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Risk Aversion, the Analysis and Improvement of Risk-Free Arbitrage Based on CBOE Data

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ABSTRACT
This paper conducts various risk-free arbitrage studies on options data in the Chicago Board Options Exchange. Although there are not many opportunities for arbitrage that violates market rules on the surface, there are many external factors that need to be considered in practice. These factors were thoroughly analyzed. This paper concluded that the B-S-M model has imperfections in determining future option prices. Based on the actual market, the model has been improved to speculate and to conjecture. Thus, investors will be able to make corresponding suggestions.
Keywords: *Option, risk, arbitrage*

1. INTRODUCTION

With modern finance development, the system of financial markets is becoming more and more perfect, and the importance of derivatives is self-evident. The pricing of derivatives is the focus of our attention. The pricing of derivatives involves financial engineering and the establishment of models. Our pricing of derivatives is based on the assumption of no-arbitrage, and the relative pricing method is also an important method. However, the environment we are facing is always relatively changing, so no pricing method can ultimately determine the price of derivatives. Only a relatively reasonable estimate can be given.

A variety of factors causes price fluctuations. The advancement of information technology has accelerated the speed at which market entities acquire, process, and respond to information, further affecting price fluctuations. Meanwhile, human factors and panic psychology have led to abnormal fluctuations in the stock market. These fluctuations include diseases and wars, an undeniable example is a panic caused by the new type of coronary pneumonia in 2019.

Options are the most basic and essential category in derivatives. An option is a contract that gives the buyer the right to purchase or sell a certain amount of an underlying asset at an agreed price (strike price). There are many different types of options depending on the rights of the option purchaser, the time limit of the exercise, and the underlying asset. The pricing of options is complex and changeable. Under the principle of no-arbitrage, we know that the price of an option has an upper limit and a lower limit. At the same time, an option with the same exercise price of the same stock also has a parity relationship. When these principles are violated, theoretically, it will occur in a short time. Risk-free arbitrage is not in line with the market. However, in the modern market, the price of individual options often violates the principle of arbitrage. Dividends and exchange will also charge a specific commission. At this time, arbitrageurs will appear. Therefore, researching arbitrage strategies is also our focus. Among them, the black-Scholes model also provides a way to arbitrage: delta hedge. The Black-Scholes model is a model for accurate options pricing based on the stochastic process and the Ito process. It gives a short-term option pricing, and delta gives the linear relationship between certain commodities. When this linear relationship is violated, we can conclude the opportunity to hedge and arbitrage.

In this article, we have studied that the options in the real market violate the above no-arbitrage criteria or have the opportunity of delta-hedge. We have calculated the proportion of options that violate these no-arbitrage criteria, frequency, and the relationship between density and profit is made. At the same time, we can use existing options to arbitrage, and thus earn the profits we want to obtain. In this article, as investors we will give and improve specific arbitrage strategies in order to maximize the return as much as possible.

2. LITERATURE REVIEW

Kaushik Amin, Joshua D. Coval and H. Nejat Seyhun, Index Option Prices and Stock Market Momentum October 2004, the article talked about the influence of stock yield to the options market. The historical data has huge impact on option pricing. Especially when the stock yield has higher returns. The article proves that the possibility of violate boundary conditions depends on the return on stock in the past. And their conclusion is run-of-mill. On the other hand, the influence of stock has nothing to do with the model, and they don't rely on any known options pricing models. This proved our opinion that it's risky to arbitrage by using modern option pricing model. This article also proposes that considering the past stock returns, a more accurate estimate of the implied volatility of the index option price can be obtained, which is helpful for us to determine the fluctuation of related parameters through the analysis of historical data when revising the BSM model. Interval (such as the volatility of the underlying asset)

Amit Goyal, Alessio Saretto, Option Returns and Volatility Mispricing, March 2007, This paper proposes a method for explaining implied volatility that does not depend on the black-scholes model, but on the option price. The article uncovered existing pricing errors in the market and also made some speculations, such as the use of alternative estimates of implied volatility (the results calculated using option prices) may make investment results more profitable. The reason for this phenomenon is that investors did not consider all factors when forming their expectations for future stocks, especially ignoring the information contained in the cross section of implied volatility and considering assets alone. The price calculated by the cross-section model is closer to the real price, which also provides as a gist.

Joshua D. Coval, Tyler Shumway, Expected Option Returns, June 2000, this article conducts a detailed study of long-term option returns. The conclusion is that option returns conform to the rules of most assets to a large extent, but these returns appear to be low considering the corresponding level of risk. In addition to risk, volatility is also an important factor to consider in option pricing, which is undoubtedly the same as the factors considered in this paper when it comes to risk-free arbitrage. This article also finds that cross-grade returns are also negative in various options markets, and are related to their correlation with the implied volatility of the S & P 100 Index. This view is also consistent with the cross section of implied volatility above (that is, the implied volatility of different markets around the world at the same time) that plays a role in the pricing of stocks and options.

Jie Cao, Bing Han Cross section of option returns and idiosyncratic stock volatility, 2013. The article finds that the average hedging return of a hedging option is negative, regardless of the type of option, [5]and decreases monotonically with the increase of the special volatility of the underlying stock, but there is a positive correlation with the systematic risk of the underlying stock. [5] The test of this paper is consistent with the financial intermediation model under constraints, which excludes the volatility risks of the conventional stock market and is more focused on the actual market. [5] Some other conclusions are that the returns on hedge options in high-yielding stocks in the past are much higher than the returns on the stock itself. Several agents of the arbitrage limit between stocks and options affect the cross-section of hedge option returns. The impact is again a very complicated process. These are different factors from standard pricing models that investors need to be considering.

Jie Cao, Aurelio Vasquez, Xiao Xiao, Xintong Zhan, Volatility Uncertainty and the Cross Section of Option Returns March,2018. This paper reached a similar conclusion to the previous ones, namely, that is (to say), there is a negative correlation between volatility and the return on future options. It is characterized by the use of three different volatility analyses to make the results more representative. The author constructs a set of tradable options portfolio strategies based on neutral call options trading, and ranks them by three volatility. After controlling common risk factors from the stock market and various volatility risk factors, these options portfolio strategies can provide positive average returns. The conclusions obtained are similar to the above articles, that is, the volatility theory has nothing to do with market behavior and cannot be explained by conventional risk theory. The main influencing factor is the behavior of market makers.

3. DATA AND METHODOLOGY

3.1. Sample selection

In this study, we used the stock data on the Chicago Board of Trade (CBOE) from January to March 2017. CBOE established an options trading market and launched standardized contracts to revolutionize options trading. [8] The establishment of the Chicago Board Options Exchange marked that the options trading has entered a new stage of standardization and standardization. The Chicago Board of Options Exchange has successively introduced call and put options for stocks, and both have achieved success. Moreover, as an on-exchange transaction, its process also meets the characteristics of standardization and standardization, which can avoid a series of problems caused by off-exchange transactions. For each transaction, the data content includes option type, option price, strike price, expiration time, volatility, and delta. In order to ensure the randomness of the results, we selected 200 different stocks from the more than one million data of CBOE from January to March, and each of them selected 100 options. In the selection process, we first number each stock and then use random numbers to pick out the corresponding number of options. After excluding stocks with less than 100 options and stocks corresponding to short-term options with trading days less than seven days, a total of 20,000 sets of data were obtained.

3.2. Research hypotheses

This article considers both transaction costs and non-transaction costs. Based on the data available, and based on the principle that investors want to complete transactions at the lowest cost, this article makes the following assumptions about market behavior and other factors

3.2.1. Relevant assumptions of trading behavior

Limited by information protection, it is impossible to obtain information on investor transactions. This article assumes that there is no bid-ask spread in the market; that is, the securities purchase price is equal to the sale price. According to the assumption of option parity theory, the trading strategy chosen in this article is the hold-to-maturity strategy. Investors buy and sell the product and hold it to the option expiration date and close the position on the option expiration date.

3.2.2. Relevant assumptions of the options market

The option transaction fee is composed of two parts, namely the transaction commission charged by the securities company where the investor's securities account is located, and the transaction handling fee charged by CBOE. Assume that the commission rate is 5 / 10,000 and 1 / 10,000 of the total transaction amount. Therefore, the corresponding cost of each option contract is six ten thousandths of the contract amount.

3.2.3. Hypothesis of the bond market

When constructing a portfolio of options parity theory strategy and delta-hedge strategy, it is necessary to consider making loans from banks or depositing in banks. This article assumes that the investor's loan and deposit is risk-free, and the basic interest rate of deposits that announced by the bank is risk-free rate as well, which is the coupon rate of the bond. Here we assume that the annual interest rate is 0.5% and calculated as simple interest.

3.2.4. The cost of selling bonds is the difference between the benchmark interest rate on deposits and loans.

Thus, investors deposit at the bank with a deposit-based interest rate equivalent to their risk-free bonds with the purchase rate as the deposit-based interest rate. Risk-free bonds.

3.3. Method analysis

We mainly focus on the risk-free arbitrage strategy for options such as the parity relationship, the upper and lower limits of the put-call price, and delta-hedge hedging risk arbitrage according to the black-Scholes model. This article discusses whether there are dividends for American options during their lifetime. When there is no dividend, an American call option will not be executed in advance. Whether an American put option is executed in advance depends on the time value of the option relative to the time value of the currency (interest). We next prove this conclusion:
For American call options, consider two portfolios:
Portfolio 1: [7] An American call option to buy one share and cash of $X\mathrm{e}^{-rT}$
Portfolio 2: one corresponding stock
Let t be any trading day before the option expiration date, and St is the value of the stock at time t, which is also the value of portfolio 2. If the call option is exercised at time t, the value of portfolio 1 is "$S_T - X + Xe^{-r(T-t)} < S_t$". If a call option is exercising when it expires, the value of option portfolio 1 is "$max(S_t, X) \geq S_t$". Therefore, we can see that the value of Portfolio 1 is smaller than the value of Portfolio 2 when it is executed in advance, and the value of Portfolio 1 is not lower than the value of Portfolio 2 if it is executed at the expiry date. Hence, it is not wise to execute American call options in advance.
For American put options with the similar method, we could know if the interest generated by the execution of the put option and the sale of stocks at the market interest rate is higher than the increase in the time value of the option in the same time period. It is exercised in advance. The put option is feasible. Specific circumstances include
1, The exercise price of the option is relatively high, but the stock market price is relatively low, that is, the put option is in a deep real value state
2, Higher risk-free interest rates in the market
3, Stock price volatility is small
After that, we discuss various situations of American option risk-free arbitrage. For example, the price is higher than the upper limit of the option price theory or lower than the lower limit of the theoretical price, does not meet the American option parity formula, and the price calculated by the black-Scholes model is biased.

3.3.1. Upper Bound

Call options: American options paid out of dividends available above can now be considered European options. Its price should meet
$c \leq S$. If this formula is not valid, the arbitrageur can quickly obtain risk-free returns by buying stocks and selling call options. Put options: For everyone who owns a put option, the most favorable situation is that the stock price falls to 0 on a trading day. At this time, the price of the put option is equal to its exercise price, so the price limit is $P \leq X$

3.3.2. Lower Bound

Call options: We consider the following two portfolios:
Portfolio A: A European call option that can buy one share and Xe^{-rT}cash
Portfolio B: one underlying stock
On the expiration date, if the stock price is higher than or equal to the strike price of the option, so the option is alive, and the value of the AB portfolio is equal. However, if it is lower than B, so the option is dead. The value of A should

not be lower than B.($c+Xe^{-rT}\geq S$, $c\geq S-Xe^{-rT}$) Because of the price can't be negative， $c\geq max(S-Xe^{-rT},0)$ Put option：If the price of the American put option paid without dividends is $P<x_s$, the arbitrageur will buy the American put option at the price of P and buy the stock at the price of S while selling the stock at X, so that $x_s-p>0$ is achieved. Risk return, so the price of American put options should satisfy $P\geq max(x_s,0)$

3.3.3. American option parity formula

Because the value of American call options and European call options without dividend payment is equal ($C=c$). Moreover, the price of American put options is higher than European put options ($P>p$). It is similar with European option parity formula $c+Xe^{-rT}=P+S$. Thus, we can get$P+S>C+Xe^{-rT}$. One more corollary to the same call and put options, consider the following two portfolios:
Portfolio 1: An American call option to buy one share of stock and X amount of cash
Portfolio 2: An American put option that can sell one share and one corresponding stock
If the put option of portfolio 2 is executed in advance at time t, then the value of portfolio 2 is X, and the value of portfolio 1 is Xe^{rT}, so the value of portfolio 1 is higher than the value of portfolio 2. If the put option is not exercised in advance, the value of portfolio 2 at time T is $max(s_T,x)$, while the value of portfolio 1 is $\max(s_T-x,0)+Xe^{rT}=\max(s_T,x)+X(e^{rT}-1)$
The value of combination 1 is higher than the value of combination 2 at any time, so the price of combination 1 should be higher than the price of combination 2, that is, $C+X>P+S$, so we can get $C+X>P+S>C+Xe^{-rT}$

Table 1. Arbitrage when the parity relationship does not hold

Arbitrage Current Portfolio	Current Cash Flow	CashFlows at the expiry of an option contract	
		$s_T\geq X$	$s_T<X$
call option	$-c$	$+s_T-X$	0
Risk-free investment Xe^{-rT}	$-Xe^{-rT}$	$+X$	$+X$
put options	$+p$	0	$-X+s_T$
Short stock	$+S$	$-s_T$	$-s_T$
Total	$-c-Xe^{-rT}+p+S>0$	0	0

3.3.4. Delta-hedge

When pricing options, the approach we take is to construct a risk-free portfolio consisting of options contracts and their corresponding underlying assets so that their return is equal to the risk-free rate of return, and then the option price is derived. The reason why a risk-free portfolio can be constructed is that the option price will be affected by the price of the underlying asset so that the loss on the option can be made upon the underlying asset vice versa. This strategy is delta-hedge. We consider the following portfolios：N_s(shares) and Nc(call option)，the total value is V, which is $V=N_sS+N_cC=N_c\left(\frac{N_s}{N_C}S+C\right)$， h stands for $\frac{N_s}{Nc}$， h is called Hedging ratio. In order to fully hedge the portfolio, V should be independent of the price change of S ， so $\frac{\partial V}{\partial s}=N_c\left(h+\frac{\partial c}{\partial s}\right)=0$， $h=-\frac{\partial c}{\partial s}$. $\frac{\partial c}{\partial s}$ is the value of delta, it describes the rate of change of the option price to the price of the underlying asset. The call option delta is positive, and the put option delta is negative.
Consider about black-Scholes

$$c=SN(d_1)-Xe^{-rT}N(d_2) \quad (1)$$

$$p=Xe^{-rT}N(-d_2)-SN(-d_1) \quad (2)$$

$$d_1=\frac{ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (3)$$

$$d_2=d_1-\sigma\sqrt{T}=\frac{ln\left(\frac{S}{X}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (4)$$

Thus, we get$\Delta_P=\frac{\partial p}{\partial s}=N(d_1)-1$ $\Delta_c=\frac{\partial c}{\partial s}=N(d_1)$
We can get profits from the wrong pricing of options. If the price of a call option is overvalued, it may be profitable only by selling the option but may also be affected by factors such as rising stock prices. However, if we use delta-hedge, when the stock price rises, the gains of the stock longs can make up for the losses on the options so that investors get a net profit.

Table 2. Adjust hedge strategy based on the price of the underlying asset

Option type	Underlying asset price rises	Underlying asset price decreases
Long call	Delta increases, Sell more assets	Delta decreases, Buy more assets
Short call	[2]Delta increases, Buy more assets	[2]Delta decreases, Sell more assets
Long put	[2]Delta decreases, Sell more assets	[2]Delta increases, Buy more assets
Short put	[2]Delta decreases, Buy more assets	[2]Delta increases, Sell more assets

3.3.5. Finding

According to our data processing, we can find that there are very few options that violate upper bond and lower bond, which can be ignored except in extreme cases. This is because of the estimated range of upper bond and lower bond in a more standardized and effective market is too large. Once it appears, the magnitude of the violation will be too large. In addition to its own arbitrage opportunities, it will also be accompanied by other various arbitrage opportunities. This is totally inconsistent with the laws of the market. To a large extent, such rare arbitrage opportunities are investors' mistakes in pricing. Therefore, in normal operation, these two arbitrage opportunities can be almost ignored.

We focus on the arbitrage opportunities brought by the option parity relationship and delta-hedge, and whether there is an opportunity to make money in violation of the no-arbitrage principle. In a real market environment, any transaction requires costs, and transaction costs play a vital role in the success of risk-free arbitrage. It is the only criterion to test the effectiveness of this arbitrage strategy.

In practice, common arbitrage costs include transaction costs, margin occupation costs, and hedging costs. Therefore, the result of the option arbitrage strategy is actually the money left after the theoretical arbitrage profit excluding all costs. Among them, transaction cost is the most important factor, whether it is execution cost or spot transaction cost, etc., all belong to transaction cost. We are talking about stock options, so the most basic transaction costs we need to consider are the intermediate fees charged by the stock exchange and the intermediate fees charged by the option exchange itself. The intermediate fees charged by various exchanges seem to be small, and in fact, the impact on option prices cannot be ignored. So in doing risk-free arbitrage we need to add the impact of such fees.

In addition, the cost of occupancy of the deposit is also a very important factor, and there are many factors that affect the cost, so the calculation results obtained in the market are generally quite different. We consider an example here. For example, in the process of implementing an arbitrage strategy, it is necessary to short the spot to establish a position. Generally, in this case, investors use securities brokers or banks to carry out financing and margin trading channels to borrow funds. Sometimes the annual interest rate is high, which usually around 10%. However, when we deposit cash flows from arbitrage strategies into the money market, we often do not get such high returns, so for investors, the interest rate of risk-free assets actually has multiple. And these spreads also affect the cost of investment, and then affect the parity formula.

See the figure below for a comparison of the effects of cost considerations and non-cost considerations on parity arbitrage opportunities.

Table 3. The difference of some digital features between two put-call parities

	Violation	Violation Rate	Magnitude Mean	Magnitude Median
Put-call Parity(0)	8707	43.535%	0.9199	0.5452
Put-call Parity(0.45)	4773	23.87%	1.096	0.791

We see that a large number of options that originally meet the arbitrage opportunity will no longer have the possibility of arbitrage, which shows that the impact of cost on the pricing of options is huge and cannot be ignored. If we do not consider the cost, blindly using the model to carry out arbitrage often leads to negative benefits, which we do not want to see. This also further shows that arbitrage opportunities are not so easy to occur, and many factors have not been taken into account. At the same time, we know that more options are in line with market laws, which makes our understanding of the model further.

We can see that the stock code AAL can be arbitrage without considering the cost. We can arbitrage through the established strategy. We seem to get the desired cash flow, but when we are deducted by the exchange from transaction costs When someone ordered to repay the loan, we will find that our income has become negative. But we have no way to remedy because we hedged the risks in the market, so at this time we can no longer get extra money from this strategy, which means that arbitrage has failed.

The results of the above analysis show that in the practical operation of risk-free arbitrage, we must be careful and consider as comprehensive as possible. Only in this way can we accurately estimate the benefits of risk-free arbitrage. Otherwise, if you just operate according to a simple model, there will be negative returns, which is an investment loss that we don't want to see. Therefore, when we use the risk-free arbitrage strategy, we must be optimistic about the current market interest rate and various impacts.

4. DISCUSSION AND IMPROVEMENT OF BLACK-SCHOLES MODEL

After the parity arbitrage opportunities, we shall discuss about the black-Scholes model and delta-hedge. From the black-Scholes model we can know:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{5}$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{ln\left(\frac{S}{X}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{6}$$

$$N(d_1) = NORMSDIST(d1) \quad (Excel\ function) \tag{7}$$

$$N(d_2) = NORMSDIST(d2) \quad (Excel\ function) \tag{8}$$

$$C = Se^{-\gamma T}N(d_1) - Xe^{-r_f T}N(d_2) \tag{9}$$

$$P = Xe^{-r_f T}N(-d_2) - Se^{-\gamma T}N(-d_1) \tag{10}$$

[3] We can use this formula to calculate the theoretical value of the option price, and then compare it with the option price in the mature market. If there is an abnormality, we can use it to hedge. When the delta is neutral, and the spread is not 0, People can buy low and sell high for arbitrage.

Table 4. Delta-hedge cash flow at the time node shown by the initial data

	Positive cash flow rate	Magnitude mean	Magnitude median	Maximum value

Put delta-hedge	77.32%	27.9482	9.1891	745.534
Call delta-hedge	84.71%	33.5314	16.5326	744.861

The above table only shows the cash flow of delta-hedge at a certain time, but the delta value is not static. To keep the asset in a neutral state at all times, investors need to adjust the option position in time. The final return is the total return after stocks and options are closed, so it brings the limitations of delta hedge. When conducting hedging adjustments, investors need to frequently adjust the number of options and stocks, and each transaction will have a certain transaction fee, which will increase as the number of transactions increases. [6] Secondly, this hedging strategy is only effective when there is a small change in the stock price. It is more complicated to perform delta-hedge on options to pay dividend stocks, and we will not go into details here.

Although in practice, the B-S-M option pricing formula is by far one of the best models for explaining the dynamics of option prices, the market has not fully followed our schedule. The price of the B-S-M model is still different from the actual market. The most typical phenomenon is that the implied volatility should be constant under the B-S-M model, but there are often volatility surfaces in reality. This proves that the B-S-M model does have many flaws and is based on many assumptions. The black-scholes model is like the parity relationship, relaxes in transaction costs. In parity relations, we have discussed the impact of this aspect, and we will not repeat it.

The relaxation of parameter assumptions will also affect our results, because the BSM model assumes that the interest rate and the return on the underlying asset are constant, but in fact this is wrong, and it is not even a deterministic function that uses time and the underlying asset price as independent variables. We should add a range of variation to the value of these parameters and then calculate the option's price range.

In addition, the BSM model assumes continuous changes in asset prices, and the BSM model assumes that the underlying asset prices continue to change, obeying a log-normal distribution. However, asset price changes are discrete and often jump, regardless of the minimum number of points and the investor's Blindly following the trend and so on, geometric Brownian motion cannot describe and capture these pairs. Moreover, jumps often occur suddenly, and hedging cannot be accomplished simply by relying on the normal distribution.

We mainly discuss about the impact of volatility on our strategy. The underlying asset volatility in the B-S-M model is a known constant, but this is incorrect. Because an asset's market is different under different circumstances, the degree of market regulation is also different, and liquidity and technological improvement are also uncertain; the implied volatility will change with the expiration time of options. In fact, one finds that volatility is itself a random variable.

We should build a stochastic model of volatility. That is, in addition to the stochastic process that the underlying asset price itself obeys, volatility should also be described using a stochastic process. Such as

$$\mathrm{d}S_t = \mu S_t \,\mathrm{d}t + \sigma_t S_t \,\mathrm{dz_t} \tag{11}$$

$$d\sigma_t^2 = p(S_t, \sigma_{t,} t)\,\mathrm{d}t + q(S_t, \sigma_t, t)\,\mathrm{dz_t} \tag{12}$$

Because its establishment process is too complicated, we will not repeat it here.
We believe that when volatility is a stochastic process, a more appropriate option pricing model would be:

$$c_t = \int_0^\infty f_t(\theta_0) h(\theta_0|\sigma_0)\, d\theta_0 \tag{13}$$

Where f () is the B-S-M formula:
T is the option expiration date:
$\theta_0 = \int_0^T \sigma_t \, dt$ h is Risk-neutral probability density

$$d_1 = \frac{\frac{\ln \mathrm{S_t}}{\mathrm{X}} + \left(r + \frac{\sigma_t^2}{2}\right)T}{\sigma_t\sqrt{T}} \tag{14}$$

$$d_2 = \frac{\frac{\ln \mathrm{S_t}}{\mathrm{X}} + \left(r - \frac{\sigma_t^2}{2}\right)T}{\sigma_t\sqrt{T}} \tag{15}$$

We believe that a density should be add in the original formula which is about probability, and the pricing formula should be recalculated under the dimensionless change. That is, the expectation of B-S-M under the stochastic model is calculated. It will be more accurate to replace the original option value with this expectation.

Therefore, in the investment process, investors should have a deeper understanding of the rate of change. Don't simply get a “determined” volatility from history and then hedge, this often results in the opposite. We believe that investors should divide history into broad sections when conducting hedging, and focus on researching some changes. Based on the current trend, re-evaluate volatility. When a position is established, it should be at any time after the position is established. Changing positions in response to uncertain changes in the market; buying and selling based on constant fluctuations in volatility. People cannot stick to the original strategy. Although this operation is too technical, it does help our investment. We also mention the risks posed by the operation in the later section.

5. RISKS AND OTHER FACTORS

At the same time, the impact of the existence of dividends on parity should also be considered, because the size of the dividend also represents the size of the risk and the difference in cash. If we use D to represent the discounted value of dividends over the life of the option. We must assume that the timing and amount of dividends paid during the option period are known. But in practice, this assumption is not very reasonable, because most options traded on the exchange do not exceed one year. Therefore, this factor is too complicated and also hinders the length of this article, so we will not go detail about that.

In actual situations, the requirements for risk-free arbitrage operations are high, because operations in accordance with existing models often cause order failures due to external factors. For example, sometimes we need to establish multiple positions in order to constitute an arbitrage adjustment button. We need to buy and sell call options, put options, and underlying assets, which is often impossible in actual transactions. The liquidity of the stock market environment or other reasons often increases our costs. Moreover, the conditions for arbitrage generally require the simultaneous delivery of various assets. If the arbitrage operation cannot be executed synchronously due to liquidity and other reasons, it will be very easy to miss opportunities, and may even expose downside risks, eventually leading to negative returns. Therefore, whether each position can be traded at the same time and the success award is established directly affects the profit of arbitrage.

At the same time, delivery also creates risks, although American options can be delivered at any time, once operations such as dividends occur, the value of our options will change directly, which will be very detrimental to arbitrage. In addition, there will be large fluctuations in the market during this process, which will result in delivery risks during the arbitrage operation. Because when carrying out an arbitrage strategy, we need to establish obligatory position and a right position, so we need to use the underlying assets required to achieve the held Yiwu positions for delivery at the contract expiration date. If the stock is on a continuous daily limit at this time, then the investor cannot normally buy the stock, and often needs a rights issue. The investor will default because of insufficient quota. In addition, in the actual operation of option risk-free arbitrage, when we multiple synthetic option power positions are established and the underlying stock needs to be held at the same time. If the stock market is in the context of stock market volatility, the stock spot will also be executed before delivery and has great risk.

Moreover, the "risk-free" in risk-free arbitrage refers to the risk of the time arbitrage portfolio, but for the operation process that requires the opening of an obligation position, it will not be reduced to the risk-free expected by its strategy. This is a risk that will exist in actual operation. [4] For example, when the price of the underlying asset encounters a sharp decline, short call options in the strategy will face to additional margin, and increasing the cost of capital occupation. The market continues to deteriorate, which can easily lead to a chain reaction and a full liquidation. It will cause the entire strategy to collapse and lead to great losses.

6. ANALYSIS OF FUTURE OPTION PRICING

According to the quotations that there are many types of factors can affect future options and stock prices, such as the cross section of implied volatility, special volatility, and historical [1] returns on stocks and options. The behavior of market makers is also an important part of it. For example, in the actual market, market change charge higher premiums on high-volatility stock options. Because these options have higher hedging costs, more difficult arbitrage, and people get more profit. In the fifth quotation, the author divided the volatility into VOL + and VOL-. The market maker charged a high premium for two types of high volatility options that have achieved volatility measurement, which affected its implied volatility, which in turn affected The entire market. From a cross-sectional perspective, prices in different markets around the world are all interrelated, but because few people notice this problem, some pricing errors have occurred. From the perspective of historical data, the returns of stock options with higher profits in the past are much higher than the returns of the stock itself, indicating that investors are mostly confident in the positive direction. If you want to accurately predict the price, you need to consider the volatility of each global market at the same time in the horizontal direction, historical data and implied volatility in a single market. The mathematical calculation and modeling are too complicated, so the option price constitutes Time series still have no more accurate mathematical models to describe.

Therefore, when investors carry out risk-free arbitrage operations, they need to be optimistic about the market and not blindly entangle with the model. They should appropriately adjust according to the model to avoid external risks as much as possible. We recommend that when investing, investors, choose a relatively stable market to avoid possible recovery risks and delivery risks. We also recommend that investor positions should be established in a timely and accurate manner.

7. CONCLUSION

In the conclusion, we found that the arbitrage opportunities obtained in violation of the parity relationship in the CBOE are plenty. This is inconsistent with market laws and no arbitrage rules, but according to our in-depth discussion, we found that the facts are not what we thought. In practice, arbitrage opportunities need to consider various external factors, we need to consider costs, and external risks caused by margin and exchange regulations. This makes our arbitrage often not in accordance with the established strategy and makes it suitable. The opportunities for arbitrage have been greatly reduced. Therefore,in the market, we need to screen out options that really meet the arbitrage opportunities, otherwise we may backfire and get negative returns.We must pay attention to some problems which cannot be calculated in the operation of the market in order to further avoid risks. In this article, we analyze a variety of risks and some factors that affect the price of options, and delve into the frequency of parity arbitrage relationships, as well as the factors that affect the frequency. Risks provide a certain direction for investors to avoid risks and reduce the possibility of losses outside of the strategy. This is very meaningful to investors, because all the investor like profit and hates risk.

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