

# Formal Representation of the Model of the Designed Software-Analytical Complex Based on the Principle of the Necessary Variety of Structural Relationships

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**Abstract**—The paper considers an applied approach to solving computer science problems in the framework of Cartesian closed categories to provide the necessary variety of structural connections when designing a software analytical complex taking into account the decision support system. The article considers the application of category theory to solve the problem of integration (presentation) of various software engineering technologies in a single form, for their formalized application as part of the overall design cycle of software and analytical systems. Universal category-theoretic semantic models of domain objects and their relations are constructed. On the basis of the proposed methodology, the creation of a digital twin for interaction with the subject area is considered. Subject area, has a systemic representation based on the fundamental laws of formal logic. The possibility of building hierarchical chains of electronic administrative regulations interacting with each other, which also correspond to the concept of categories, that is, some combination of regulations is a category, is shown. Proposed is a measure of estimation of proximity of a model to a real object, which is necessary for formation of a digital twin of the system. The results of the research were used in the development of the process of providing state and municipal services to citizens of the Russian Federation in electronic form. They improve the efficiency and quality of organizational and functional process management.

**Keywords**—*virtual object, digital double, formal model, category theory, state electronic services*

## I. INTRODUCTION

The development of science and technology, the globalization of the economy, and integration processes in civil society necessitate the creation of increasingly sophisticated defense, educational, industrial, transportation, energy, and other systems. The application of systems engineering standards allows us to describe the concepts and management of activities to create systems of any scale and purpose. So, over the past 5 years, the volume of stored

information has increased 100 times. The volume of data of the whole world will increase 10 times by 2025.

Accordingly, the volume of data and information presented in electronic form are increasing. The number of automated systems associated with the management of production resources (ERP systems) is also increasing. Therefore, the control problem in this case is faced with the problem of processing heterogeneous data. And also there is an increase in the number of written software systems, the disadvantage of these software products is that they are not fully developed for certain enterprise processes. As a result, for a particular subject area, it is necessary to categorize, abstract, and integrate these systems into software systems. In order to fully provide the necessary information.

Thus, there are many management tasks, many software systems, there is the problem of choice of software tools and capabilities integrated use of these software tools, through the use of analytical systems. Formalization of control tasks subject area and transfer them to appropriate program-analytical complex (PAC), provide information according to management action. It technologies offer new opportunities, focusing on that there is a change (adjustment) regulatory framework - reengineering of business processes subject area (FOR). As a consequence of the growing complexity of the structure of software systems, the time delay of their design and re-engineering. All this is happening against the background of increasing interest with public services in electronic form.

There is a task of unified life cycle life cycles subject area, information systems, and software systems.

The number of re-engineering processes of state regulation during this time increased by 80%, project development SMEV - 12 times. Growth of PC is accompanied by a lack of return on funding (mostly reverse engineering), and the inexpediency of increase of it professionals. All this is

accompanied by intensity the number of running state services. Given the specificity of this information and a greater engagement with the population, it is necessary to resolve the issue of traceability and the possibility of repetition of any previously issued information.

As a result, requires integration in terms of semantic heterogeneity, incompleteness and inaccuracies of the data.

The use of set theory and category theory is considered as one of the ways associated with the formalization of these approaches.

It is proposed to use methods of category theory, standards, system engineering ISO15288.

Proposed to apply the principle of convolution of a logical semantic model of the studied subject area, containing a priori the relations of contradiction with the available syntactic information network model (network topology). The result is formalization of the existing contradictions in the form of squares of Descartes. To eliminate such potential conflict domain model should be supplemented with cybernetic squares. Thus, it is possible to obtain consistent meta-model-driven subject area. In the applied aspect, it is possible to realize a two-step procedure: 1) search for conflicting squares; 2) elimination of the revealed contradictions by completing the metamodel with additional squares to the cybernetic control system with feedback.

## II. CONCEPTUAL APPROACH TO SOLVING THE PROBLEM

It is known that knowledge of purposeful activity in any subject area being studied is presented, first of all, in the form of ontological descriptions of various completeness, adequacy, and accuracy - semantic models. For example, in production activities, these include systems of normative reference and reporting information (documentation), presented in digitized form, they can serve as the virtual part in the subject area under study. With further formalization of this knowledge, a corresponding digital double can be compiled, which could interact with the real part of the subject area.

In the applied aspect, an example of the subject area of provision of state and municipal services to citizens of the Russian Federation in electronic form is considered (No. 210-FL of 27.07.2010). Every year, the quality of provision of state electronic services (hereinafter referred to as "SES") grows due to the development of the functionality of the unified portal of state services (hereinafter referred to as "UPSS"). However, like any system, each of the types of state services (hereinafter referred to as "SS") lives and develops: the number and composition of sub-services changes, changes are made in the legislative base for the provision of a specific type of SS, etc. Such changes entail the reengineering of the process of providing SES and changes in the relevant software. Conflicting (non-staff) situations that require optimal management decisions to be taken in the process of providing the UPSS are also not excluded, and their elimination as incidents requires reengineering the previously developed SES. For the purpose of forecasting and rapid response to such incidents it became necessary to provide a detailed presentation and formal description of this subject area as a system mathematical model (digital twin). Such a model will ensure

the indentability and traceability of the SES life cycle processes in accordance with the principles of system engineering.

Let's consider process of rendering/receipt of SES from the point of view of system engineering. The given process contains set of objects and relations between them, is many-sided and multidimensional, contains a considerable quantity both of relations within system, and external interrelations. Let's consider an example of designing of process of reception by the citizen of the SES: electronic submission by the citizen of the application for granting of service, tracing of a course of its performance and reception of result of service in the software and tool environment of UPSS.

In turn, the process of providing the SES by the agency is linked to external functions. For example, during the processing of an application for a SES, an office often uses an interagency electronic collaboration system (IECS) to obtain additional information on the applicant, which has its own structure with many internal and external relationships. Thus, the interaction between the applicant and the office is achieved through the UGI, the interaction of a set of offices among themselves through a number of information transfer services. The activities of the Applicant and the Office(s) are carried out in the organizational and functional structure of the UPSS as a digital factory, where interaction with all objects of the system under consideration and participants of the project of the process of rendering/receipt of the SES is realized [1].

As well as in any information system, for its uninterrupted work there is a regular technical support of the portal where, as a result of emergency situations, this or that participant of the process can address. Technical support is provided by the developer, but there are still incidents, the main of which is the lack of timely response to the request or its absence and, as a result, the lack of processing/resolution of the problem of the participant in the delivery/reception of the SES process.

The main reason for such a large spectrum/multiple of incidents/precedents in the subject area under consideration is the large number of facilities and relationships, and the lack of regular identification and traceability of processes.

Thus, the life cycle of processes in this subject area is characterized by a set of its states at any point in time, and this system - the process of SES representation will be presented in the form of a categorical fractal in the LC.

Under a fractal we understand the following definition: a fractal is a certain structure, consisting of parts, which by some rule are similar to the structure itself. Thus, we are talking about the ways of chaotic structures to join the system [2].

To describe the relations in the system, mathematical category theory was applied as a formal language for the consistent integration of different mathematical and applied models [2].

Category is a collection of abstract objects connected by morphisms (arrows) which designate admissible transformations of some objects in others. Application of category theory allows to define briefly the basic concepts

without "sinking" in details of description of composition and structure of component and system models. In this article under the category we will understand the subject area within which the rendering of this or that kind of SES is carried out.

In terms of category theory, fractal properties will meet certain conditions. Objects are transformed into a program module, which is defined by a chain of relationships, i.e. the lower the level of the fractal, the more relationships.

### III. METHOD OF FORMIRE ARCHYTHITECTURE ARHYTHITECTURE OF THE RESEARCH FOLLOW-UP

The LC processes adopted in ISO/IEC/IEEE 15288 (enterprise, project and technical processes) largely determine the efficiency of system formation and application, predetermining the achievement of enterprise goals. And the agreement processes are represented by working relationships, which take place through agreements that are of most interest to us. [3].

There is a correspondence ratio between the two objects. Compliance is the process of agreement, which is characterized in ISO/IEC/IEEE 15288 by the relationship of the two objects of the process agreement. According to ISO/IEC/IEEE 15288, the agreement process consists of acquisition processes and delivery processes.

Let us enter the following designations:

$SR = \{SR_1, SR_2, \dots, SR_m\}$  – many of the features of the service registry;

$SPPR = \{SPPR_1, SPPR_2, \dots, SPPR_h\}$  – multiple facility management systems;

$P = \{P_1, P_2, \dots, P_j\}$  – multiple information features of the SES portal;

$Z = \{Z_1, Z_2, \dots, Z_i\}$  – profile of the applicant;

$V = \{V_1, V_2, \dots, V_n\}$  – datasets of departments;

$S = \{S_1, S_2, \dots, S_k\}$  – many information characteristics of services of interdepartmental interaction systems.

Fig. 1 shows, among other things, the model of objects' interaction in the process of receiving by the applicant a state electronic service (left pyramid) with the use of interagency interaction (right pyramid).

The whole list of state services is contained in the register of "state services" ( $SR$ ), according to which the applicant is entitled to receive the electronic service by accessing the SES portal ( $Z$ ), or the personal office on the Office's website ( $V_1$ ).

Since 2010, according to the Federal Law-210, the conversion of the SU into electronic form has been underway. A system of interdepartmental interaction has also been developed which makes it possible to shorten the list of documents required from an applicant to provide him/her with a service and to receive them from other agencies through interdepartmental interaction channels.

The model of interaction of objects in the process of interdepartmental interaction is presented in the right pyramid in Figure 1.

Considering the interaction of information characteristics of objects  $Z$  (information data of the service applicant) and  $W_1$  (data conversion object to the type required by the system) can be represented by a composition  $Z \circ W_1$  with morphisms  $p_1: Z \circ W_1 \rightarrow Z$  and  $p_2: Z \circ W_1 \rightarrow W_1$  such that for any information object  $P$  (portal SES) with morphisms  $f_6: P \rightarrow Z$  and  $f_7: P \rightarrow W_1$  there is only one morphism  $g_1: P \rightarrow Z \circ W_1$ , which is determined by the commutative diagram.

The interaction of objects is considered in the same way  $R$  (developer information) with  $V$  (government information systems)  $R \circ V$  with morphisms  $p_3: R \circ V \rightarrow R$  and  $p_4: R \circ V \rightarrow V$  such that for any object  $W_1$  with morphisms  $f_8: W_1 \rightarrow R$  and  $f_9: W_1 \rightarrow V$ , there is only one morphism  $g_2: W_1 \rightarrow R \circ V$  which is determined by the commutative diagram.

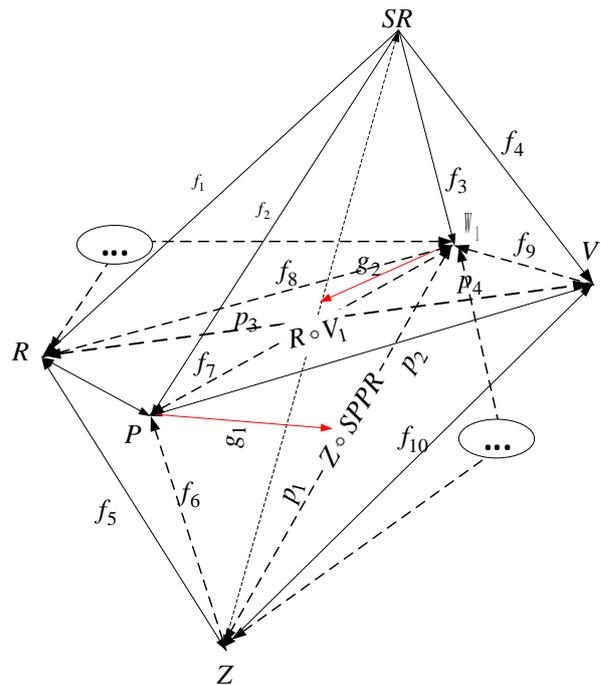


Fig. 1 Categorical fractal control SES

There is a possibility to describe all processes occurring within the limits of administrative regulations for absolutely all stages with use of hierarchy of Khomsky and the offered categorical approach.

As a result, we come to the structurization of object relations in the form of functional relations. At further selection of objects within large objects, we receive reduction in them of the unstructured part and increase in the general structured part of the subject area by certain

rules that allows to maximise traceability and identifiability in processes of reception/indication of SES, and also to provide efficiency of accepted decisions in case of occurrence of incidents. It opens the possibility of tracing the entire chain of relationships to a specific performer.

As a result, we come to the fact that as a result of such decomposition, the complexity of objects decreases and the complexity of relationships increases. In a limit, from the point of view of modeling, it turns out full coverage of information area by information objects and relations between them, thus relations between objects are formalized and structured, can be described on the basis of functional relations of objects of categories, i.e. can be categorized.

Thus, we can speak that the relations between two objects in some cases can be presented in the form of a new object, which can be either real, or virtual (Figure 1). Virtual object is an object obtained as a result of relations, which is completed to the Cartesian closed category, which also does not contradict the necessary diversity of U. Ashby. As a result of transformations we get self-similarity of scale invariance - fractals [2, 4, 5].

On the basis of carried out researches [6 – 9] and rules of synthesis of self-organizing structure, the process of agreement based on composition of objects-processes can be defined by logical addition of some fractals in the form of commutative triangles into Cartesian squares.

So, in the commutative triangle  $(P, W_1, Z)$  a virtual object is formed from the composition  $(Z \circ W_1)$ , which represents, for example, the process / possibility of the applicant contacting the support service of the GU portal in the event of a malfunction of the first, as a result of which a database (accounting) of system errors / bugs is formed. Next, in the triangle  $(R, W_1, V)$  a virtual object is formed from a composition  $(R \circ V)$ , giving the possibility to work out the database of errors and formation on their basis of the database of decisions on the revealed incidents (fig.1). Thus, we can speak about a possibility of formation of a program analytical complex for the decision of problems on work with incidents/precedents in the course of rendering/receipt of SES where, taking into account interconnection of all relations, any incident can be localized by minimization of quantitative changes.

Incidents will be understood as perturbations of external and internal environment in a considered category of rendering/receiving of this or that kind of SES. For example, nuances of the provision of a particular category of citizens (including geographical factor and others), system/technical errors of the UPSS, changes in the regulatory framework for a particular type of SES, and others, which are not accounted for/specified in administrative regulations.

In view of the above, the development of a new SES can be represented as a set of objects structured into a category. In the course of the conducted researches it is established that relations between objects of different categories correspond to the notion of a functional relation. As a result, we come to a set of functions that can be combined as a separate category. I.e. relations between fractal elements are built on the level of categorical relation.

Thus, the subobjects that make up the fractal can be described as functor relations, i.e. Elements of different categories relate to each other through a functor. As a result, the categories are interconnected by the relation of "self-similarity" for a specific subject area. For example, in the subject area under consideration, the relationship of "self-similarity" will be associated with different types of SES.

The proposed conclusions are valid for any triangle under consideration.

Using the rules of fractal construction of systems can be applied in designing the structure of software analytical complexes, for example, based on fractal graphs and fractoid transformations [10]. The combination of a graph and a fractal is a fractal graph with fractal properties: self-similarity, scale invariance [11]. A variety of fuzzy calculations are fractal calculations, which are represented by an algebraic structure - a fractoid. Using functor relations, it is possible to describe fractal processes by combining "iterated mappings" into the previously described chains. In this case, the fractal itself does not act as a geometrical place of points, but as an object of a categorical relationship, we call it "categorical fractal" [12,13].

The proposed method allows, when describing the subject area in terms of category theory, to come to the possibility of choosing another software product without reengineering the entire system. So, for example, disturbance effects in the form of incidents are formed in the processes of implementation and provision / receipt of SES, which in turn are one of the reasons for changing many of the rules of interaction between the stages of the LC project and the launch of the iterative process of converting GIs into electronic form, which, in turn, can be achieved by applying functor relations. Moreover, the reengineering task is reduced to a minimax problem already at the stage of working with models [14].

The state of the object as a whole in the studied category can be determined in terms of its "mixed" real and virtual states (Figure 2) [9,15].

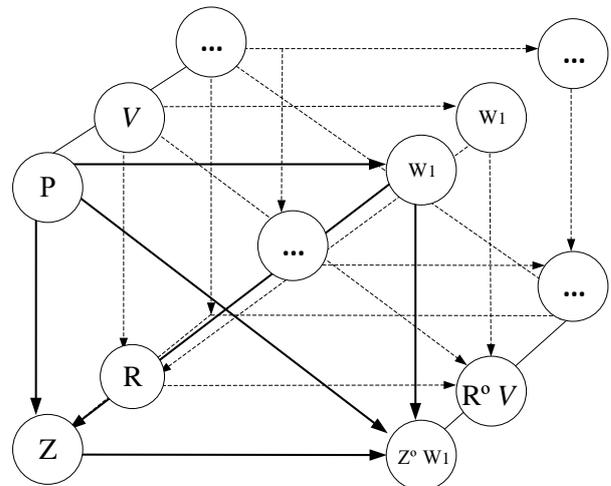


Fig. 2 "The mixed state of the object (analog qubit)

When considering a commutative triangle  $P, W_1, Z$ , we get the real state of the object and the augmented state of the object  $Z$ , virtual state of an object  $W_1$  and augmented state of the real object  $W_1 \circ Z$ , whose objects are categories, can be reduced to a multidimensional matrix (Fig. 3). It is possible not to completely change the structure of the software complex, but only a separate part of it, adding a new module, transforming the matrix of relations. The possibility of further building the system, i.e. openness of the system.

The semantics of the information itself can be defined as the uncertainty of knowledge or relationships. Properties that are not taken into account in the domain model are represented as a separate domain and are characterized by unaccounted for properties, which makes it possible to consider it as an environment. As a result, each subject area integrates with the environment and has the ability to be considered distributed / expandable. Thus, it opens the possibility of building interaction between different GEMs by using the tools of an interdepartmental electronic interaction system (hereinafter referred to as SMEV), which allows the interaction of two or more structures.

Figure 3 shows a functional diagram of a digital double. We introduce the following definitions:

The control object is the process of rendering a GEM (Fig. 2), the attribute values of which will be the input parameters of a digital double;

$W_1$  - object for converting data to the form required by the system (virtual object);

$W_2$  - object - a database and knowledge management system;

$W_3$  - the object - a digital model - builds on the available data a model of the process of functioning of the modeling object. Provides the ability to design and predict the behavior of the entire model;

$W_4$  - the object - the process control system - "Control Panel", implements new relationships between objects based on data from the Optimization Block and the Digital Model.

Carrying out similar considerations, we come to the construction of a possible scheme for implementing a digital double based on interacting real and virtual objects (Fig. 3), which would correspond to the initial requirements.

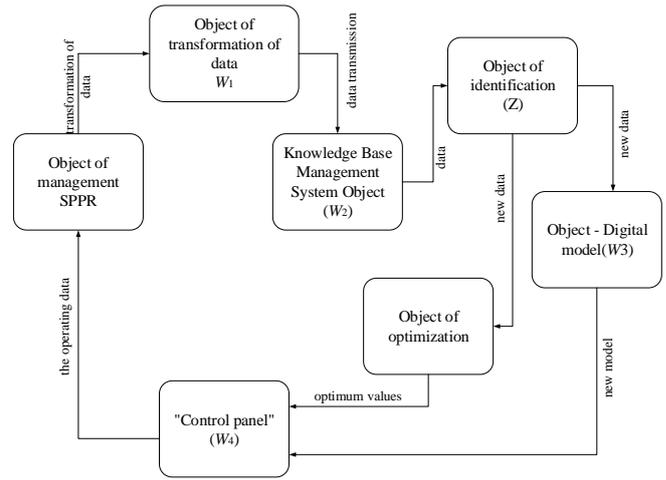


Fig. 3 Functional diagram of a digital double

Representation of relations in the form of information objects allows us to consider them as a "digital double", which extends the "electronic" transparency in many aspects, including managerial and technological ones.

#### IV. CONCLUSION

1. A conceptual approach to the construction of the architecture of the PAA metastructure based on the set of real and virtual objects and their consistent and contradictory relationships determined by the semantic model (ontology) of the software under study according to the logic of the Cartesian square is proposed.

2. Based on the mathematical theory of categories, a detailed conceptual model of systems engineering is defined, defined by the international standard ISO 15288 for software for the representation of power plants, which allows the use of methods and technologies for design and process control of power supply units.

3. The possibility of building hierarchical chains of electronic administrative regulations interacting with each other that also correspond to the concept of categories is shown, that is, a certain union of regulations is a category.

4. A measure is proposed for assessing the proximity of the model to a real object, necessary for the formation of a digital double of the system.

5. Given the interconnectedness of all relations, any incident can be localized by minimizing quantitative changes.

6. Due to the extensibility of the model, it is possible to complete the interaction between different SEIs, as well as to implement the interaction of several structures among themselves. This makes it possible to create a single digital platform that overcomes the fragmentation of departmental systems and is based on a single data set.

7. It allows for identification in the processes of receiving/reporting to the GEO, as well as tracking the entire chain of relationships to a specific implementer.

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