

## Research Article

# An Extended TODIM Method with Unknown Weight Information Under Interval-Valued Neutrosophic Environment for FMEA

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## ABSTRACT

Failure mode and effect analysis (FMEA) is a powerful risk assessment tool to eliminate the risk and improve the reliability. In this article, a novel risk prioritization model based on extended TODIM (an acronym in Portuguese of interactive and multiple attribute decision-making) method under interval-valued neutrosophic environment is proposed. Firstly, the interval-valued neutrosophic sets (IVNSs) are adopted to deal with uncertainty and indeterminate information. Secondly, in order to obtain objective weights of risk factors, integration of the new similarity degree and entropy measures are applied for risk factor weighting. Moreover, the extended TODIM method is present to reduce fuzziness in process of decision-making by using an improved score function, and it also attach importance to the psychological behavior of team members, which can get a more reasonable ranking. Finally, a numerical case of steel company is provided to illustrate the feasibility of the FMEA model, and a comparison analysis with other conventional methods are further performed to indicate its effectiveness.

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## 1. INTRODUCTION

In current society of production activities, risk is an unavoidable problem, nothing can completely eliminate the risk in the reality environment. Although it is inevitable, it can be achieved within the acceptable range by reducing the probability of its occurrence. The purpose of risk assessment is to prevent accidental failures through appropriate techniques, the most famous of which is the failure mode and effect analysis (FMEA) method [1]. FMEA was proposed by the NASA in the 1960s at the first time and applied in the aerospace industry to improve the reliability of military products [2]. Subsequently, FMEA has been widely applied in manufacture [3], agriculture [4], aerospace [5] and geothermal power plant [6]. The most important step of the conventional FMEA method is to rank the failure mode according to the value of the risk priority number (RPN) [7], where RPN is the product of the three risk factors: occurrence (O), severity (S) and detection (D). Although the conventional RPN method to evaluate and prioritize for failure modes is feasible, it still suffers from many drawbacks. Firstly, it is difficult to describe information with clear numbers in the actual environment. Secondly, the weight of each risk factor is considered equally important. Thirdly, the different combinations of O, S and D may produce exactly the same RPN value, but their hidden risk implications may be entirely different, which may lead to an incorrect ranking.

To overcome these limitations associated with the traditional RPN method, Zadeh (1965) [8] firstly proposed the theory of fuzzy sets, which was an important tool to narrate fuzzy information. Then, Atanassov (1986) [9] introduced the concept of intuitionistic fuzzy sets (IFSs) by increasing non-membership. Yager [10] proposed Pythagorean fuzzy sets (PFS) which it enlarged the range of the sum of membership degree and nonmembership degree. However, decision makers (DM) could be hesitant when describing the membership degree of an object. Torra [11] developed hesitant fuzzy sets (HFSs) to fully display DM's hesitant. The aforementioned sets can only handle incomplete information but not the indeterminate information and inconsistent information which exist commonly in real situations. The emergence of the neutrosophic sets (NSs) just make up for the deficiency. Smarandache [12] proposed NSs, which are an extension of IFSs. Wang *et al.* [13] presented the notion of single-valued neutrosophic sets (SVNSs), which can use the truth-membership function, the indeterminacy-membership function and the falsity-membership function by three real numbers in the interval [0,1] to express the ambiguity of information, and these are independent of each other. Wang *et al.* [14] further extended SVNS to interval numbers, and proposed interval neutrosophic sets (INSs). Recently, interval-valued neutrosophic set (IVNS) has obtained extensive attention and was applied to the fields of multi-attribute decision-making (MADM). Broumi and Smarandache [15] investigated the correlation coefficient for INS. Bausys *et al.* [16] presented a VIKOR (ViseKriterijumska Optimizacija I Kompromisno Resenje) method in IVNS environment to address MADM problem. Garg *et al.* [17] used a nonlinear

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programming (NP) model based on the technique for order preference by similarity to ideal solution (TOPSIS) to solve decision-making problems in which information were given in the form of IVNNs. Ye *et al.* [18] was aim to study the distance and similarity of IVNS and proposed some entropy measures of IVNS based on the distances. Zhou *et al.* [19] investigated some Frank aggregation operators of IVNNs and applied to MADM. Moreover, some applications are focused on the aggregation operators [20–22] and information measures [23,24]. IVNS is relatively less compared with the IFSs ranges of application and research. Therefore, IVNSs are the more perfect and meaningful means to represent uncertainty information than FSs, IFSs and SVNNS.

In addition, a lot of multi-criteria decision-making (MCDM) methods have been applied in FMEA. In [25], ELECTRE method was employed to assign failure modes to predefined and ordered risk classes. Liu *et al.* [26] adopted the MULTIMOORA approach to analyze the rank of FMEA under fuzzy environment. Moreover, including analytic hierarchy process (AHP) [27], EDAS [28], and so on. Unfortunately, these are approaches based on completely rationalization. It is common phenomenon that a lot of experts have different attitudes toward uncertain and unknown information in reality environment. Therefore, considering the risk attitudes are significant for determining risk priority of FMEA. The TODIM method, proposed by Gomes and Lima [29], was an effective tool based on prospect theory for capturing psychological behavior of experts. The TODIM method has been widely used in aspects of MADM and risk assessment. Gomes *et al.* [30] presented an evaluation study for residential properties using TODIM method. Qin *et al.* [31] used triangular intuitionistic fuzzy numbers (TIFNs) and extended TODIM method to select renewable energy alternatives. A novel TODIM method, considering reference dependence and loss aversion at the same time, was proposed by Jiang *et al.* [32] to address interval MADM problems. Zhu *et al.* [33] established a comprehensive FMEA model based on the Bonferroni mean and TODIM method. Wang *et al.* [34] established a projection-based TODIM method with multi-valued neutrosophic sets (MVNSs) for personnel selection. Wang *et al.* [35] integrated Choquet integral and an extended generalized TODIM for risk evaluation and prioritization of failure modes. Yuan *et al.* [36] applied a combined ANP-Entropy method to determine weights of risk factors, and used the TODIM method to rank the overall risk level. Due to group emergency decision-making (GEDM) plays an important role in dealing with urgent situations, Wang *et al.* [37] used experts' psychological behavior to GEDM process. Further, in the existing fuzzy TODIM method, converting fuzzy information to crisp value may lead to significant loss of information, Wang *et al.* [38] proposed a novel fuzzy TODIM method based on alpha level sets to solve it. Therefore, it is beneficial to utilize the superiority of TODIM method in representing the rationality of experts.

According to above literature review, we find that acquiring the weight of each risk factor is one of the key factors in FMEA, and few researches have been conducted to extend the TODIM method within the context of IVNSs for FMEA. Therefore, this paper proposes the following solutions: First, IVNNs are used to characterize the fuzzy and uncertainty evaluation information in FMEA. Secondly, a new similarity method is developed based on cross-entropy in IVNS environment. Then, we form a systematic thought to determine the final weights by combining similarity degree and entropy measures. Finally, the improved score function is applied in TODIM method, which reduce the fuzzy information in the

process of decision, making the priority of failure modes are more reasonable and effective. Besides, a case study on steel company is used to demonstrate the application and effectiveness of the proposed method.

The rest of the paper is organized as follows: Section 2 reviews some basic concepts regarding IVNS and relevant decision methods, Section 3 proposes the improved FMEA model. Section 4 introduces the FMEA model into an illustrative example of steel company. Comparative analysis is presented in Section 5. Conclusion is finally drawn in Section 6.

## 2. PRELIMINARIES

### 2.1. IVNS

**Definition 1** [13] Let X be a universe of discourse. A SVNNS M over X is an object having the form

$$M = \{ \langle x, T_M(x), I_M(x), F_M(x) \rangle \mid x \in X \} \quad (1)$$

where  $T_M(x) : X \rightarrow [0, 1]$ ,  $I_M(x) : X \rightarrow [0, 1]$  and  $F_M(x) : X \rightarrow [0, 1]$  with  $0 \leq T_M(x) + I_M(x) + F_M(x) \leq 3$  for all  $x \in X$ . The values  $T_M(x)$ ,  $I_M(x)$  and  $F_M(x)$  denote the truth-membership degree, the indeterminacy-membership degree and the falsity-membership degree of  $x$  to M, respectively. For a SVNNS M in X, the triplet  $(T_M(x), I_M(x), F_M(x))$  is called the single-valued neutrosophic numbers (SVNNs). For convenience, we can simply use  $x = (T_x, I_x, F_x)$  to represent a SVNN as an element in the SVNNS M.

**Definition 2** [14] Let X be a universe of discourse, with a class of elements in X denoted by x. An interval-valued neutrosophic numbers (IVNNs) A in X is summarized by a truth-membership function  $T_A(x)$ , an indeterminacy-membership function  $I_A(x)$  and a falsity-membership function  $F_A(x)$ . Then an IVNN A can be denoted as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \} \quad (2)$$

For each point x in X,  $T_A(x) = [T_A^L(x), T_A^U(x)]$ ,  $I_A(x) = [I_A^L(x), I_A^U(x)]$ ,  $F_A(x) = [F_A^L(x), F_A^U(x)] \subseteq [0, 1]$  and  $0 \leq T_A^U(x) + I_A^U(x) + F_A^U(x) \leq 3$ . For convenience, we can simply use  $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$  to represent an IVNN as an element in the IVNS A.

**Definition 3** [39] Let  $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$  and  $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$  be two IVNNs, and  $\lambda > 0$ ; then the operations for the IVNNs are defined as follows:

1.  $\lambda x_1 = \left( \left[ 1 - (1 - T_1^L)^\lambda, 1 - (1 - T_1^U)^\lambda \right], \left[ (I_1^L)^\lambda, (I_1^U)^\lambda \right], \left[ (F_1^L)^\lambda, (F_1^U)^\lambda \right] \right);$
2.  $x_1^\lambda = \left( \left[ (T_1^L)^\lambda, (T_1^U)^\lambda \right], \left[ 1 - (1 - I_1^L)^\lambda, 1 - (1 - I_1^U)^\lambda \right], \left[ 1 - (1 - F_1^L)^\lambda, 1 - (1 - F_1^U)^\lambda \right] \right)$
3.  $x_1 \oplus x_2 = \left( [T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L * I_2^L, I_1^U * I_2^U], [F_1^L * F_2^L, F_1^U * F_2^U] \right)$

4.  $x_1 \otimes x_2 = ([T_1^L * T_2^L, T_1^U * T_2^U], [I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U])$
5.  $x_1^c = ([F_1^L, F_1^U], [1 - I_1^U, 1 - I_1^L], [T_1^L, T_1^U])$

**Definition 4** [40] Two IVNSs A and B in  $X = \{x_1, x_2, \dots, x_n\}$  are denoted by  $A = \{\langle x_i, [T_A^L(x_i), T_A^U(x_i)], [I_A^L(x_i), I_A^U(x_i)], [F_A^L(x_i), F_A^U(x_i)] \rangle\}$  and  $B = \{\langle x_i, [T_B^L(x_i), T_B^U(x_i)], [I_B^L(x_i), I_B^U(x_i)], [F_B^L(x_i), F_B^U(x_i)] \rangle\}$ , then we define the Hamming distance for A and B.

$$d_h(A, B) = \frac{1}{6} \sum_{i=1}^n \left( \begin{aligned} &|T_A^L(x_i) - T_B^L(x_i)| + |T_A^U(x_i) - T_B^U(x_i)| \\ &+ |I_A^L(x_i) - I_B^L(x_i)| + |I_A^U(x_i) - I_B^U(x_i)| \\ &+ |F_A^L(x_i) - F_B^L(x_i)| + |F_A^U(x_i) - F_B^U(x_i)| \end{aligned} \right) \quad (3)$$

We need to take the weights of the element  $x_i (i = 1, 2, \dots, n)$  into account. Therefore, let  $w = \{w_1, w_2, \dots, w_n\}$  become the weight vector of the elements  $x_i (i = 1, 2, \dots, n)$ , then weight Hamming distance is defined as follows:

$$d_{wh}(A, B) = \frac{1}{6} \sum_{i=1}^n w_i \left( \begin{aligned} &|T_A^L(x_i) - T_B^L(x_i)| + |T_A^U(x_i) - T_B^U(x_i)| \\ &+ |I_A^L(x_i) - I_B^L(x_i)| + |I_A^U(x_i) - I_B^U(x_i)| \\ &+ |F_A^L(x_i) - F_B^L(x_i)| + |F_A^U(x_i) - F_B^U(x_i)| \end{aligned} \right) \quad (4)$$

**Definition 5** [41] Let  $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$  be an IVNN; the score function is defined as follows:

$$S = \frac{4 + T^L + T^U - (I^L + I^U) - (F^L + F^U)}{6} \quad (5)$$

The score function can intuitively compare the value of IVNNs, we will cannot differentiate  $x_1$  and  $x_2$  if  $T_1 = I_1 + F_1, T_2 = I_2 + F_2$  and  $T_1 \neq T_2, I_1 \neq I_2, F_1 \neq F_2$ . Therefore, we further propose an improved method as follows:

**Definition 6** Let  $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$  be an IVNN, the improved score function by considering the percentage of the indeterminacy-membership function:

$$S' = \frac{4 + T^L + T^U - (F^L + F^U) - K(I^L + I^U)}{6} \quad (6)$$

where  $K = \frac{F}{T+F}$ .

If a and b are IVNNs, then its comparison rule are as follows:

1. If  $S'(a) > S'(b)$ , then  $a > b$ .
2. If  $S'(a) = S'(b)$  then  $a = b$ .

**Definition 7** [39] Let  $x_j = (j = 1, 2, \dots, n)$  be a series of the IVNNs, and  $w = (w_1, w_2, \dots, w_n)^T$  be the weight vector of  $x_j = (j = 1, 2, \dots, n)$ ; then an interval-valued neutrosophic weighted averaging (IVNWA) operator is a mapping IVNWA,  $X^n \rightarrow X$ , where

$$\begin{aligned} \text{IVNWA}(x_1, x_2, \dots, x_n) &= \bigoplus_{j=1}^n (w_j x_j) \quad (7) \\ &= \left( \begin{aligned} &\left[ 1 - \prod_{j=1}^n (1 - T_j^L)^{w_j}, 1 - \prod_{j=1}^n (1 - T_j^U)^{w_j} \right], \\ &\left[ \prod_{j=1}^n (I_j^L)^{w_j}, \prod_{j=1}^n (I_j^U)^{w_j} \right], \left[ \prod_{j=1}^n (F_j^L)^{w_j}, \prod_{j=1}^n (F_j^U)^{w_j} \right] \end{aligned} \right) \end{aligned}$$

## 2.2. A Cross-Entropy Measure of IVNNS

**Definition 8** [19] Let  $Q^X = \langle [T_{ij}^{L(X)}, T_{ij}^{U(X)}], [I_{ij}^{L(X)}, I_{ij}^{U(X)}], [F_{ij}^{L(X)}, F_{ij}^{U(X)}] \rangle_{m \times n}$  ( $X=1,2$ ) be two IVNNs matrices. The cross-entropy between  $Q^1$  and  $Q^2$  is defined as

$$\begin{aligned} \text{INS}(Q^1, Q^2) &= \frac{1}{2mn} \left\{ \sum_{i=1}^m \sum_{j=1}^n \left[ \sqrt{\frac{(T_{ij}^{L(1)})^2 + (T_{ij}^{L(2)})^2}{2}} \right. \right. \\ &- \left. \left( \frac{\sqrt{T_{ij}^{L(1)}} + \sqrt{T_{ij}^{L(2)}}}{2} \right)^2 + \sqrt{\frac{(I_{ij}^{L(1)})^2 + (I_{ij}^{L(2)})^2}{2}} \right. \\ &- \left. \left( \frac{\sqrt{I_{ij}^{L(1)}} + \sqrt{I_{ij}^{L(2)}}}{2} \right)^2 + \sqrt{\frac{(1 - I_{ij}^{L(1)})^2 + (1 - I_{ij}^{L(2)})^2}{2}} \right. \\ &- \left. \left( \frac{\sqrt{1 - I_{ij}^{L(1)}} + \sqrt{1 - I_{ij}^{L(2)}}}{2} \right)^2 + \sqrt{\frac{(F_{ij}^{L(1)})^2 + (F_{ij}^{L(2)})^2}{2}} \right. \\ &- \left. \left( \frac{\sqrt{F_{ij}^{L(1)}} + \sqrt{F_{ij}^{L(2)}}}{2} \right)^2 \right] + \sum_{i=1}^m \sum_{j=1}^n \left[ \sqrt{\frac{(T_{ij}^{U(1)})^2 + (T_{ij}^{U(2)})^2}{2}} \right. \\ &- \left. \left( \frac{\sqrt{T_{ij}^{U(1)}} + \sqrt{T_{ij}^{U(2)}}}{2} \right)^2 + \sqrt{\frac{(I_{ij}^{U(1)})^2 + (I_{ij}^{U(2)})^2}{2}} \right. \\ &- \left. \left( \frac{\sqrt{I_{ij}^{U(1)}} + \sqrt{I_{ij}^{U(2)}}}{2} \right)^2 + \sqrt{\frac{(1 - I_{ij}^{U(1)})^2 + (1 - I_{ij}^{U(2)})^2}{2}} \right. \\ &- \left. \left( \frac{\sqrt{1 - I_{ij}^{U(1)}} + \sqrt{1 - I_{ij}^{U(2)}}}{2} \right)^2 + \sqrt{\frac{(F_{ij}^{U(1)})^2 + (F_{ij}^{U(2)})^2}{2}} \right. \\ &- \left. \left( \frac{\sqrt{F_{ij}^{U(1)}} + \sqrt{F_{ij}^{U(2)}}}{2} \right)^2 \right] \left. \right\} \quad (8) \end{aligned}$$

### 2.3. Classic TODIM Method

The classic TODIM method decision-making steps can be summarized as follows [42]:

Step 1: Construction decision matrix  $R = [\tilde{r}_{ij}]_{m \times n}$ , where  $\tilde{r}_{ij}$  represents the evaluation value of  $i$ th alternative according to  $j$ th criterion.

Step 2: Determine the relative weight of each attribute  $C_j$  relative to the reference attribute.

$$w_j^* = \frac{w_j}{w^*} \quad (9)$$

where  $w_j$  is the weight of attribute  $C_j$ ,  $w^* = \max\{w_j | j = 1, 2, \dots, n\}$ .

Step 3: Calculate the dominance degree of  $\eta_i$  over each alternative  $\eta_t$  based on  $C_j$ . Let  $\theta$  be the attenuation factor of the losses. Then

$$\vartheta(\eta_i, \eta_t) = \sum_{j=1}^n \phi_j(\eta_i, \eta_t), \forall (i, t) \quad (10)$$

Among them,

$$\phi_j(\eta_i, \eta_t) = \begin{cases} \sqrt{w_{jr} (z_{ij} - z_{tj}) / \sum_{j=1}^n w_{jr}} & \text{if } z_{ij} - z_{tj} > 0 \\ 0 & \text{if } z_{ij} - z_{tj} = 0 \\ -\frac{1}{\theta} \sqrt{\sum_{j=1}^n w_{jr} (z_{tj} - z_{ij}) / w_{jr}} & \text{if } z_{ij} - z_{tj} < 0 \end{cases} \quad (11)$$

where  $\phi_j(\eta_i, \eta_t)(z_{ij} - z_{tj} > 0)$  means gain and  $\phi_j(\eta_i, \eta_t)(z_{ij} - z_{tj} < 0)$  indicates loss.

Step 4: Compute the overall value of  $\vartheta(\eta_i)$  with formula (12):

$$\vartheta(\eta_i) = \frac{\sum_{t=1}^m \vartheta(\eta_i, \eta_t) - \min_i \left\{ \sum_{t=1}^m \vartheta(\eta_i, \eta_t) \right\}}{\max_i \left\{ \sum_{t=1}^m \vartheta(\eta_i, \eta_t) \right\} - \min_i \left\{ \sum_{t=1}^m \vartheta(\eta_i, \eta_t) \right\}} \quad (12)$$

Step 5: To choose the best alternative by rank the values of  $\vartheta(\eta_i)$ , the alternative with maximum value is the best choice.

### 3. THE PROPOSED FMEA MODEL

Assume that there are  $l$  cross-functional team member  $TM_k (k = 1, 2, \dots, l)$  in a risk assessment group responsible for the risk evaluation of  $m$  failure modes  $FM_i (i = 1, 2, \dots, m)$  in terms of  $n$  risk factors  $RF_j (j = 1, 2, \dots, n)$ . Because FMEA team members frequently come from different fields and professional experience, each team member should be given different weighting factor  $\tau_k, k = 1, 2, \dots, l$  (where  $\sum_{k=1}^l \tau_k = 1$ ) to reflect their relative importance. Figure 1 shows that the specific process of the proposed FMEA model.

Step 1: Identify potential failure modes.

The potential failure modes in a risk analysis system are determined according to different team members commented.

Step 2: Evaluate failure modes using linguistic terms expressed in IVNNs.

In the paper, we suppose that all the risk evaluation information about failure modes by using the linguistic terms shown in Table 1 (adopted from [28]), the risk scores can be converted into their corresponding IVNNs. the linguistic rating of the  $i$ th failure mode with respect to  $j$ th risk factor provided by the team member  $TM_k$  can be denoted as  $r_{ij}^k = ([T_{ijk}^L, T_{ijk}^U], [I_{ijk}^L, I_{ijk}^U], [F_{ijk}^L, F_{ijk}^U])$ . Then, an interval-valued neutrosophic fuzzy assessment matrix is constructed as  $R^k = [r_{ij}^k]_{m \times n} (k = 1, 2, \dots, l)$  for each team member of the FMEA.

Step 3: Aggregate the FMEA team members' assessment matrix  $R$ . After obtaining the risk assessment information and relative weights of the FMEA team members, to aggregate all individual risk assessment matrices  $R^k (k = 1, 2, \dots, l)$  into the collective risk assessment matrix  $R = [r_{ij}]_{m \times n}$  by the INWA operator as follows:

$$R_{ij} = INWA(r_{ij1}, r_{ij2}, \dots, r_{ijn}) = \sum_{k=1}^l \tau_k r_{ijk} \quad (13)$$

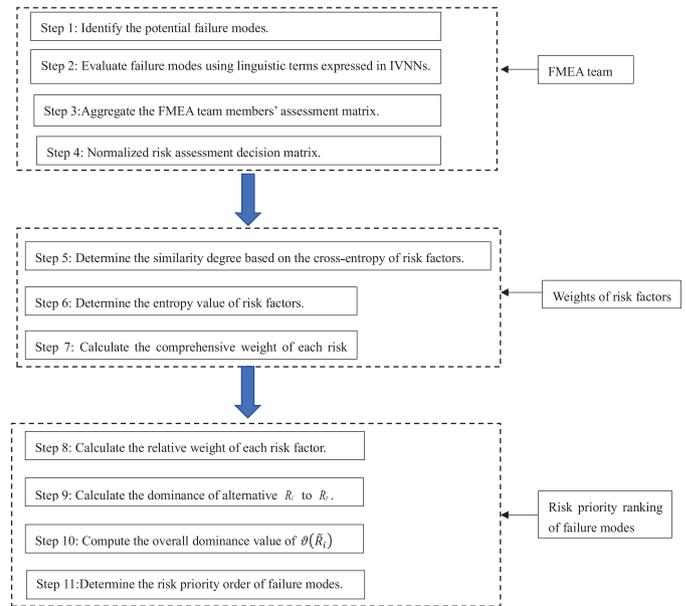


Figure 1. The flowchart of the risk prioritization method based on extended TODIM.

Table 1 | Linguistic variables for rating failure modes [28].

Linguistic variables	<T, I, F>
Extremely low (EL)	< [0.05,0.2], [0.6,0.7], [0.75,0.9]>
Very low (VL)	< [0.15,0.3], [0.5,0.6], [0.65,0.8]>
Low (L)	< [0.25,0.4], [0.4,0.5], [0.55,0.7]>
Medium low (ML)	< [0.35,0.5], [0.3,0.4], [0.45,0.6]>
Medium (M)	< [0.4,0.6], [0.1,0.2], [0.4,0.6]>
Medium high (MH)	< [0.45,0.6], [0.3,0.4], [0.35,0.5]>
High (H)	< [0.55,0.7], [0.4,0.5], [0.25,0.4]>
Very high (VH)	< [0.65,0.8], [0.5,0.6], [0.15,0.3]>
Extremely high (EH)	< [0.75,0.9], [0.6,0.7], [0.05,0.2]>

where  $r_{ij} = \langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \rangle (i = 1, 2, \dots, m; j = 1, 2, \dots, n)$  and  $T_{ij}^L = 1 - \prod_{k=1}^l (1 - T_{ij}^L)^{\tau_k}$ ,  $T_{ij}^U = 1 - \prod_{k=1}^l (1 - T_{ij}^U)^{\tau_k}$ ,  $I_{ij}^L = \prod_{k=1}^l (I_{ij}^L)^{\tau_k}$ ,  $I_{ij}^U = \prod_{k=1}^l (I_{ij}^U)^{\tau_k}$ ,  $F_{ij}^L = \prod_{k=1}^l (F_{ij}^L)^{\tau_k}$ ,  $F_{ij}^U = \prod_{k=1}^l (F_{ij}^U)^{\tau_k}$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ )

Step 4: Normalized risk assessment decision matrix. Normalize decision matrix  $R^k = (r_{ijk})_{m \times n}$  into  $R^k = (\tilde{r}_{ijk})_{m \times n} = \langle [\tilde{T}_{ijk}^L, \tilde{T}_{ijk}^U], [\tilde{I}_{ijk}^L, \tilde{I}_{ijk}^U], [\tilde{F}_{ijk}^L, \tilde{F}_{ijk}^U] \rangle$  by Eq. (14):

$$\tilde{r}_{ijk} = \begin{cases} \langle [T_{ijk}^L, T_{ijk}^U], [I_{ijk}^L, I_{ijk}^U], [F_{ijk}^L, F_{ijk}^U] \rangle & C_j \text{ is benefit type} \\ \langle [F_{ijk}^L, F_{ijk}^U], [1 - I_{ijk}^U, 1 - I_{ijk}^L], [T_{ijk}^L, T_{ijk}^U] \rangle & C_j \text{ is cost type} \end{cases} \quad (14)$$

Step 5: Determine the weights of risk factors in similarity degree based on cross-entropy.

(1) Determine absolute positive ideal solution.

Let  $\tilde{R}^+ = (\chi_1, \chi_2, \dots, \chi_n) (i = 1, 2, \dots, n)$  is called an interval-valued neutrosophic absolute positive ideal solution (INAPIS), where  $\chi_i = \langle [1, 1], [0, 0], [0, 0] \rangle$  is the  $n$ th largest IVNN.

(2) Compute cross-entropy of  $\tilde{r}_{ij}$  from  $INS(\tilde{r}_{ij}, \chi_i)$  by Eq. (8).

(3) Compute the similarity degree of each factor.

Usually,  $S(\tilde{r}_{ij}, \chi_i)$  is used to represent the similarity between  $\tilde{r}_{ij}$  and  $\chi_i$ . Therefore, the similarity degree  $S(\tilde{r}_{ij}, \chi_i)$  is calculated as

$$S(\tilde{r}_{ij}, \chi_i) = 1 - \frac{INS(\tilde{r}_{ij}, \chi_i)}{\sum_{j=1}^n INS(\tilde{r}_{ij}, \chi_i)} \quad (15)$$

The similarity between  $R^+ = (\chi_1, \chi_2, \dots, \chi_n)$  and  $R = (\tilde{r}_{1j}, \tilde{r}_{2j}, \dots, \tilde{r}_{mj})$  is calculated as follows:

$$S_j = \sum_{i=1}^m S(\tilde{r}_{ij}, \chi_i) \quad (16)$$

(4) Determine weights of risk factors.

It is obvious that there is a greater degree of similarity with the interval absolute positive ideal solution, the attribute should have a bigger weight. Hence, the weight of the  $j$ th ( $j = 1, 2, \dots, n$ ) risk factor can be obtained as follows:

$$\omega_j = \frac{S_j}{\sum_{j=1}^n S_j} \quad (17)$$

Step 6: Determine the entropy value of risk factors.

Shannon entropy [43] is a measure of information uncertainty. It is well suitable for measuring the relative contrast intensities of criteria to represent the importance.

(1) Compute the improved score function  $S'(\tilde{r}_{ij})$  of  $\tilde{r}_{ij}$  by Eq. (6).

(2) Normalize the score function  $S'(\tilde{r}_{ij})$  by  $P_{ij} = S'(\tilde{r}_{ij}) / \sum_{i=1}^m S'(\tilde{r}_{ij})$

(3) Calculate the entropy measure of risk factors

$$e_j = -\frac{1}{\ln m} \sum_{i=1}^m P_{ij} \ln P_{ij} \quad (18)$$

$$\varpi_j = \frac{1 - e_j}{n - \sum_{j=1}^n e_j} \quad (19)$$

Step 7: Calculate the comprehensive weight of each risk factor.

$$w_j = \lambda \omega_j + (1 - \lambda) \varpi_j (j = 1, 2, \dots, n) \quad (20)$$

where  $\lambda$  is a parameter,  $0 < \lambda < 1$  (in this paper, we set  $\lambda = 0.5$ ).

Step 8: Calculate the relative weight of each risk factor.

$$w_{jt} = \frac{w_j}{w_t} \quad (21)$$

where  $w_t = \max\{w_j | j = 1, 2, \dots, n\}$ .

Step 9: Calculate the dominance of alternative  $R_i$  to  $R_t$ .

$$\vartheta(R_i, R_t) = \sum_{j=1}^n \phi_j(R_i, R_t), \forall (i, t) \quad (22)$$

Among them,

$$\phi_j(R_i, R_t) = \begin{cases} \sqrt{w_{jt} (d_{wh}(w_j \tilde{r}_{ij}, w_j \tilde{r}_{tj})) / \sum_{j=1}^n w_{jt}} & \text{if } S'(\tilde{r}_{ij}) - S'(\tilde{r}_{tj}) > 0 \\ 0 & \text{if } S'(\tilde{r}_{ij}) - S'(\tilde{r}_{tj}) = 0 \\ -\frac{1}{\theta} \sqrt{\left(\sum_{j=1}^n w_{jt}\right) d_{wh}(w_j \tilde{r}_{ij}, w_j \tilde{r}_{tj}) / w_{jt}} & \text{if } S'(\tilde{r}_{ij}) - S'(\tilde{r}_{tj}) < 0 \end{cases} \quad (23)$$

where  $d_{wh}(w_j \tilde{r}_{ij}, w_j \tilde{r}_{tj})$  is the weighted distance of the IVNNs  $\tilde{r}_{ij}$  and  $\tilde{r}_{tj}$ ,  $S'(\tilde{r}_{ij})$  and  $S'(\tilde{r}_{tj})$  are improved scoring functions of  $\tilde{r}_{ij}$  and  $\tilde{r}_{tj}$ .

Step 10: Compute the overall dominance value of  $\vartheta(R_i)$  as follows:

$$\vartheta(R_i) = \frac{\sum_{t=1}^m \vartheta(R_i, R_t) - \min_i \left\{ \sum_{t=1}^m \vartheta(R_i, R_t) \right\}}{\max_i \left\{ \sum_{t=1}^m \vartheta(R_i, R_t) \right\} - \min_i \left\{ \sum_{t=1}^m \vartheta(R_i, R_t) \right\}} \quad (24)$$

Step 11: Determine the risk priority order of failure modes. For FMEA, the greater the value of  $\vartheta(R_i)$ , the higher the risk priority of the failure modes will be. As the result, all the failure modes that have been listed in FMEA can be ranked according to the ascending order of the overall dominance value.

### 4. AN ILLUSTRATIVE EXAMPLE

In this section, we introduced a previous case to demonstrate the feasibility of the proposed approach for risk evaluation in FMEA. A case about steel production process risk management has ten options of sheet steel production process(adopted from [44]). The failure modes of this case were evaluated by Deshpande and Modak in a factory in the past which had three team members  $TM_k(k = 1, 2, 3)$  have given the risk evaluation. The criteria are related to their occurrence probability, severity of the associated effects and detection to each failure mode as shown in Figure 2.

Step 1: Identify all potential failure modes. There are ten failure modes have been identified according to Figure 2. As shown in Table 2.

Step 2: Evaluate failure modes using linguistic terms expressed in IVNNs.

The evaluate information for each failure mode in terms of the risk factors provided by FMEA team members are shown in Table 3 [45]. Therefore, it is simple to transform linguistic information

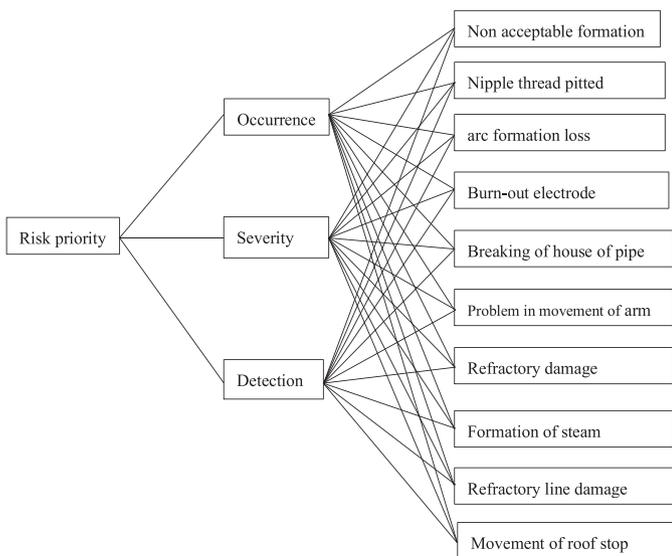


Figure 2. Hierarchical structure of the risk analysis problem.

into IVNNs according to Table 1 which are shown in Appendix A Table 13–15.

Step 3: Aggregate each team member’s evaluation information into a group evaluation matrix.

Due to these team members have different background knowledge and experience, the relative importance weights of the three FMEA team members are known as  $\tau_k = (0.35, 0.4, 0.25)$  according to literature [45]. Therefore, the aggregated the risk assessment information for each team member can be obtained according to Eq. (13) and the result is shown in Table 4.

Step 4: Three risk factors are benefit criterion and do not need to be normalized. Thus, the normalized operation should not be conducted.

Step 5: Determine the similarity degree based on the cross-entropy of risk factors.

(1) Because INAPIS is  $FM_i^+ = (\chi_1, \chi_2, \dots, \chi_n) (i = 1, 2, \dots, n)$ , where  $\chi_i = \langle [1, 1], [0, 0], [0, 0] \rangle$ .

(2) Based on Eq. (8), we can calculate the cross-entropy. The results are shown in Table 5.

(3) Combining the data in Table 5 with Eqs. (15) and (16), we can calculate the similarity of failure modes under each risk factor, and the result is denoted as  $S_j = (5.6795, 7.357, 6.9662)$ .

(4) Finally, according to Eq. (17), the weight of each risk factor is  $\omega_1 = 0.28, \omega_2 = 0.37, \omega_3 = 0.35$ .

Step 6: Determine the entropy value of risk factors. Based on Eqs. (4), (18) and (19), the weight of each risk factor can be derived as  $\varpi_1 = 0.48, \varpi_2 = 0.09, \varpi_3 = 0.43$ .

Step 7: Calculate the final weight of each risk factor. Combined with above weights, we can get the final weight of each

Table 2 | Ten related failure modes [44].

The Nodes of Failure Modes	The Description of Failure Modes
FM1	Nonacceptable formation
FM2	Nipple thread pitted
FM3	Arc formation loss
FM4	Burn-out electrode
FM5	Breaking of house of pipe
FM6	Problem in movement of arm
FM7	Refractory damage
FM8	Formation of steam
FM9	Refractory line damage
FM10	Movement of roof stop

Table 3 | Linguistic evaluations on failure modes by the team members [45].

Risk Factors	Occurrence			Severity			Detection			
	Team Members	TM1	TM2	TM3	TM1	TM2	TM3	TM1	TM2	TM3
FM1		L	EL	L	H	VH	L	L	ML	L
FM2		L	EL	L	H	H	MH	VH	H	VH
FM3		EL	EL	VL	VH	EH	H	L	ML	ML
FM4		EL	EL	VL	ML	ML	ML	L	EL	VL
FM5		L	L	L	ML	ML	ML	H	VH	MH
FM6		L	L	L	ML	ML	ML	H	VH	MH
FM7		L	L	L	ML	M	MH	L	EL	L
FM8		EL	EL	VL	VH	EH	H	ML	ML	ML
FM9		L	L	L	ML	ML	ML	H	VH	MH
FM10		L	ML	ML	H	VH	MH	H	ML	ML

**Table 4** | Group assessment matrix.

Failure Modes	Occurrence	Severity	Detection
FM1	< [0.1756,0.3268], [0.4704,0.572], [0.6226,0.774] >	< [0.5376,0.6967], [0.4373,0.5378], [0.2482,0.4101] >	< [0.2917,0.4422], [0.3565,0.4573], [0.5076,0.6581] >
FM2	< [0.1756,0.3268], [0.4704,0.572], [0.6226,0.774] >	< [0.5268,0.6776], [0.3722,0.4729], [0.2719,0.4229] >	< [0.613,0.7648], [0.4573,0.5578], [0.184,0.3366] >
FM3	< [0.0761,0.2263], [0.5733,0.6735], [0.7236,0.8739] >	< [0.6742,0.8323], [0.5086,0.609], [0.1098,0.2741] >	< [0.3166,0.4671], [0.3318,0.4325], [0.4827,0.6333] >
FM4	< [0.0761,0.2263], [0.5733,0.6735], [0.7236,0.8739] >	< [0.35,0.5], [0.3,0.4], [0.45,0.6] >	< [0.1494,0.2766], [0.4974,0.5987], [0.6492,0.8003] >
FM5	< [0.25,0.4], [0.4,0.5], [0.55,0.7] >	< [0.35,0.5], [0.3,0.4], [0.45,0.6] >	< [0.5721,0.7259], [0.407,0.5086], [0.2217,0.377] >
FM6	< [0.25,0.4], [0.4,0.5], [0.55,0.7] >	< [0.35,0.5], [0.3,0.4], [0.45,0.6] >	< [0.5721,0.7259], [0.407,0.5086], [0.2217,0.377] >
FM7	< [0.25,0.4], [0.4,0.5], [0.55,0.7] >	< [0.3962,0.5675], [0.1933,0.3031], [0.4031,0.5733] >	< [0.1756,0.3268], [0.4704,0.572], [0.6226,0.774] >
FM8	< [0.0761,0.2263], [0.5733,0.6735], [0.7236,0.8739] >	< [0.6742,0.8323], [0.5086,0.609], [0.1098,0.2741] >	< [0.35,0.5], [0.3,0.4], [0.45,0.6] >
FM9	< [0.25,0.4], [0.4,0.5], [0.55,0.7] >	< [0.35,0.5], [0.3,0.4], [0.45,0.6] >	< [0.5721,0.7259], [0.407,0.5086], [0.2217,0.377] >
FM10	< [0.3166,0.4671], [0.3318,0.4325], [0.4827,0.6333] >	< [0.5721,0.7259], [0.407,0.5086], [0.2217,0.377] >	< [0.4285,0.5819], [0.3318,0.4325], [0.3663,0.5206] >

**Table 5** | The cross-entropy from  $\tilde{r}_{ij}$ .

	Occurrence	Severity	Detection
$INS (FM_i, \chi_i)$	$FM_i (i=1,2,\dots,10)$	$FM_i (i=1,2,\dots,10)$	$FM_i (i=1,2,\dots,10)$
$\chi_1 (i = 1)$	0.0266	0.0156	0.0201
$\chi_2 (i = 2)$	0.0266	0.0145	0.0147
$\chi_3 (i = 3)$	0.0332	0.0147	0.0189
$\chi_4 (i = 4)$	0.0332	0.0172	0.0284
$\chi_5 (i = 5)$	0.0224	0.0172	0.0143
$\chi_6 (i = 6)$	0.0224	0.0172	0.0143
$\chi_7 (i = 7)$	0.0224	0.014	0.0266
$\chi_8 (i = 8)$	0.0332	0.0147	0.0172
$\chi_9 (i = 9)$	0.0224	0.0172	0.0143
$\chi_{10} (i = 10)$	0.0189	0.0143	0.0158

risk factor according to Eq. (20) (suppose  $\lambda = 0.5$ ), The results are  $w_1 = 0.403, w_2 = 0.21, w_3 = 0.387$ .

Step 8: Calculate the relative weight of each risk factor.

Since  $w_{\max} = \max\{w_j | j = 1, 2, 3\} = w_1 = 0.403$ , the reference weight is  $w_t = 0.403$ . So, we can calculate the relative weight  $w_{jt}$  of each risk factor  $w_j$  to reference weight  $w_t$  based on Eq. (21). The result is as follows:  $w_{1t} = 1, w_{2t} = 0.521, w_{3t} = 0.96$ .

Step 9: Calculate the dominance of each failure mode  $FM_i (i = 1, 2, \dots, 10)$  over each failure mode  $FM_t (t = 1, 2, \dots, 10)$  with respect to  $RF_j (j = 1, 2, 3)$  via Eqs. (3), (4) and (23) by taking  $\theta = 2.5$ . The results are listed in Tables 6–8.

Step 10: Based on Eq. (22), the overall dominance degree matrix is shown in Table 9.

Step 11: Derive the risk prioritization.

According to Eq. (24), we can compute the overall dominance value of each failure mode  $\vartheta (FM_t) (i = 1, 2, \dots, 10)$ , the final risk priority ranking of overall failure modes based on numerical result is shown in Table 10.

## 5. COMPARATIVE ANALYSIS AND DISCUSSION

### 5.1. Analysis of the Influence on the Attenuation Coefficient $\theta$

The improved TODIM method is a MADM method with parameters. The parameter  $\theta$  is the attenuation coefficient of the loss, which

**Table 6** The dominance degree matrix under risk factor O.

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8	FM9	FM10
FM1	0	0	0.1269	0.1269	-0.1084	-0.1084	-0.1084	0.1269	-0.1084	-0.1503
FM2	0	0	0.1269	0.1269	-0.1084	-0.1084	-0.1084	0.1269	-0.1084	-0.1503
FM3	-0.1277	-0.1277	0	0	-0.1674	-0.1674	-0.1674	0	-0.1674	-0.1972
FM4	-0.1277	-0.1277	0	0	-0.1674	-0.1674	-0.1674	0	-0.1674	-0.1972
FM5	0.1077	0.1077	0.1663	0.1663	0	0	0	0.1663	0	-0.1043
FM6	0.1077	0.1077	0.1663	0.1663	0	0	0	0.1663	0	-0.1043
FM7	0.1077	0.1077	0.1663	0.1663	0	0	0	0.1663	0	-0.1043
FM8	-0.1277	-0.1277	0	0	-0.1674	-0.1674	-0.1674	0	-0.1674	-0.1972
FM9	0.1077	0.1077	0.1663	0.1663	0	0	0	0.1663	0	-0.1043
FM10	0.1493	0.1493	0.1959	0.1959	0.1036	0.1036	0.1036	0.1959	0.1036	0

**Table 7** The dominance degree matrix under risk factor S.

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8	FM9	FM10
FM1	0	-0.0728	-0.1361	0.0875	0.0875	0.0875	0.0882	-0.1361	0.0875	-0.0701
FM2	0.0379	0	-0.1544	0.0789	0.0789	0.0789	0.0797	-0.1544	0.0789	-0.0836
FM3	0.0708	-0.0803	0	0.1126	0.1126	0.1126	0.1131	0	0.1126	0.0675
FM4	-0.1682	-0.1516	-0.2164	0	0	0	-0.1026	-0.2164	0	-0.1732
FM5	-0.1682	-0.1516	-0.2164	0	0	0	-0.1026	-0.2164	0	-0.1732
FM6	-0.1682	-0.1516	-0.2164	0	0	0	-0.1026	-0.2164	0	-0.1732
FM7	-0.1696	-0.1531	-0.2174	0.0534	0.0534	0.0534	0	-0.2174	0.0534	-0.1745
FM8	0.0708	0.0803	0	0.1126	0.1126	0.1126	0.1131	0	0.1126	0.0675
FM9	-0.1682	-0.1516	-0.2164	0	0	0	-0.1026	-0.2164	0	-0.1732
FM10	0.0365	0.0435	-0.1298	0.0901	0.0901	0.0901	0.0908	-0.1298	0.0901	0

**Table 8** The dominance degree matrix under risk factor D.

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8	FM9	FM10
FM1	0	-0.2003	-0.0634	0.1513	-0.1822	-0.1822	0.1344	-0.0964	-0.1822	-0.1275
FM2	0.1975	0	0.1941	0.2265	0.0821	0.0821	0.2157	0.1895	0.0821	0.1605
FM3	0.0625	-0.1968	0	0.1637	-0.1784	-0.1784	0.1482	-0.0726	-0.1784	-0.1107
FM4	-0.1534	-0.2297	-0.166	0	-0.2263	-0.2263	-0.0701	-0.1812	-0.2263	-0.1995
FM5	0.1796	-0.0833	0.176	0.2232	0	0	0.2121	0.1708	0	0.1379
FM6	0.1796	-0.0833	0.176	0.2232	0	0	0.2121	0.1708	0	0.1379
FM7	-0.1363	-0.2188	-0.1503	0.0691	-0.2151	-0.2151	0	-0.1671	-0.2151	-0.1868
FM8	0.095	-0.1921	0.0716	0.1786	-0.1733	-0.1733	0.1647	0	-0.1733	-0.1021
FM9	0.1796	-0.0833	0.176	0.2232	0	0	0.2121	0.1708	0	0.1379
FM10	0.1258	-0.1628	0.1091	0.1967	-0.1398	-0.1398	0.1842	0.1007	-0.1398	0

**Table 9** The overall dominance degree matrix.

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8	FM9	FM10
FM1	0	-0.2731	-0.0726	0.3657	-0.2031	-0.2031	0.1142	-0.1056	-0.2031	-0.3479
FM2	0.2354	0	0.1666	0.4323	0.0526	0.0526	0.187	0.162	0.0526	-0.0734
FM3	0.0056	-0.2442	0	0.2763	-0.2332	-0.2332	0.0939	-0.0726	-0.2332	-0.2404
FM4	-0.4493	-0.509	-0.3824	0	-0.3937	-0.3937	-0.3401	-0.3976	-0.3937	-0.5699
FM5	0.1191	-0.1272	0.1259	0.3895	0	0	0.1095	0.1207	0	-0.1396
FM6	0.1191	-0.1272	0.1259	0.3895	0	0	0.1095	0.1207	0	-0.1396
FM7	-0.1982	-0.2642	-0.2014	0.2888	-0.1617	-0.1617	0	-0.2182	-0.1617	-0.4656
FM8	0.0381	-0.3198	0.0716	0.2912	-0.2281	-0.2281	0.1104	0	-0.2281	-0.2318
FM9	0.1191	-0.1272	0.1259	0.3895	0	0	0.1095	-0.1207	0	-0.1396
FM10	0.3116	0.03	0.1752	0.4827	0.0539	0.0539	0.3786	0.1668	0.0539	0

value is usually between 1.0 and 2.5 [46]. In order to analyze the influence of the parameter  $\theta$  on the results of the ranking, we take different values for  $\theta$  in Step 9. Table 11 lists the ranking results of the ten alternatives based on different  $\theta$  values.

From Table 11 we can see that the ranking results of the failure modes are consistent and there is no change with the attenuation coefficient  $\theta$  changes.

## 5.2. Comparative Analyses

In order to further verify the validity and rationality of our proposed method in FMEA, we used the above example to analyze other risk assessment methods, including the traditional RPN method, the IVIF-MULTIMOORA method [45] and the WASPAS-IVIF method [47], the same FMEA problem is also solved by applying the TrFN-TODIM method [35], converting the linguistic terms of

failure modes to Trapezoidal fuzzy number (shown in Appendix A Table 16) and calculating the risk priority. The sorting results of all the failure modes have been obtained by the five methods, which are shown in Table 12.

No matter which method is used, it can be clearly seen from the table that FM10 has the highest risk priority, FM4 has the lowest risk priority, and FM7 has the ninth risk priority in all the five methods. Moreover, the ranking results of WASPAS-IVIF and our proposed method are completely consistent. This finding indicates that there is relatively homogeneity and demonstrates the effective and feasibility between the proposed FMEA model and other methods.

But there are still some differences can be explained as following causes. Firstly, we can clearly see that the ranking orders of FM2, FM5, FM3 and FM8 are different between the traditional RPN and others four methods. FM2 has higher risk priority in other four approaches than FM5. This is mainly because the uncertain and fuzzy information is not considered in the conventional RPN method. The failure modes FM3 and FM8 have the same risk ranking in RPN. In contrast, the proposed method can distinguish the two failure modes and find that failure mode FM8 has a higher risk priority than FM3. Secondly, the failure modes FM5 and FM3 have different risk priority between IVIF-MULTIMOORA and the

proposed FMEA model. The IVIF-MULTIMOORA method may be unreasonable because it does not attach importance to the influence of team members' psychological character. we can see that failure mode FM5 has higher priority than FM3 in our proposed method, this is because the failure mode FM5 has higher occurrence than FM3 in Table 3 and the weight of risk factor O is the largest. Thus, our proposed FMEA method is more closed of the practical problems. Lastly, the TrFN-TODIM method and our proposed method have minor difference in order of FM1 and FM3. we propose extended TODIM method using the new score function to reduce the uncertain decision information, and FM3 has higher occurrence and severity degree than FM1 in Table 3. Therefore, the former should have a higher risk priority.

The comparative results aforementioned indicates that the proposed risk evaluation method can obtain more reasonable risk priority ranking.

### 6. CONCLUSION

In this article, a new FMEA model based on IVNS and extended TODIM approach is presented to cope with the risk evaluation and prioritization problems. The major advantages of this study are listed below:

**Table 10** | The final ranking order of failure modes.

Failure Modes	$\vartheta (FM_i)$	Final Ranking
FM1	0.5240	8
FM2	0.9207	2
FM3	0.5326	7
FM4	0	10
FM5	0.7997	3
FM6	0.7997	3
FM7	0.4128	9
FM8	0.5608	6
FM9	0.7997	3
FM10	1	1

**Table 11** | Different parameter values  $\theta$  corresponding ranking results.

	$\theta = 1.0$		$\theta = 1.25$		$\theta = 1.5$		$\theta = 2.0$		$\theta = 2.25$		$\theta = 2.5$	
	$\vartheta$	Ranking	$\vartheta$	Ranking	$\vartheta$	Ranking	$\vartheta$	Ranking	$\vartheta$	Ranking	$\vartheta$	Ranking
FM1	0.4604	8	0.4719	8	0.4821	8	0.5038	8	0.5128	8	0.5240	8
FM2	0.8633	2	0.874	2	0.8852	2	0.9067	2	0.9135	2	0.9207	2
FM3	0.4715	7	0.482	7	0.4932	7	0.5149	7	0.5214	7	0.5326	7
FM4	0	10	0	10	0	10	0	10	0	10	0	10
FM5	0.7264	3	0.7369	3	0.7481	3	0.7698	3	0.7885	3	0.7997	3
FM6	0.7264	3	0.7369	3	0.7481	3	0.7698	3	0.7885	3	0.7997	3
FM7	0.3433	9	0.3546	9	0.3658	9	0.3867	9	0.4018	9	0.4128	9
FM8	0.4913	6	0.5023	6	0.5135	6	0.5347	6	0.5496	6	0.5608	6
FM9	0.7264	3	0.7369	3	0.7481	3	0.7698	3	0.7885	3	0.7997	3
FM10	1	1	1	1	1	1	1	1	1	1	1	1

**Table 12** | Ranking the failure modes by different approaches.

Approaches	Ranking
IVIF-MULTIMOORA [45]	FM10>FM2>FM8>FM3>FM5=FM6=FM9>FM1>FM7>FM4
WASPAS-IVIF [47]	FM10>FM2>FM5=FM6=FM9>FM8>FM3>FM1>FM7>FM4
Trapezoidal-TODIM [35]	FM10>FM2>FM5=FM6=FM9>FM8>FM1>FM3>FM7>FM4
RPN	FM10>FM5=FM6=FM9>FM2>FM3=FM8>FM1>FM7>FM4
The proposed method	FM10>FM2>FM5=FM6=FM9>FM8>FM3>FM1>FM7>FM4

1. The IVNNs are powerful tool to describe the uncertainty, indeterminacy and fuzziness element in the risk evaluation process.
2. A new similarity measure based on cross-entropy is proposed to calculate the weights of risk factors in IVNS environment. In order to obtain more objective weights, we combine similarity degree and entropy measures to determine the final weights.
3. The improved score function by considering the percentage of the indeterminacy-membership function is applied in TODIM method to reduce vagueness of decision information, and extended TODIM method further shows its obvious advantage, it pays attention to team members' psychological behavior in risk ranking, which will help to obtain a more reasonable risk evaluation result.
4. The practical case study and comparison analysis are conducted to demonstrate the effectiveness of the proposed framework.

The future research can focus on various operators rather than traditional MADM method to calculate risk priority of failure modes, such as HM operator, BM operator, and so on. Moreover, we should consider more risk factors to get more reliable FMEA.

## CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest.

## AUTHORS' CONTRIBUTIONS

Jianping Fan established the research direction and content. Dandan Li conducted the literature review and wrote the entire manuscript. Meiqin Wu conducted the review and editing. All authors have read and agreed to the published version of the manuscript.

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## Appendix A

Dear reviewers:

Thank you very much for your valuable comments on our paper. We have carefully answered the questions according to your requirements. All the changes in the article are marked in red. The revised article has become better because of your suggestions.

Reviewer #1:

1. Authors should revise expressions of English.  
For this issue, we have revised.
2. Try to use consistent expressions, such as “D-S theory” and “D-S evidence theory,” “two-dimensional belief function” and “TDBF”  
For this issue, we have revised.

3. The introduction part needs to be strengthened and should be summarized after reviewing previous studies.  
For this issue, we have revised.
4. Some recently published works, such as those published in 2020, should be reviewed to highlight the contribution of the manuscript.

For this issue, we have revised.

Reviewer #2:

1. Grammatical mistakes and the quality of English have been the concern of the paper. Check thoroughly.  
For this issue, we have revised.
2. The TODIM methodology has already been tailored by other authors in FMEA such as “An improved reliability model for FMEA using probabilistic linguistic term sets and TODIM

**Table 13** | The IVNNs of FMEA team member TM1.

Failure Modes	Occurrence	Severity	Detection
FM1	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>
FM2	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>
FM3	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>
FM4	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>
FM5	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>
FM6	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>
FM7	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>
FM8	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>
FM9	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>
FM10	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>

IVNN, interval-valued neutrosophic number; FMEA, failure mode and effect analysis.

**Table 14** | The IVNNs of FMEA team member TM2.

Failure Modes	Occurrence	Severity	Detection
FM1	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>
FM2	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>
FM3	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.75,0.9],[0.6,0.7],[0.05,0.2]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>
FM4	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>
FM5	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>
FM6	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>
FM7	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.4,0.6],[0.1,0.2],[0.4,0.6]>	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>
FM8	<[0.05,0.2],[0.6,0.7],[0.75,0.9]>	<[0.75,0.9],[0.6,0.7],[0.05,0.2]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>
FM9	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>
FM10	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>

IVNN, interval-valued neutrosophic number; FMEA, failure mode and effect analysis.

**Table 15** | The IVNNs of FMEA team member TM3.

Failure Modes	Occurrence	Severity	Detection
FM1	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>
FM2	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.45,0.6],[0.3,0.4],[0.35,0.5]>	<[0.65,0.8],[0.5,0.6],[0.15,0.3]>
FM3	<[0.15,0.3],[0.5,0.6],[0.65,0.8]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>
FM4	<[0.15,0.3],[0.5,0.6],[0.65,0.8]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.15,0.3],[0.5,0.6],[0.65,0.8]>
FM5	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.45,0.6],[0.3,0.4],[0.35,0.5]>
FM6	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.45,0.6],[0.3,0.4],[0.35,0.5]>
FM7	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.45,0.6],[0.3,0.4],[0.35,0.5]>	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>
FM8	<[0.15,0.3],[0.5,0.6],[0.65,0.8]>	<[0.55,0.7],[0.4,0.5],[0.25,0.4]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>
FM9	<[0.25,0.4],[0.4,0.5],[0.55,0.7]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.45,0.6],[0.3,0.4],[0.35,0.5]>
FM10	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>	<[0.45,0.6],[0.3,0.4],[0.35,0.5]>	<[0.35,0.5],[0.3,0.4],[0.45,0.6]>

IVNN, interval-valued neutrosophic number; FMEA, failure mode and effect analysis.

**Table 16** | Group assessment matrix by using Trapezoidal fuzzy number.

Failure Modes	Occurrence	Severity	Detection
FM1	(0.018,0.06,0.108,0.15)	(0.654,0.7,0.756,0.802)	(0.1,0.148,0.252,0.31)
FM2	(0.018,0.06,0.108,0.15)	(0.713,0.775,0.87,0.933)	(0.876,0.910,954,0.763)
FM3	(0,0.0025,0.0075,0.01)	(0.924,0.945,0.972,0.993)	(0.128,0.178,0.297,0.348)
FM4	(0,0.0025,0.0075,0.01)	(0.18,0.22,0.36,0.4)	(0.011,0.038,0.071,0.098)
FM5	(0.03,0.1,0.18,0.25)	(0.18,0.22,0.36,0.4)	(0.797,0.835,0.91,0.945)
FM6	(0.03,0.1,0.18,0.25)	(0.18,0.22,0.36,0.4)	(0.797,0.835,0.91,0.945)
FM7	(0.03,0.1,0.18,0.25)	(0.357,0.405,0.553,0.601)	(0.011,0.038,0.071,0.098)
FM8	(0,0.0025,0.0075,0.01)	(0.924,0.945,0.972,0.993)	(0.18,0.22,0.36,0.4)
FM9	(0.03,0.1,0.18,0.25)	(0.18,0.22,0.36,0.4)	(0.797,0.835,0.91,0.945)
FM10	(0.128,0.178,0.297,0.348)	(0.797,0.835,0.91,0.945)	(0.38,0.43,0.55,0.6)

method.” Since there is already TODIM methodology introduced for FMEA, the author/s must highlight the novelty and prominent of the paper.

For this issue, we have added in “1 introduction” and “6 conclusion.”

3. The motivation of this paper should reorganized, the author/s must provide a much clearer explanation for the purpose of this paper.

For this issue, we have added in “1 introduction.”

4. The author/s should add more details about the extended TODIM, such as, why these methods you applied to extend the TODIM? How do you extend the TODIM?

For this issue, we have added in corresponding part.

5. What is the background of the 3 experts? Please describe more about this. Why did the authors just select only 3 experts? It is

better to use 5 or 7 experts in the decision-making team. What is difference between the decision-making results and the 5 expert decision-making result? The author/s can make a Comparison.

For this issue, we have explained in “4 An illustrative example.”

6. Who are the experts? The process of data research cannot be found. How does the author make statistics and deal with these language data? What is the basis? These should be clear.

For this issue, we have added in “3 The proposed FMEA model.”

7. In reference [33], the generalized TODIM method is proposed to determine risk priority ranking order, the author/s can make a comparison.

For this issue, we have added in “5 Comparative Analysis and Discussion.”