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The Rational Material Distribution Problem for Building Floor Wall Structures

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***Abstract*—Let us consider the problem of rational layout of load-bearing structures when designing a multi-storey building. The problem is reduced to the distribution of materials for wall sections with their specified geometry, which is described by a multi-connected orthogonal polygon. The width of the walls on the entire floor is presumed to be fixed. A mathematical model of the problem has been developed that allows taking into account restrictions on certain structural elements of walls that arise in practice. A heuristic algorithm for minimizing the total cost of construction subject to the requirements for the core stiffness is proposed.**

***Keywords*—multi-connected orthogonal polygon, optimization problem, heuristic algorithm**

I. Discrete optimization problems related to multi-connected orthogonal polynomials

In practice, multi-connected orthogonal polygons (MOP) are often found in various discrete optimization problems. Namely, it refers to the construction industry, shipbuilding, some scheduling problems, corrugation problems, etc. One of the most important applied problems is the problem of splitting a MOP into rectangular parts.

The decomposition (or a splitting) can be divided into two types, depending on the resulting rectangles. If the resulting rectangles cannot overlap with each other, then the decomposition is a partition. If the resulting rectangles overlap with each other, then the decomposition is a cover. Both partitioning and covering approaches have been discussed in previous studies such as [1 – 6] for partitioning problems and [7 – 10] for covering problems. A number of works are devoted to rectilinear polygons with holes to be covered or decomposed [11 – 15]. Such polygons are named multi-coherent or multi-connected.

Various target functions can be associated with this problems: sometimes it is advisable to minimize the number of rectangles, in other cases it is necessary to reduce the total length of the joints. The problem of splitting a MOP is well-researched with a large number of the topic-related works presented in the public domain [[12 – 16].

One of the important questions that should be solved at the stage of setting the problem is how to describe the input data, and in particular, the MOP. As a rule, the points of the envelope rectangle are processed, which by its width and length, the lower-left corner is considered the origin of coordinates; areas that do not belong to a MOP, but are included in the envelope rectangle, are called obstacles, and each of them has the shape of a rectangle. Most often, a MOP is described by a set of rectangles that do not intersect with each other. In some tasks, it is convenient for these rectangles to form a MOP (that is, they represent a certain partition of a MOP), in others, for these rectangles to be obstacles in the area of the envelope rectangle.

The way to describe rectangles can also be different:

- coordinates of the lower-left corner in the coordinate system of a MOP, the length and width of the rectangle;

- segment of the centerline of the rectangle, directed along the larger side of the rectangle and specified by the coordinates of its beginning and end, and the width of the rectangle.

The variety of approaches to describing input data allows us to develop more flexible attitudes to solving the problem of decomposition of a MOP.

Note that the classical problem of partitioning a MOP involves selecting the boundaries of rectangles from a set. So, the original area of the envelope rectangle is covered with a set of horizontal and vertical lines obtained by continuing the edges of rectangles that are obstacles. The borders of all rectangles in the split will lie on one of these lines.

When solving the problem of distribution of materials of load-bearing structures, the division of a MOP into rectangles occurs differently. The applied value of the problem dictates some significant limitations.

The material of some walls is determined solely by certain technological requirements. For example, an elevator shaft shall be produced of reinforced concrete structures, and the walls through which the ventilation channels pass are made of brick. For the rest of the walls, it is necessary to offer a rational distribution of reinforced concrete and brick structures based on both the requirements of a building load resistance and reducing the cost of construction.

Additional requirements are imposed on the location and material of the walls with account for technological and design features. For example, since brick walls are joined to reinforced concrete with the help of expensive anchorages, it is undesirable and unprofitable to have a perpendicular connection of the reinforced concrete wall to the brick wall, in which the first " wedges” into the second, dividing it into two parts (Fig. 1 a)). In this case, you should "pull out" the reinforced concrete structures and dock them with the brick wall in one place instead of two (Fig. 1 b)).

Also undesirable is the "staggered" arrangement of types of wall structures, i.e. frequent alternation of reinforced concrete and brick types on a relatively small length of the wall (Fig. 2).

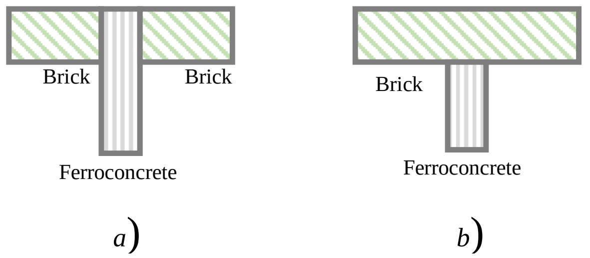

Fig. 1. a) Wedging a reinforced concrete wall into a brick wall; b) Desirable perpendicular joining of a brick and reinforced concrete wall

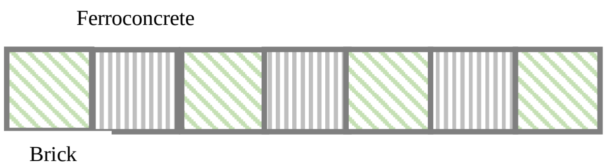

Fig. 2. A checkerboard arrangement of different designs

II. DESCRIPTION OF THE PROBLEM

The problem of forming a rational plan for the placement of load-bearing structures can be reduced to a class of discrete optimization problems. In this case, given the geometry of the location of the walls on the floor of the building, it is legitimate to consider the problem in the context of the distribution of materials of the wall sections. As you know, it is reinforced concrete structures that are load-bearing and are part of the core of the building's rigidity.

Since walls perform the main load-bearing function in a building, we will call a critical situation when the load that falls on a section of wall exceeds the maximum allowable value for this section, taking into account its location relative to other load-bearing elements and the material of its manufacture.

The main goal of solving the problem of rational distribution of wall materials on the building floor is to reduce construction costs in the absence of critical situations.

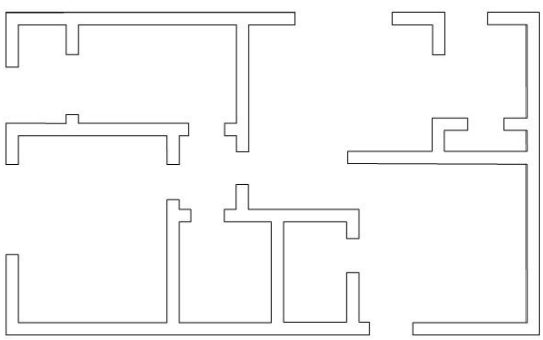

Fig. 3. Geometry of walls on the floor

The layout plan of the walls (Fig. 3) is specified and is similar for each floor. Let us consider a case, when we can use either brick or reinforced concrete to construct walls of a building.

The center line of a rectangle is the section that connects the middle of the opposite sides of the rectangle and is located along its longer side.

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In many CAD systems, for example, in Autodesk Revit, a wall layout plan is defined by the center lines of rectangular sections of walls. Often the brickwork thickness of some walls can be reduced without the occurrence of critical situations, which reduces the cost of construction and increases the usable area of the room. However, this step can be performed after determining the wall materials. Thus, at the stage of solving the problem of rational distribution of materials for wall structures, we assume that the thickness of all walls is the same.

It is easy to see that the arrangement of wall sections on a building floor is a special case of a multi-connected orthogonal polygon. Multi-connectivity is achieved by isolating connectivity components from each other with zero holes inside a polygon. The entire multi-connected orthogonal polygon can be divided into fixed-width rectangles.

The development of a building design is usually carried out using specialized software tools that enable to accurately determine the physical aspects of a future construction. The application of such tools is conditioned by high demands to building strength. As an example, MicroFe, Ansys, PC Lira-software, Autodesk Revit, Autodesk AutoCAD, and others can be named. One of the most important physical characteristics of strength used in the design of the location of building walls and their material are the values of the loads per each point of the floor. To calculate load, the entire floor area is divided into fragments using the finite element method. In this case, the load value can be considered the same inside each such fragment. The calculation of load on the floor area is automated in many specialized environments.

III. ABSTRACT MODEL

Suppose we are given a MOP P that characterizes the geometry of the walls on the floor. We will consider the geometric problem on the area of the envelope rectangle of MOP P. Then all the points considered in the mathematical model must be inside this rectangle.

In the process of searching for a rational distribution of wall materials, it is necessary to take into account possible emergencies. Given the complexity and labour intensity of the process of accurately calculating the loads experienced by each point on the floor surface with the selected distribution of wall materials, it is advisable to include a tool for recalculation of the loads when creating a brick or reinforced concrete structure on the floor in the calculation module. This will significantly reduce computational time while solving the problems. We assume that during the construction of a reinforced concrete wall, the load on the nearby rectangular sections of the walls located in a certain given neighborhood of the considered area is decreased by a fixed, predetermined percentage. Of course, the final verification of the obtained solution for the absence of critical situations should be performed taking into account all the subtleties of the load calculation process and carried out exclusively using specialized tools.

The paper considers a heuristic algorithm for solving the problem of rational distribution of materials of wall structures on the building floor. In our work we save the Pareto-optimal set of solutions, from which the expert will be able to select the most appropriate one.

IV. DESCRIPTION OF THE MATHEMATICAL MODEL

A. Input data:

1. A multi-connected orthogonal polygon P that corresponds to the location of the walls on the floor of the building, as well as an enclosing rectangle of dimension $H\times W$, where H is the height of the rectangle, W is the length of the rectangle.

2. A set of center lines of rectangular sections of walls that form MP P, and we assume that the width of the walls is always the same and equal to $w^{'}$ (Fig.4). The segments of the center lines correspond to the geometry of the walls on the floor of the building. Denote by the set of segments of axial lines of the walls, for which you specify a structural type,

$$E^0 = \{e_i\} = \langle \bar{x}_i, \bar{y}_i, \underline{x_i}, \underline{y_i} \rangle,$$

where: $\bar{x}_i, \bar{y}_i$ – coordinates of the end of the segment,

$\underline{x_i}, \underline{y_i}$ – coordinates of the beginning of the segment, and $\bar{x}_i > \underline{x_i}$, $\bar{y}_i > \underline{y_i}$; $i \in \overline{1,n}$, n is the number of segments,

$$\bar{x}_i, \underline{x_i} \in [0, W]; \ \bar{y}_i, \underline{y_i} \in [0, H], i = \overline{1,n}.$$

The set of segments of the center lines of the walls, for which their construction type is predetermined, is denoted by

$$E^{'} = \{e_i\} = \{\bar{x}_i, \bar{y}_i, \underline{x_i}, \underline{y_i}; i \in \overline{n+1, n+m}\}.$$

Here m is the number of walls with a predefined material type.

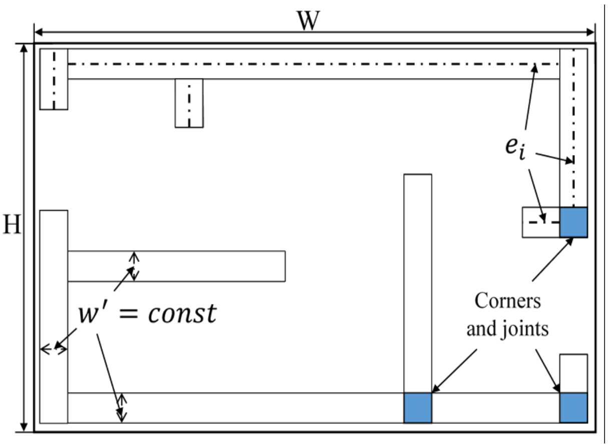

Fig. 4. The multi-connected orthogonal polygon and some characteristics

3. The set E of all walls defined by the center lines and forming the MOP P is denoted by $E = E^0 \cup E^{'}$.

4. Matrix $G_{W\times H}$ containing the load value $G[x,y]$ applied to the point with coordinates (x, y). for each point of the MOP envelope rectangle P. The original content of the matrix G is calculated using a specialized environment.

5. Critical load value $\overline{G}$, which sets the upper limit for acceptable load values.

6. Load recalculation parameters for wall sections when creating a reinforced concrete wall section:

$p < 1$ – percentage of load reduction on nearby rectangular sections when generating a reinforced concrete wall section, ε-load conversion radius.

7. The number of solutions that should be saved in the Pareto-optimal set.

8. C_k and C_{gb} the cost of a structural unit with a brick and a reinforced concrete wall with a width of w', respectively. It is known that $C_{gb} \gg C_k$.

9. Step h of dividing the original rectangular walls into sections and the minimum allowed length h^0 of the wall section of the same structural type.

B. Find a lot of corners and joints

1. Let's form two sets of additional points $T^{in} = \{(x, y)\}$ and $T^{out} = \{(x, y)\}$ as follows.

For each centerline $e_i, i = \overline{1, n+m}$ no more than four points are considered.

- Two "internal" points:
- if $\bar{x}_i = \underline{x_i}$, then the points $(\bar{x}_i, \bar{y}_i - \frac{w^{'}}{2})$ and $(\underline{x_i}, \underline{y_i} + \frac{w^{'}}{2})$ are added to the set T^{in};
- if $\bar{y}_i = \underline{y_i}$, then the points $(\bar{x}_i - \frac{w^{'}}{2}, \bar{y}_i)$ and $(\underline{x_i} + \frac{w^{'}}{2}, \underline{y_i})$ are added to the set T^{in}.
- Two "external" points:
- if $\bar{x}_i = \underline{x_i}$ and $\bar{y}_i + \frac{w^{'}}{2} \le H$ then a point $(\bar{x}_i, \bar{y}_i + \frac{w^{'}}{2})$ is added to the set T^{out};
- if $\bar{x}_i = \underline{x_i}$ and $0 \le \underline{y_i} - \frac{w^{'}}{2}$ then a point $(\underline{x_i}, \underline{y_i} - \frac{w^{'}}{2})$ is added to the set T^{out};
- if $\bar{y}_i = \underline{y_i}$ and $0 \le \underline{x_i} - \frac{w^{'}}{2}$ then a point $(\underline{x_i} - \frac{w^{'}}{2}, \underline{y_i})$ is added the set T^{out};
- if $\bar{y}_i = \underline{y_i}$ and $\bar{x}_i + \frac{w^{'}}{2} \le W$ then a point $(\bar{x}_i + \frac{w^{'}}{2}, \bar{y}_i)$ is added to the set T^{out}.

2. Then the set $T = T^{in} \cap T^{out}$ is the set of points that are the centers of the corner joints between the walls. Note that

some joints are not currently included in the set T, since there may be situations of T-shaped wall joining.

Add the centers of T-shaped wall joints to the set of T:

$$\mathrm{T} = \mathrm{T} \cup \{(x_j, y_j): (x_j, y_j) \in \mathrm{T}^{\mathrm{out}} \text{ and } (x_j, y_j) \in e_i, \\ i \in [1; n]\}, \quad j = \overline{1, q}, \quad |\mathrm{T}^{\mathrm{out}}| = q.$$

In this way, the set T will include all the butt sections of the walls whose material can be changed, and will not include butt sections with a predefined material type.

Forming a set of rectangular sections of walls

Let us form a set of rectangles with non-predefined types

$$R = R^{st} \cup R' = \{r_z\} \\ = \{\langle x_z, y_z, k_z, l_z, f_z, num_z, left_z, right_z, top_z, bottom_z, g_z \rangle\}$$

where R^{st} – set of rectangular sections of walls corresponding to joints, R' –a set of rectangles obtained on straight walls that do not intersect with other walls,

x_z, y_z – coordinates of the rectangle center,

num_z – room of the centerline of the rectangle, part of which is this; $num_z = i: (x_z, y_z) \in e_i, i \in [1; n]$;

k_z – type of construction material for the section of wall described by this rectangle:

$$k_z = \begin{cases} 1, & if \text{ the wall is made of brickwork} \\ 0, & if \text{ the wall is made of reinforced concrete} \end{cases}$$

initially, for each rectangle, $k_z = 0$;

l_z – the length of this rectangular section of the wall; we assume that this is the length of the side of the rectangle parallel to the center line e_{num_z};

f_z – direction of the centerline $\mathrm{e}_{\mathrm{num}_z}$:

$$f_z = \begin{cases} 0, \text{if } \overline{y}_{num_z} = \underline{y}_{num_z} \\ 1, \ f \ \overline{x}_{num_z} = \underline{x}_{num_z} \end{cases}.$$

$left_z, right_z, top_z, bottom_z$ – numbers of

rectangles adjacent to the given one:

$$left_z = a: x_z - \left(f_z \cdot \frac{w'}{2} + (1 - f_z) \cdot \frac{l_z}{2}\right) = \\ = x_a + \left(f_a \cdot \frac{w'}{2} + (1 - f_a) \cdot \frac{l_a}{2}\right);$$

$$right_z = a: x_z + \left(f_z \cdot \frac{w'}{2} + (1 - f_z) \cdot \frac{l_z}{2}\right) = \\ = x_a - \left(f_a \cdot \frac{w'}{2} + (1 - f_a) \cdot \frac{l_a}{2}\right);$$

$$top_z = a: y_z - \left((1 - f_z) \cdot \frac{w'}{2} + f_z \cdot \frac{l_z}{2}\right) = \\ = y_a + \left((1 - f_a) \cdot \frac{w'}{2} + f_a \cdot \frac{l_a}{2}\right);$$

$$bottom_z = a: y_z + \left((1 - f_z) \cdot \frac{w'}{2} + f_z \cdot \frac{l_z}{2}\right) = \\ = y_a - \left((1 - f_a) \cdot \frac{w'}{2} + f_a \cdot \frac{l_a}{2}\right).$$

In the absence of adjacent rectangles on either side, corresponding values of $left_z$, $right_z$, $top_z, bottom_z$ are equal to 1.

g_z – the value of the load that falls on the center of a rectangular section of the wall, taking into account the location of nearby structures, initially $g_z = G[x_z, y_z]$.

The set of wall sections corresponding to the joints is defined as follows:

$$R^{st} = \{x_z, y_z, k_z, l_z, f_z, num_z, left_z, right_z, top_z, bottom_z, \\ g_z: (x_z, y_z) \in T, \ l_z = w', z = \overline{1, q}\}, |R^{st}| = q.$$

The set of wall sections corresponding to straight sections of walls that do not have joints with other walls is defined as follows:

$$R' = \\ \{x_z, y_z, k_z, l_z, f_z, num_z, left_z, right_z, top_z, bottom_z, g_z: \\ (x_z, y_z) \notin T, \ l_z \le h, z = \overline{q + 1, t + q}\}, \ |R'| = t$$

$$R^{st} \cap R' = \emptyset, \ \bigcup_{z=1}^{q+t} r_z = P.$$

We need to define a rational set of values $k_z, z = \overline{1, q + t}$ such that the following restrictions are met:

$$g_z \le \bar{G}, \forall z \in [1; q + t].$$

The target function determines the cost of project implementation:

$$F = \sum_{z=1}^{q+t} \left(C_k \cdot (1 - k_z) + C_{gb} \cdot k_z\right) \to min$$

V. Algorithm for Finding a Plan for the Rational Distribution of Material of Wall Sections

While

$$\exists z^0 \in [1; q + t]: G[x_{z^0}, y_{z^0}] > \overline{G}:$$

Change the construction material type and load for the element $r_{z^0}: k_{z^0} = 1, g_{z^0} = g_{z^0} \cdot p$.

Adjust the load matrix: for all elements

$$r_z \in R: (x_{z^0} - x_z)^2 + (y_{z^0} - y_z)^2 \le \varepsilon^2$$

put

$$g_z = g_z \cdot \left(p + \frac{\sqrt{(x_{z^0} - x_z)^2 + (y_{z^0} - y_z)^2}}{\varepsilon} \cdot (1 - p)\right);$$

Combining similar sections of walls:

2.1. While

$$\exists z_1, z_2 \in [1; q+t]: \left(z_1 = left_{z_2} \lor z_1 = bottom_{z_2}\right) \land \left(k_{z_1} = k_{z_2}\right)$$

2.1.1 Combine the parts: given

$$x = \frac{1}{2}(x_{z_1} + x_{z_2}), y = \frac{1}{2}(y_{z_1} + y_{z_2}).$$

If $z_1 = left_{z_2}$:

$$left = left_{z_1}, right = right_{z_2}, top = -1,$$

$$bottom = -1, f = 0;$$

If $z_1 = bottom_{z_2}$:

$$left = -1, right = -1, top = right_{z_2},$$

$$bottom = left_{z_1}, f = 1;$$

If $f = 0$ and $\bar{y}_{num_{z_1}} = \underline{y}_{num_{z_1}}$

then $num = num_{z_1}$ elze $num = num_{z_2}$.

$$g = \frac{1}{2}(g_{z_1} + g_{z_2}).$$

A new section of the wall is created:

$r = \langle x, y, k_{z_1}, l_{z_1} + l_{z_2}, f, num, left, right, bottom, top, g \rangle$,

and let's include it in the set R, excluding two elements with numbers z_1 and z_2 from it:

$$R = R \cup r \setminus \left(r_{z_1} \cup r_{z_2}\right), q = q - 1.$$

Elimination of undesirable structural patterns: determination of acceptable lengths of homogeneous sections of walls.

While $\exists z^0 \in [1; q+t]: l_{z^0} < h^0$:

3.1 If the section of the wall is made of brick, that is,

$k_{z^0} = 0$, the number z^1 of the adjacent section of the wall of the minimum length is determined, change the value

$k_{z^0} = 1$, and then combine these two sections similarly to paragraph 2.1.1, additionally recalculate the load value:

3.1.1 Change the construction material type and load for the element r_{z^0}:

$k_{z^0} = 1, g_{z^0} = g_{z^0} \cdot p.$

3.1.2 Adjust the load matrix: for all elements

$r_z \in R: (x_{z^0} - x_z)^2 + (y_{z^0} - y_z)^2 \le \varepsilon^2$ put

$$g_z = g_z \cdot \left(p + \frac{\sqrt{\left(x_{z^0} - x_z\right)^2 + \left(y_{z^0} - y_z\right)^2}}{\varepsilon} \cdot (1 - p) \right);$$

If the section of the wall is made of reinforced concrete, i.e., $k_{z^0} = 1$, the number z^1 of the adjacent section of the wall of the minimum length is determined, change the value of $k_{z^1} = 1$, and then combine these two sections and recalculate the loads values similarly to paragraph 3.1.

The presented algorithm makes it possible to obtain the desired distribution of materials for wall sections on each floor when all the requirements for the core of the building stiffness are met. The variety of obtained solutions is stipulated by the invariance of the method for determining the number z^0 in paragraphs 1 and 3 of the algorithm.

The resulting solution is processed in the MicroFe environment or PC Lira-soft, where the values of the loads applied to each point of the floor are analyzed. The designer can adjust the values of the algorithm parameters in order to obtain the desired solutions. Note that a decrease in the value of the parameter h^0 will lead to an increase in the computational time of the algorithm, since a much larger set of valid solutions will be considered. In this case, more diverse solutions will be obtained from the position of placing different types of wall sections.

A decrease in p, as well as an increase in ε, can lead to repeated consideration of unacceptable solutions in which the load on some sections exceeds the maximum allowable one. At the same time, for values of p close to 1, as well as for small values of ε, it is possible to obtain results that have a more significant deviation from the best possible solution.

V. CONCLUSION

The work presents both a mathematical simulation model and a solution algorithm for rational distribution of materials during the construction of walls of a single floor of a building. The introduced data structures enable to consider adjacent wall sections.

For small values of h, a great variety of plans for material distribution among wall sections can be obtained.

The correction of the load provided by the algorithm is a preliminary one and the obtained solutions shall be verified by specialized media. Alternatively, speicalized media can be applied at each step of a partial solution generation, which would improve the accuracy of the load calculation, but would drastically increase the calculation time.

Currently, a computing experiment is being carried out.

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