha_{k+1} - \left(p_{k+1} - M_{k+1} \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \right) \bmod p_{k+1} \\ \vdots \\ S_{k+r} = \left| \alpha_{k+r} - \alpha^*_{k+r} \right|^+_{p_{k+r}} = \\ = \left(\alpha_{k+r} - \left(p_{k+r} - M_{k+r} \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \right) \bmod p_{k+r} \end{cases} \quad (15)$$

If the error syndrome is zero, RNS code contains no errors. Otherwise, the value of the error syndrome shows the location of the error and its depth. Error correction is performed according to

$$\alpha_i = (\tilde{\alpha}_i - \Delta\alpha_i) \bmod p_i \qquad (16)$$

where Δa_i - the depth of error for the i-base.

IV. RESEARCH RESULTS AND DISCUSSION

Suppose it is necessary to execute Haar wavelet transform over an input vector of data from eight elements x(n) = (145, 239, 1002, 564, 231, 11, 94, 123). In positional number system a matrix of Haar direct conversion will look like (17)

$$H_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}, \qquad (17)$$

where $\frac{1}{\sqrt{8}}$ is a normalizing multiplier.

To execute such wavelet transform in the field of prime number it is necessary to find the base of such field in which congruence relation is solvable, indicated in the expression (18).

$$X \bmod p \equiv \sqrt{2} \bmod p \qquad (18)$$

To perform an integer transform in residue number system we identified the bases of RNS p_1=7; p_2 = 17; p_3 = 23. All bases are coprime numbers because all of them are the prime numbers by themselves. In the field of each base the congruence relation described in (18) is solvable and hence in each of these fields the expression $\sqrt{2}$ can be represented in the form of integer number. Such expression will have the following values in the fields p_1=7; p_2 = 17; p_3=23 respectively $\sqrt{2} = 3 \bmod 7$; $\sqrt{2} = 6 \bmod 17$; $\sqrt{2} = 5 \bmod 23$ transform for each of the bases.

For p_1 = 7 Haar matrix will look like

$$H_8 = 6 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 6 & 6 & 6 & 6 \\ 3 & 3 & 4 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 4 & 4 \\ 2 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 5 \end{bmatrix}. \qquad (19)$$

For p_2 = 17 Haar matrix will look like

$$H_8 = 10 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 16 & 16 & 16 & 16 \\ 6 & 6 & 11 & 11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 6 & 11 & 11 \\ 2 & 15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 15 \end{bmatrix}. \qquad (20)$$

For p_3 = 23 Haar matrix will look like

$$H_8 = 10 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 22 & 22 & 22 & 22 \\ 5 & 5 & 18 & 18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 5 & 18 & 18 \\ 2 & 21 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 21 \end{bmatrix} \qquad (21)$$

Let's conduct the valuation of the matrices (19) - (21). Then, using the matrix (19-22), we calculate the results of the large-scale analysis for the input vector

x(n) = (145, 239, 1002, 564, 231, 11, 94, 123),

represented in RNS

$$x(n)_{(7,17,23)} = \begin{bmatrix} (5,9,7)(1,1,9)(1,16,13)(4,3,12)(0,10,1) \\ (4,11,11)(3,9,2)(4,4,8) \end{bmatrix}.$$

In RNS code DWT coefficients are equal. For p_1 = 7 will look like

$$|W|_7^+ = |H_8 x(n)|_7^+ =$$

$$\begin{bmatrix} 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 1 & 1 & 1 & 1 \\ 4 & 4 & 3 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 4 & 3 & 3 \\ 5 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 2 \end{bmatrix} \times \begin{bmatrix} 5 \\ 1 \\ 1 \\ 4 \\ 0 \\ 4 \\ 3 \\ 4 \end{bmatrix} \bmod 7 = \begin{bmatrix} 6 \\ 0 \\ 4 \\ 2 \\ 6 \\ 6 \\ 1 \\ 2 \end{bmatrix}$$

For p_2 = 17 will look like

$$|W|_{17}^{+}=\left|H_8\,x(n)\right|_{17}^{+}=$$

$$=\begin{bmatrix}10&10&10&10&10&10&10&10\\10&10&10&10&7&7&7&7\\9&9&8&8&0&0&0&0\\0&0&0&0&9&9&8&8\\3&14&0&0&0&0&0&0\\0&0&3&14&0&0&0&0\\0&0&0&0&3&14&0&0\\0&0&0&0&0&0&3&14\end{bmatrix}\times\begin{bmatrix}9\\1\\16\\3\\10\\11\\9\\4\end{bmatrix}^{T}\bmod 17=\begin{bmatrix}1\\1\\4\\4\\7\\5\\14\\15\end{bmatrix}$$

For $p_3 = 23$ will look like

$$|W|_{23}^{+}=\left|H_8\,x(n)\right|_{23}^{+}=$$

$$=\begin{bmatrix}7&7&7&7&7&7&7&7\\7&7&7&7&16&16&16&16\\12&12&11&11&0&0&0&0\\0&0&0&0&12&12&11&11\\14&9&0&0&0&0&0&0\\0&0&14&9&0&0&0&0\\0&0&0&0&14&9&0&0\\0&0&0&0&0&0&14&9\end{bmatrix}\times\begin{bmatrix}7\\9\\13\\12\\1\\11\\2\\8\end{bmatrix}\bmod 23=\begin{bmatrix}4\\18\\7\\1\\18\\14\\21\\8\end{bmatrix}$$

To perform the inverse Haar DWT transform in RNS code the matrix inverse to the matrix (19) - (21) are used. Implementation of these matrices allows you to recover the original input signal presented in RNS.

For $p_1 = 7$ will look like

$$|x(n)|_{7}^{+}=\left|H_8^{T}W(n)\right|_{7}^{+}=$$

$$=\begin{bmatrix}6&6&4&0&5&0&0&0\\6&6&4&0&2&0&0&0\\6&6&3&0&0&5&0&0\\6&6&3&0&0&2&0&0\\6&1&0&4&0&0&5&0\\6&1&0&4&0&0&2&0\\6&1&0&3&0&0&0&5\\6&1&0&3&0&0&0&2\end{bmatrix}\times\begin{bmatrix}6\\0\\4\\2\\6\\6\\1\\2\end{bmatrix}\bmod 7=\begin{bmatrix}5\\1\\1\\4\\0\\4\\3\\4\end{bmatrix}$$

For $p_2 = 17$ will look like

$$|x(n)|_{17}^{+}=\left|H_8^{T}W(n)\right|_{17}^{+}=$$

$$=\begin{bmatrix}10&10&9&0&3&0&0&0\\10&10&9&0&4&0&0&0\\10&10&8&0&0&3&0&0\\10&10&8&0&0&14&0&0\\10&7&0&9&0&0&3&0\\10&7&0&9&0&0&14&0\\10&7&0&8&0&0&0&3\\10&7&0&8&0&0&0&14\end{bmatrix}\times\begin{bmatrix}1\\1\\4\\4\\7\\5\\14\\15\end{bmatrix}\bmod 17=\begin{bmatrix}9\\1\\16\\3\\10\\11\\9\\4\end{bmatrix}$$

For $p_3 = 23$ will look like

$$|x(n)|_{23}^{+}=\left|H_8^{T}W(n)\right|_{23}^{+}=$$

$$=\begin{bmatrix}7&7&12&0&14&0&0&0\\7&7&12&0&9&0&0&0\\7&7&11&0&0&14&0&0\\7&7&11&0&0&9&0&0\\7&16&0&12&0&0&14&0\\7&16&0&12&0&0&9&0\\7&16&0&11&0&0&0&14\\7&16&0&11&0&0&0&9\end{bmatrix}\times\begin{bmatrix}4\\18\\7\\1\\18\\14\\21\\8\end{bmatrix}\bmod 23=\begin{bmatrix}7\\9\\13\\12\\1\\11\\2\\8\end{bmatrix}$$

Then in PNS code the input vector

$$x(n)_{(7,17,23)}=\begin{bmatrix}(5,9,7)(1,1,9)(1,16,13)(4,3,12)(0,10,1)\\(4,11,11)(3,9,2)(4,4,8)\end{bmatrix}.$$

Consider the application of the developed algorithm for detection and error correction when performing Haar DWT in RNS code. To correct a single error that distorts only one residue of RNS code we introduced two reference bases. As the reference bases let's use $p_4 = 31$ and $p_5=41$. As a result the operating range of RNS code is $P=\prod_{i=1}^{3}p_i=7\cdot 17\cdot 23=2737$. Consider RNS system that uses 3 operating bases and one reference base p4 = 31. Then the full range of P1 = 84847. For this system the orthogonal bases are equal

$$B_1=24242=K_1P+B_1^{*}=8P+2346;$$

$$B_2=59892=K_2P+B_2^{*}=21P+2415;$$

$$B_3=66402=K_3P+B_3^{*}=24P+714;$$

$$B_4=19159=K_4P=7P.$$

To calculate the residue of the reference base p_4 it is necessary to determine the value $M_4=(K_4)^{-1}\bmod p_4=7^{-1}\bmod 31=9$.

Consider RNS system that uses 3 operating bases and one reference base p5 = 41. Then the full range of P1 = 112217. For this system the orthogonal bases are equal

$$B_1 = 16031 = K_1 P + B_1^* = 5P + 2346;$$

$$B_2 = 46207 = K_2 P + B_2^* = 16P + 2415;$$

$$B_3 = 39032 = K_3 P + B_3^* = 14P + 714;$$

$$B_5 = 10948 = K_5 P = 4P.$$

To calculate the residue of the reference base p_5 it is necessary to determine the value $M_5 = (K_5)^{-1} \bmod p_5 = 4^{-1} \bmod 31 = 31$.

Consider the wavelet coefficient that is represented in RNS code W = (6,10,10) = 1574. This code can be represented using two reference bases p_4 = 31 and p_5 = 41. Thus we obtain W = (6,10,10,24,16). Suppose that in the process of calculating DWT coefficients no error occurred.

We use algorithm and calculate the residues for the reference bases, using the residues of information bases (14). For the first reference base p_4 = 31, so we obtain

$$\alpha_4^* = \left(p_4 - M_4 \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \bmod p_4 =$$

$$= \left| 31 - 9 \begin{pmatrix} 6 \cdot 8 + 10 \cdot 21 + 10 \cdot 24 + \\ + \left[\dfrac{6 \cdot 2346 + 10 \cdot 2415 + 10 \cdot 714}{2737} \right] \end{pmatrix} \right|_{31}^{+} = 24$$

For the second reference base p_5 = 41

$$\alpha_5^* = \left(p_5 - M_5 \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \bmod p_5 =$$

$$= \left| 41 - 31 \left(6 \cdot 5 + 10 \cdot 16 + 10 \cdot 14 + \left[\frac{6 \cdot 2346 + 10 \cdot 2415 + 10 \cdot 714}{2737} \right] \right) \right|_{31}^{+} = 16$$

Then, according to (15) the error syndrome is

$$\begin{cases} S_4 = \left| \alpha_4 - \alpha_4^* \right|_{p_4}^{+} = \left| 24 - 24 \right|_{31}^{+} = 0 \\ S_5 = \left| \alpha_5 - \alpha_5^* \right|_{p_5}^{+} = \left| 16 - 16 \right|_{41}^{+} = 0 \end{cases}$$

Since the error syndrome is zero, the RNS code contains no error.

Suppose there was an error in the first RNS base p1 = 7 and its depth is $\Delta\alpha_1 = 1$. Then DWT coefficient in RNS code has the form

$$\tilde{W} = (\alpha_1 + \Delta\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (0, 10, 10, 24, 16)$$

Using developed algorithm we calculate the residues for reference bases, using the residues of information bases (14). For the first reference base p_4 = 31 we obtain

$$\alpha_4^* = \left(p_4 - M_4 \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \bmod p_4 =$$

$$= \left| 31 - 9 \left(0 \cdot 8 + 10 \cdot 21 + 10 \cdot 24 + \left[\frac{0 \cdot 2346 + 10 \cdot 2415 + 10 \cdot 714}{2737} \right] \right) \right|_{31}^{+} = 5.$$

For the second reference base p5 = 41 we have

$$\alpha_5^* = \left(p_5 - M_5 \left(\sum_{i=1}^{k} \alpha_i K_i + R^* \right) \right) \bmod p_5 =$$

$$= \left| 41 - 31 \left(0 \cdot 5 + 10 \cdot 16 + 10 \cdot 14 + \left[\frac{0 \cdot 2346 + 10 \cdot 2415 + 10 \cdot 714}{2737} \right] \right) \right|_{31}^{+} = 35.$$

Then according to (15) the error syndrome is equal to

$$\begin{cases} S_4 = \left| \alpha_4 - \alpha_4^* \right|_{p_4}^{+} = \left| 24 - 5 \right|_{31}^{+} = 19 \\ S_5 = \left| \alpha_5 - \alpha_5^* \right|_{p_5}^{+} = \left| 16 - 35 \right|_{41}^{+} = 22 \end{cases}$$

Since the error syndrome does not equal zero, then RNS code contains an error. We determined the correlation between the error syndrome and its depth $\Delta\alpha_i$, i = 1, 2, 3, 4, 5. Table 1 shows the values of the error syndrome and its depth for the first RNS base p_1 = 7.

Table 1. Values Of The Error Syndrome And Depth For Modulo p_1 = 7

Depth of error $\Delta\alpha_1$	S_4	S_5
1	19	22
2	7	3
3	26	25
4	14	6
5	24	38
6	12	19

Then use the expression (16) and correct the error base in RNS code

$$\alpha_1 = (\tilde{\alpha}_1 - \Delta\alpha_1) \bmod p_7 = (0 - 1) \bmod 7 = 6.$$

To research the effectiveness of discrete wavelet transforms implemented in modular codes, in order to improve the resiliency of infocommunication systems with OFDM, simulation was performed using Matlab 2017.

A pseudo-random sequence was generated from which a set of 64 samples of 8 bits each was formed. The formation took place in the frequency band of 20 MHz and with a duration of 12.8 µs. Then an additive Gaussian noise was generated, which was added to each of the generated signals. The received signals were demodulated, and then the ratio of the erroneously received bytes L * to the total number of transmitted L ( ) was calculated.

The results of the conducted studies of the OFDM system using FFT, OFDM system realizing the implementation of hardboard in the modular code and the

OFDM system realizing the implementation of hardboard in the redundant COD code are presented in Figure 1.

So with a signal to noise ratio (SNR) of 4 dB, the Byte Error Rate of a DFT-based OFDM system is Rosh = 0.41. When using Haar fiberboard implemented in the MC, the error probability is provided by Byte Error Rate Roche = 0.1. The implementation of the OFDM system based on the corrective modular code provides a Byte Error Rate of 0.042

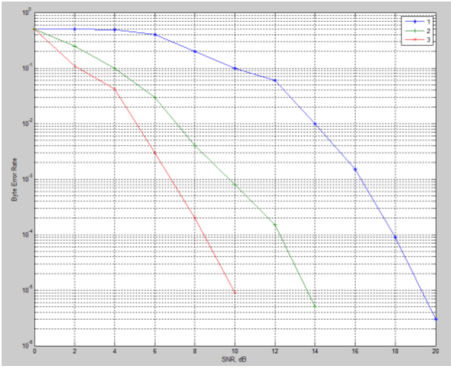

Figure 1 – Estimation of noise immunity of OFDM systems.

1 is a model of an OFDM system using an FFT,

2 - model OFDM system using DVP in modular code;

3 model OFDM system using fiberboard in redundant MC.

To achieve an error probability of equal to Roche = 10-5 in OFDM systems based on FFT, it is necessary to provide a signal to noise ratio equal to 19 dB, using the developed algorithm for realizing a fiberboard in modular code - it will take 13.5 dB, and application of the developed mathematical model based on correcting codes RNS- 10 dB. Thus, the integration of the properties of correcting modular codes and discrete wavelet transforms makes it possible to increase the noise immunity when performing orthogonal frequency multiplexing of signals by 1.9 times in comparison with the OFDM-DFT system and 1.35 times in comparison with the OFDM-fiberboard system constructed in modular codes.

V. CONCLUSION

The article presents an algorithm of large-scale analysis using RNS codes. This example shows that the use of modular code allows maximizing DWT accuracy by performing integer calculations. RNS code enables to increase the reliability of the results of discrete wavelet transform in infotelecommunication systems OFDM. Due to introduction of two reference bases the resulting redundant code is able to eliminate all single errors in RNS code residues. New modular technologies in the problems of digital signal processing allow not only to increase the accuracy of calculation, but also to ensure the correct result by means of parallelization at the level of operation and processing of low-bit data. Thus, the integration of the properties of correcting modular codes and discrete wavelet transforms makes it possible to increase the noise immunity when performing orthogonal frequency multiplexing of signals by 1.9 times in comparison with the OFDM-DFT system and 1.35 times in comparison with the OFDM-fiberboard system constructed in modular codes.

ACKNOWLEDGMENT

This work was supported by the Russian Foundation for Basic Research, project No. 18 ‑ 07-01020.

REFERENCES

[1] Ananda Mohan Residue Number Systems. Theory and Applications. Springer International Publishing Switzerland. 2016

[2] [Che95] Chervyakov, N.I., Veligosha, A.V., Kalmykov, I.A., Ivanov, P.E. 1995 Digital filters in a system of residual classes Izvestiya Vysshikh Uchebnykh Zavedenij. Radioelektronika, 1995, 38 (8), pp. 11-20

[3] Gordenko, D.V., Rezenkov, D.N., & Sarkisov, A.B. (2014). Methods and Algorithms of Reconfiguration of Non-Positional Computational Structures for Providing the Failure Robustness of the Special Processors. Stavropol: Fabula Publishe

[4] Mohan P.V. Residue Number Systems. Algorithms and Architectures. Springer. 2002

[5] Omondi A. and Premkumar B. 2007. Residue Number Systems: Theory and Implementation. Imperial College Press. UK 2007

[6] Hans-Georg Stark Wavelets and signal processing. Springer International Publishing Switzerland. 2005

[7] Ingrid Daubechies. Ten Lectures on Wavelets. CBMS-NSF Regional Conference Series in Applied Mathematics (1992)

[8] Joilson (2011) Alves Junior Using redundant residue number system in increase routing dependability on mobile ad hoc networks// Journal of Selected Areas in Telecommunications (JSAT). January Edition, 2011. pp.67-73

[9] Munteanu C., Rosa A. Gray-Scale Image Enhancement as an Automatic Process Driven by Evolution // IEEE Transactions on Systems, Man, and Cybernetics. – 2004. – V. 34. – P. 1292–1298

[10] Paul S. Addison. The Illustrated Wavelet Transform Handbook. Institute of Physics Publishing (2002)

[11] Ramazan Gençay, Faruk Selçuk and Brandon Whitcher, An Introduction to Wavelets and Other Filtering Methods in Finance and Economics, Academic Press, 2001

[12] Wavelets 4 Dummies: Signal Processing, Fourier Transforms and Heisenberg https://georgemdallas.wordpress.com/2014/05/14/wavelets-4-dummies-signal-processing-fourier-transforms-and-heisenberg

[13] Katkov, K.A., & Kalmykov, I.A. (2013). Application of Parallel Technologies in Navigation Management under the Conditions of Artificial Ionospheric Disturbances. World Applied Sciences Journal, 26(1), 108-113

[14] Yatskiv, V., Yatskiv, N., Su, Jun, Sachenko, A., & Zhengbing, Hu (2013). The use of a modified correction code based on a reside number system in WSN. In Proc. 7-th IEEE Int. Conf. Intelligent Data Acquisition and Advanced Computing Systems, (IDAACS 2013), vol. 1, pp. 513-516. Berlin, Germany

[15] Kalmykov, I.A., Katkov, K.A., Naumenko, D.O., Sarkisov, A.B., & Makarova, A.V. (2014). Parallel Modular Technologies in Digital Signal Processing. Life Science Journal, 11(11s), 435-438. Retrieved April 9, 2016 from http://www.lifesciencesite.com

[16] Kalmykov, I.A., Katkov, K.A., Timoshenko, L.I., Dunin, A.V., & Gish, T.A. (2015). Application of Modular Technologies in the Large-Scale Analysis of Signals. Journal of Theoretical and Applied Information Technology, 80(3), 391-400. Retrieved April 9, 2016 from

[17] Stepanova E.P., Kalmykov I.A., Toporkova E.V., Kalmykov M.I., Katkov R.A., Rezenkov D.N. (2016) Application of the codes of a polynomial residue number system, aimed at reducing the effects of failures in the AES cipher // Journal of Digital Information Management, April 2016. Vol. 14. N.2. PP.114-123

[18] Liew T.H., Yang L-L. (1999) Soft-decision redundant residue number system based error correction coding // Proceeding of vtc'99 22 september 1999, Amsterdam, Nederland

[19] Hamidreza Mahyar (2012) Reliable and High-Speed KASUMI Block Cipher by Residue Number System Code// World Applied Sciences Journal 17 (9): 1149-1158, 2012.

[20] Grinsted, A., Moore, J.C., Jevrejeva, S. (2004) Application of the cross wavelet transform and wavelet coherence to geophysical time series, Nonlin. Processes Geophys., 11, 561–566, doi:10.5194/npg-11-561-2004