

Analysis of Interactions in Structural System Representations

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Abstract—Interactions between the elements of a functioning system in its structural representation are considered. This raises questions of system modeling for the development of models and algorithms for structuring relationships and decision rules on sets of information. This is necessary for generating output documents. The definitions of interactions of conflict, cooperation and neutrality between the subsystems in the process of achieving their local goals are introduced. The competition that occurs between goals is resolved by choosing a compromise in the framework of solving problems of vector optimization and game theory. Systems participate in the formation of integral properties of the subsystem and can enter into relations of conflict, cooperation and neutrality with other subsystems and conversely. The proposed approach allows us to evaluate the quality of the entire system as a whole. Some structural and topological characteristics used in graph theory that are suitable for describing such systems, which are of interest for problems of analyzing binary relations of subsystems. These include the degree of centralization. It characterizes the uniformity of connections in the system, structural redundancy. Characterizes the measure of redundancy of the structure by connections in the system. Structural compactness—this property is proposed to be evaluated by a number of indicators: the diameter, center and radius of the subsystem, a relative indicator that characterizes the structural proximity of subsystems to each other. A methodology for structural analysis of such a system with an example of its implementation using a computer is proposed.

Keywords—analysis, structural representation of systems, subsystems, conflict, cooperation, neutrality, graph theory, computer implementation

I. INTRODUCTION

First, consider some system $S = \{S_1, S_2, S_3, \dots, S_N\}$, which consists of N active elements (subsystems) S_i , $i = 1, 2, \dots, N$. While S is acting, it is accordingly moving towards some target W , and another of its parts S_i to achieve, in turn, the local target W_i . Next, we describe the mutual effects of subsystems on each other and on system S .

The description of the S system is applicable using a theoretical-multiple approach. We introduce X -set of inputs, V -

set of outputs, and C - set of global states of system S , X, V, C - implementation of these sets, and $X = X_1 \times X_2 \times \dots \times X_m$, $V = V_1 \times V_2 \times \dots \times V_n$, where \times is the symbol of Cartesian product and $C = \{C_1, C_2, \dots, C_k\}$ [1].

Therefore, it is possible to write down:

$$S : X \times C \rightarrow V, S \subset (X \times C) \times V \quad (1)$$

and $(X, V) \subset S \Rightarrow \exists C [R(C, X) = V], X \in X, V \in V, C \in C$, where $R(C, X)$ - some function. It is called a global system response. Such a description can be called "input - state - output."

Suppose that the targets W, W_1, W_2, \dots, W_n are measurable. Then, for W on the set X , the utility function $q(X)$ can be introduced. If $x^1, x^2 \in X$ and $q(x^1) > q(x^2)$, then $x^1 \succ x^2$ (\succ - better) if possible to achieve W . Thus, system S strives to achieve goal W faster. These findings apply to W_i as well.

II. THE DESCRIPTION OF THE SYSTEM

Enter the definitions of conflict ratio ($>I_k$), cooperation ($>I_c$) and neutrality ($>I_n$) for the two parts of the system S_i and S_j as they approach their local W_i and W_j targets. Thus [2]:

- the S_i subsystem conflicts with the subsystem S_j ($S_i > I_k S_j$), if

$$q_j(S_j, S_i) < q_j(S_j, \sim S_i), \quad (2)$$

where the designation $\sim S_i$ assumes that there is no S_i subsystem surrounded by S_j subsystem. The definition means that the presence of the subsystem S_i in the environment of the subsystem S_j reduces the efficiency of achieving the goal W_j by the subsystem S_j in the sense of q_j criterion;

- the S_i subsystem cooperates with the subsystem S_j ($S_i > I_c S_j$) if

$$q_j(S_j, S_i) > q_j(S_j, \sim S_i), \quad (3)$$

that is, the presence of the subsystem S_i in the environment of the subsystem S_j increases the efficiency of achieving the goal W_j by the subsystem S_j in the sense of the criterion q_j ;

- S_i and S_j subsystems are neutral ($S_i > I_n S_j$) if

$$q_j(S_j, S_i) = q_j(S_j, \sim S_i), \quad (4)$$

that is, the presence of the subsystem S_i in the environment of the subsystem S_j does not affect the efficiency of achieving the goal W_j by the subsystem S_j in the sense of the criterion q_j .

Also it is worth assuming that system S acts on a time interval of $t \in [t_0, t_k] = T$. In that case $X(t) : \{X(t) / t \in T\}$, $Y(t) : \{Y(t) / t \in T\}$, $C(t) : \{C(t) / t \in T\}$ [3-6]. The action of each subsystem $S_i(t)$ can be shown as the "input-state-output" expression for some nonlinear system in the form of differential equality:

$$C'_i(t) = F_i(C_i(t), X_i(t)), \quad Y_i(t) = R_i(C_i(t), X_i(t)), \quad (5)$$

where $C_i(t) = \{c_i(t)\}$ is the set of global states of the subsystem S_i , $X_i(t) = X_{i1}(t) \times X_{i2}(t) \times \dots \times X_{imi}(t)$ is the input part of the entire S_i system element, $Y_i(t) = Y_{i1}(t) \times Y_{i2}(t) \times \dots \times Y_{imi}(t)$ is the output part of the entire S_i system element, with $X_{ik}(t) = \{X_{ik}(t), k = \overline{1, mi}\}$, $Y_{ir}(t) = \{Y_{ir}(t), r = \overline{1, ni}\}$ - the set of system inputs and outputs S_i . F_i and R_i -vector - columns of nonlinear dependencies. $C'_i(t)$ -opens changes of states in a uniform system eventually.

Using function of usefulness for the S_i system with some value of process time $t \in [t_0, t_k] = T$ the possibility of implementation of the purpose of W_i : $q_i(X_i(t)) = q_i(X_i(t), R_i(C_i(t), X_i(t)))$, where $X_i(t) \in X_i(t)$, $C_i(t) \in C_i(t)$. Note that $i = \overline{O, N}$, is a representation of the external environment in the form of the S_0 system.

Let, $\forall i, j = \overline{O, N}$ will be a derivative (6) [7] where $\Delta X_j(t, s_i) = (\Delta X_j^0(t, s_i), \Delta X_j^1(t, s_i), \dots, \Delta X_j^N(t, s_i))$ - a difficult vector - increment, received by an entrance $S_j(t)$ as a result of action of an argument of a subsystem of $S_i(t)$ in a certain period of time t : $q_i(X_i(t) + \Delta X_i(t, s_i)) > q_i(X_i(t))$; $\Delta X_j^\lambda(t, s_i)$ - a component of increment which is created by a subsystem of $S_i(t)$ during activity of a subsystem $S_i(t)$; $i, j, \lambda = \overline{O, N}$.

$$q'_j(t, s_i) = \frac{\partial q_j(t)}{\partial s_i(t)} = \lim_{\Delta X_i(S_i) \rightarrow 0} [q_j(X_j^0(t, s_i) + \Delta X_j^0(t, s_i), \dots, X_j^N(t, s_i) + \Delta X_j^N(t, s_i)) - q_j(X_j^0(t), \dots, X_j^N(t))] / \Delta X_i(t, s_i) \quad (6)$$

Then, given (3), (4), (5) at time t :

- the subsystem of $S_i(t)$ clashes with a subsystem of $S_j(t)$ ($S_i(t) > I_n S_j(t)$) if $q'_j(t, s_i) < 0$;

- the subsystem of $S_i(t)$ promotes functioning of a subsystem of $S_j(t)$ ($S_i(t) > I_n S_j(t)$) if $q'_j(t, s_i) = 0$.

- functioning of a subsystem of $S_j(t)$ does not depend on $S_i(t)$ ($S_i(t) > I_n S_j(t)$) if $q'_j(t, s_i) = 0$.

The same definitions are fair also for $\forall i, j = \overline{O, N}$ and also for $(S(t), S_i(t))$.

Obviously, in this approach, the formation of increment vectors for each of the subsystems, and for the entire system, depends on the structure of their relationships, which determines their interaction during the operation of the system.

Having calculated the relations $>I_{ks}, >I_{cs}, >I_{hs}$ between all pairs of elements of the set $S^M(t) \forall t \in T$, where $S^M(t) = \{S_0(t), S_1(t), \dots, S_N(t)\}$, it can be described by the directed graph $G(t) = G(S^M, E, t)$, where $S^M = S^M(t)$ is the set of vertices, $E = \{e_{ij}(t)\}$ is the set of arcs.

Knowing the values (weight) of the arcs, the adjacency matrix is filled. It is the source for the program described below. This program allows you to solve the problems of static analysis of interactions in the structural representation of systems. To write it, the standard components of the application development tool in the Windows environment were used. This made it possible to create a user-friendly interface with a fairly convenient menu and multi-tasking mode of operation. In addition, the program provides the ability to output the necessary information to the PMP and save graphic images in JPG format.

For example, we fill in the adjacency matrix with random values, and pay attention to the interaction of conflicting subsystems.

In order to correctly understand their interaction, it is necessary to select $\forall t \in T$ from $S^M(t)$ a subset of the values $S^{>I_h}(t)$, $S^{>I_c}(t)$ and $S^{>I_k}(t)$. For them, the binary relations $>I_h, >I_c, >I_k$ respectively, are true.

This is the same as distinguishing from $G(t) = G(S^M, E, t) \forall t \in T$ sub graphs: $G(>I_k) = G^{>I_k}(S^{>I_k}, E^{>I_k}, t)$, $G(>I_c) = G^{>I_c}(S^{>I_c}, E^{>I_c}, t)$, $G(>I_h) = G^{>I_h}(S^{>I_h}, E^{>I_h}, t) \subset G(S^M, E, t)$.

The following binary relations $>I_{ks}, >I_{cs}, >I_{hs}$ can be defined by adjacency matrices for the graph $G = G(S^M, E, t)$. We denote them by $K = [\kappa_{ij}]$, $K^K = [\kappa_{ij}^k]$, $K^C = [\kappa_{ij}^c]$, $K^H = [\kappa_{ij}^h]$ of order $n \times n$, $n \leq N+1$, moreover:

- $\kappa_{ij} \neq 0$ if $e_{ij} \in E \wedge e_{ij} \neq \emptyset, \Rightarrow \kappa_{ij} = e_{ij}$;
- $\kappa_{ij}^k \neq 0$ if $e_{ij} \in E^{>I_k} \wedge e_{ij} \neq \emptyset, \Rightarrow \kappa_{ij}^k = e_{ij} < 0$;
- $\kappa_{ij}^c \neq 0$ if $e_{ij} \in E^{>I_c} \wedge e_{ij} \neq \emptyset, \Rightarrow \kappa_{ij}^c = e_{ij} > 0$;
- $\kappa_{ij}^h \neq 0$ if $e_{ij} \in E^{>I_h} \wedge e_{ij} \neq \emptyset, \Rightarrow \kappa_{ij}^h = e_{ij} = 0$.

The value of the variable is equal to the weight of the arc that connects S_i to S_j and the value κ_{ij} is equal to the weight of the arc that connects S_j to S_i , therefore, the expression $K = K^K + K^C + K^H$ is true.

For the next vertex S_i :

$$\pi_o(S_i) = \sum_{j=1}^n \omega_{ij}, \quad \pi_e(S_i) = \sum_{i=1}^n \omega_{ij}, \quad (7)$$

here ω_{ij} is an element of the zero-identity matrix $\Omega(\mathfrak{R})$ where \mathfrak{R} is the generalized relation $\mathfrak{R} = (>I_k \vee >I_c \vee >I_h)$. Suppose if $\omega_{ij} \in \Omega(>I_k)$, and $e_{ij} \in E^{>I_k}$ and $e_{ij} \neq \emptyset$, then $\omega_{ij} = 1$, otherwise $\omega_{ij} = 0$.

The values of $\pi_o(S_i)$ and $\pi_e(S_i)$ in graph theory are called the semi-degree of outcome and semi-degree of entry, respectively, for a certain vertex S_i . The value of $\pi_o(S_i)$ reflects the number of arcs that exit from the vertex S_i , and $\pi_e(S_i)$ - respectively, the number of arcs in S_i .

Suppose $\pi_o(S_i) = 0$, then it is possible to isolate elements of S_i (finite subsystems), the action of which does not affect the creation of a relation \mathfrak{R} from the point of view of the graph in question $G^{>I_k} \vee G^{>I_c} \vee G^{>I_h}$, the value $\pi_o(S_i) > 0$ shows the number of subsystems on which S_i acts during formation \mathfrak{R} .

Let $\pi_o(S_i) = \pi_e(S_i) = 0$, therefore, the vertex will be isolated for the relation \mathfrak{R} .

Due to the fact that the sums $\pi_o(S_i)$ and $\pi_e(S_i)$ show only the place of S_i in the general structure $G^{\mathfrak{R}}$, it is worth introducing quantities that reveal the degree of influence of S_i on the formation of the ratio \mathfrak{R} . For the next vertex S_i , we introduce sums of the form (8).

The sum $\lambda_o(S_i)$ reflects (as a percentage of the total \mathfrak{R} for S^M) the degree of influence of S_i on the remaining parts of the relations \mathfrak{R} . The sum $\lambda_e(S_i)$ is different.

Based on the foregoing, it is obvious that $(\pi_o(S_i), \lambda_o(S_i))$ and $(\pi_e(S_i), \lambda_e(S_i))$ show the importance (weight) of the S_i subsystem during the construction of relations \mathfrak{R} between the subsystems of the set S^M .

$$\lambda_o(S_i) = \frac{\sum_{j=1}^n k_{ij}}{\sum_{j=1}^n \sum_{i=1}^n k_{ij}} \cdot 100\%, \lambda_e(S_i) = \frac{\sum_{i=1}^n k_{ij}}{\sum_{j=1}^n \sum_{i=1}^n k_{ij}} \cdot 100\% \quad (8)$$

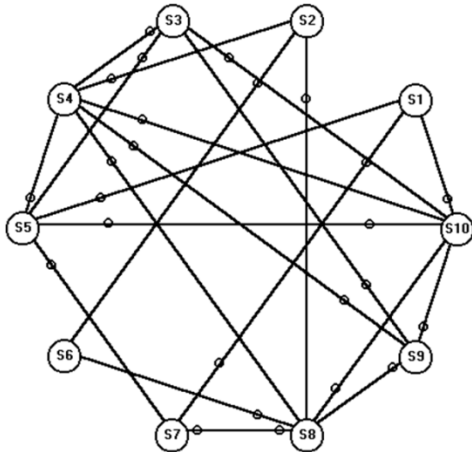


Figure 1 shows the $K^{\mathfrak{K}}$ matrix with the calculated $\pi_o, \lambda_o, \pi_e, \lambda_e$.

We introduce the definitions of symmetry and transitivity of binary relations.

III. SYMMETRY AND TRANSITIVITY

A symmetric relation is the relation between S_i and S_j if $S_i \mathfrak{R} S_j$ and $S_j \mathfrak{R} S_i$.

A transitive relation is the relation \mathfrak{R} between S_i, S_j, S_k , if it satisfies: $S_i \mathfrak{R} S_j, S_j \mathfrak{R} S_k, S_i \mathfrak{R} S_k$.

Thus, the relation \mathfrak{R} is neither symmetrical nor transitive, but for each graph it seems possible to distinguish its symmetric and transitive parts (sub graphs), moreover $\forall S_i \in G$ $\mathfrak{R} : [(S_i \in G^{\mathfrak{R}_{ss}}) \wedge (S_i \in G^{\mathfrak{R}_{tr}}) \vee ((S_i \notin G^{\mathfrak{R}_{ss}}) \wedge (S_i \in G^{\mathfrak{R}_{tr}})) \vee ((S_i \in G^{\mathfrak{R}_{ss}}) \wedge (S_i \notin G^{\mathfrak{R}_{tr}})) \vee ((S_i \notin G^{\mathfrak{R}_{ss}}) \wedge (S_i \notin G^{\mathfrak{R}_{tr}}))]$ and $\forall e_{ij} \in E^{\mathfrak{R}} : [((e_{ij} \in E^{\mathfrak{R}_{ss}}) \wedge (e_{ij} \in E^{\mathfrak{R}_{tr}})) \vee ((e_{ij} \notin E^{\mathfrak{R}_{ss}}) \wedge (e_{ij} \in E^{\mathfrak{R}_{tr}})) \vee ((e_{ij} \in E^{\mathfrak{R}_{ss}}) \wedge (e_{ij} \notin E^{\mathfrak{R}_{tr}})) \vee ((e_{ij} \notin E^{\mathfrak{R}_{ss}}) \wedge (e_{ij} \notin E^{\mathfrak{R}_{tr}}))]$.

The matrices K^{Kir} and K^{Kss} and the corresponding sub graphs are shown in Fig. 2 and Fig. 3.

Next, we analyze the structural - topological criteria of interest for the binary relations $>I_k, >I_c, >I_h$.

IV. DEGREE OF CENTRALIZATION

Determined by centrality index

$$\gamma = \frac{\sum_{i=1}^n (\pi_{max} - \pi_i(S_i))}{(n-1) \cdot (\pi_{max} - 1)}, \quad (9)$$

where $\pi(S_i) = \pi_o(S_i) + \pi_e(S_i)$; $\pi_{max} = \max_i \pi(S_i)$.

Index γ is used to assess the amount and degree of centralization of the structural definition of a binary relation. When $\gamma = 0$, the bonds are uniform, but if $\gamma = 1$, then the structure describes the relationship \mathfrak{R} which tends to the maximum degree of centralization.

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10		
S1							-48			1	3,8	
S2						-49	-80			2	10	
S3				-42	-11				-99	3	12	
S4		-42					-15	-62	-78	4	16	
S5	-30		-45			-25			-85	4	15	
S6										0	0	
S7	-53							-72		2	9,9	
S8						-42	-32		-75	3	12	
S9			-77	-52				-11		-89	4	18
S10	-34				-6						2	3,2
	3	1	1	3	2	2	3	4	1	5	0	99,9
	9,3	3,3	6,1	11	1,4	7,2	8,4	14	4,9	34	99,6	12,5

Fig. 1. General graph of conflict and its adjacency matrix: - graphic engine image

V. STRUCTURAL REDUNDANCY

Describes some difference in the number of bonds $|E^{\mathcal{R}}|$, that are in the definition of a binary relation, and the number of bonds $|E^{\mathcal{R}}|_{min}$, at a minimum, are necessary for the graph to become connected. We introduce

$$\varphi = (|E^{\mathcal{R}}| - |E^{\mathcal{R}}|_{min}) / |E^{\mathcal{R}}|_{min} \quad (10)$$

The value φ shows a measure of the redundancy of the structure of relations. If $\varphi > 0$, then the structure is maximally redundant (such as a full graph), if $\varphi = 0$, then it is minimal; $\varphi < 0$ - the structure of the relationship is not connected.

VI. STRUCTURAL COMPACTNESS

The property is evaluated by a number of the following indicators:

- diameter of the structure - $d(G(\mathcal{R})) = \max_{i,j} d(S_i, S_j)$,

where $d(S_i, S_j)$ is the distance between $S_i, S_j \in \mathcal{S}^{\mathcal{R}}$;

- center and radius of the structure - the vertex S_{00} will be central if $\forall S_i \in \mathcal{S}^{\mathcal{R}} : (\max_{S_j} d(S_i, S_j) \geq \max_{S_j} d(S_{00}, S_j))$ and $d(S_{00}, S_j)$ is the radius;

- a relative indicator, reflects the structural relationship of the subsystems with each other –

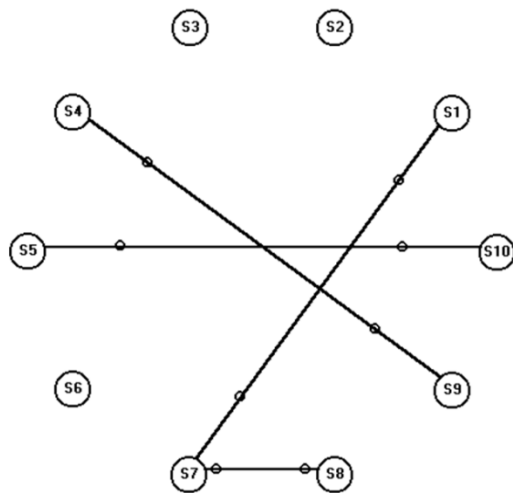
$$\varepsilon_{om} = (\varepsilon / \varepsilon_{min}) - 1, \varepsilon = \sum_{i=1}^n \sum_{j=1}^n d(S_i, S_j), \varepsilon_{min} = n(n-1).$$

$$\varepsilon_{om} = \frac{\sum_{i=1}^n \sum_{j=1}^n d(S_i, S_j)}{n \cdot (n-1)} \quad (11)$$

All described characteristics are shown in table 1.

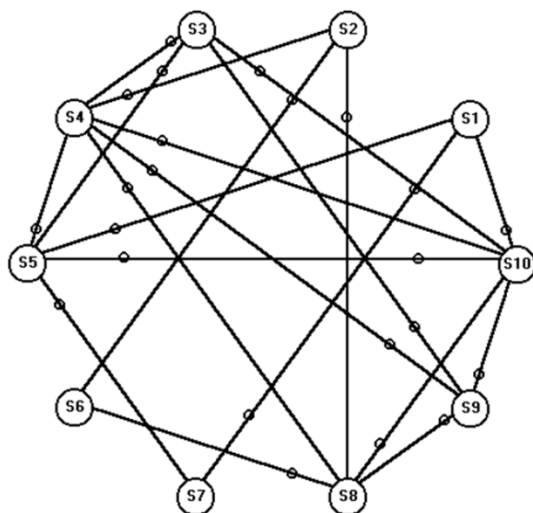
TABLE I. CHARACTERISTICS

	Centrality Index	Structural compactness Indicator	Structural redundancy Indicator
General graph of conflict	0,5	2	1,5
Symmetric graph conflict	0,74	0,11	-0,71
Transitive graph conflict	0,44	1,6	0,83





	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10		
S1							-48			1	12	
S2										0	0	
S3										0	0	
S4									-62	1	15	
S5									-85	1	21	
S6										0	0	
S7	-53							-72		2	31	
S8							-32			1	7,8	
S9				-52						1	13	
S10					-6					1	1,5	
	1	0	0	1	1	0	2	1	1	1	0	101
	13	0	0	13	1,5	0	20	18	15	21	102	4,1

Fig. 2. Graph of a symmetric conflict and its adjacency matrix: - graphic engine image



	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10		
S1							-48			1	4,2	
S2						-49	-80			2	11	
S3				-42	-11					-99	3	13
S4		-42					-15	-62	-78	4	17	
S5	-30			-45			-25			-85	4	16
S6										0	0	
S7	-53									1	4,6	
S8						-42				-75	2	10
S9			-77	-52				-11		-89	4	20
S10	-34				-6						2	3,5
	3	1	1	3	2	2	2	3	1	5	0	99,3
	10	3,7	6,7	12	1,5	8	6,4	9,3	5,4	37	100	11,5

Fig. 3. Transitive conflict graph and its adjacency matrix:  — graphic engine image 

VII. CONCLUSION

In the structural-parametric approach, conflict is considered as a specific way of interacting systems. As a result, a super-system is formed that has different properties than each of the conflicting systems separately. Consequently, the problem of constructing a conflict model is reduced to the development of ways to formally describe the relations that bind the participants in the conflict.

Using this axiomatics removes a number of limitations that are characteristic of the structural-parametric approach. It opens a method to an analytical description of the conflict with full consideration of its phenomenology, i.e. to the creation of a mathematical theory of the conflict.

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