Advances in Engineering Research, volume 197
Proceedings of the 2020 9th International Conference on Applied Science, Engineering and Technology (ICASET 2020)

Research on Optimal Decision of Open-Ended Fund Investment Based on Mathematical Model

Luling Duan[a] Zhiheng Lin[b,*]

Department of Mathematics and Information Science, Guangxi College of Education, Nanning, Guangxi, China

[a]*duanluling2006@163.com*

[*b]*Corresponding author Email: nnlin@126.com*

ABSTRACT
Adopting the idea of mathematical modelling, this paper studies four problems of open-end fund investment. For problem one and problem two, (0-1) programming model was used to find the maximum profit under certain investment fund conditions. For problem three, after increasing the factors of investment risk, considering the conditions of problem one and problem two, two-objective optimization model was set up, and hierarchical method was used to solve it. Through Lingo software programming, the maximum profit and the optimal investment scheme were obtained. For problem four, considering the need to keep a proper amount of cash to reduce the risk of customers who are unable to cash in, taking into account the four situations obtained after consulting experts, using the total profit difference between simultaneous investment and non- simultaneous investment, a model is established and the optimal solution is obtained.
Keywords: *0—1 programming, Enumeration method, Double objective optimization, Open end fund, Total profit.*

1. RAISE THE PROBLEMS

An open-end fund has a total amount of 1.8 billion yuan, which can be used for investment. At present, there are eight projects to choose from. And each project can be repeatedly invested (i.e. several shares are invested at the same time)[1] . Based on the expert experience, the total amount of investment for each project should not be too high (there is an upper limit). The amount of investment required for these projects and the profit of each project after one year of investment have been given, as shown in Table 1.

Table 1 Capital required for investment project and expected profit after one year
Unit: 10,000 yuan

Project No.	1	2	3	4	5	6	7	8
Amount of investment	6700	6600	4850	5500	5800	4200	4600	4500
Profit	1139	1056	727.5	1265	1160	714	1840	1575
Upper limit	41000	33000	34000	29000	35000	26000	27000	25000

Please help the company solve the following problems:
(l) According to the data provided in Table 1, which projects should be invested to maximize the profit in the first year?
(2) In the specific investment of these projects, there will be mutual influence between the projects. After consulting relevant experts, the company obtained the following reliable information: when we investment A_1 and A_3 projects at the same time, their estimated profits are 10.05 million yuan and 10.185 million yuan respectively; when we investment A_4 and A_5 projects at the same time, their estimated profits are 10.45 million yuan and 12.76 million yuan respectively; when we investment A_2 , A_6 , A_7 and A_8 projects at the same time, their expected

profits are 13.53 million yuan, 8.4 million yuan, 16.1 million yuan and 13.5 million yuan respectively. How should the fund be invested[2]?

Table 2 Client–server experimental results

Project No.	1	2	3	4	5	6	7	8
Risk loss rate (%)	30	14.5	22	30	33	5.5	39	33

(3) If we consider the investment risk, how should we invest to maximize the return and the minimize risk? The total risk of an investment project can be measured by the risk of the project with the largest amount of investment. The risk loss rate of investment projects predicted by experts is shown in Table 2.
(4) In general, open-end funds should keep an appropriate amount of cash[3] in case of customers cashing at any time for the unexpired fund (cash in advance, the customers bear certain losses). In this case, considering the four pieces of information from experts, how to make decisions on the fund to maximize investment profit after one year with the minimum risk ?

2. MODEL ESTABLISHMENT AND SOLUTION

In order to establish the model[4], we need to make the following assumptions:
(1) Assume that in the process of investment, the subscription fee, selling fee and redemption fee are included in the investment amount of the project.
(2) Assume that in the process of investment, it is not affected by policies and government conditions.
(3) Assume that the profit of each project and the investment risk rate are not affected by external factors.
(4)Assume that investors' preference for each project is the same under the same profit.
(5)Assume that the profit from the investment project is pure profit, excluding the profit from the retained funds in the form of deposit.
(6) Assume that under the condition of minimum investment risk rate and maximum profit, single project investment is not considered.
Meanwhile, we explain the symbols used in the paper.
Assume x_i represents the investment share of the i project $(i=1,2,\cdots 8)$, and a_i represents the profit one year after the investment in the i project; b_i represents the investment amount of the i project; c_i represents the profit of the i project when considering the mutual influence of investment; d_i represents the investment upper limit of the i project; p_i represents the risk loss rate; ΔZ represents the total profit difference between simultaneous investment and non-simultaneous investment; Z represents the total profit; G represents the total investment amount.

2.1. Model establishment and solution of problem 1

For problem one, there are only two kinds of decisions for each investment project: investment or no investment. Its fundamental is to solve how to select projects for investment, so as to maximize profits in one year, and the choice of project types is the problem of portfolio. Therefore, we consider introducing 0-1 variable to establish (0-1) planning model to obtain the maximum profit of one year after investment. Assume that

$$y_i=\begin{cases}1, & \text{Invest in project } i\\ 0, & \text{Don't nvest in project } i\end{cases}, \qquad (i=1,2,\cdots,8)$$

Model 1 is established: $\max=\sum_{i=1}^{8} y_i x_i a_i$

s.t.

$$\sum_{i=1}^{8} y_i x_i b_i \le 180000; \qquad (1)$$

$$x_i b_i \le d_i;$$

$$x_i \in N;$$

$$y_i \in \{0,1\}.(i=1,2,...,8);$$

Using Lingo programming, we can get the following conclusion[5]: The investment shares of project $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ are as follows: .

$$x_1=6, x_2=1, x_3=0, x_4=5, x_5=6, x_6=6, x_7=5, x_8=5$$

At this time, the maximum profit is 435.34 million yuan.

2.2.Model establishment and solution of problem 2

For problem 2, we need to consider the interaction between projects on the basis of problem one. We continue

to use the (0-1) planning model to solve the problem of the maximum profit. In addition, considering that there may be a big gap between the actual data and only one model, we use the second method (enumeration method) to establish the model. Finally, the two results are compared to choose a better investment plan.

Method 1: introduce three (0-1) variables

Assume

that $e=\begin{cases}1, & \text{Invest in both } A_1 \text{ and} A_3\\ 0, & \text{Don't invest in both } A_1 \text{ and} A_3\end{cases}$

$$f=\begin{cases}1, & \text{Invest in both } A_4 \text{and} A_5\\ 0, & \text{Don't invest in both} A_4 \text{and} A_5\end{cases}$$

$$g=\begin{cases}1, & \text{Invest in both } A_2, A_6, A_7 \text{and} A_8\\ 0, & \text{Don't invest in both } A_2, A_6, A_7 \text{and} A_8\end{cases}$$

（1）For A_1 and A_3, the solution is as follows:

$$m_1=e(c_1x_1+c_3x_3)+(1-e)(a_1x_1+a_3x_3)$$

（2）For A_4 and A_5, the solution is as follows:

$$m_2=f(c_4x_4+c_5x_5)+(1-f)(a_4x_4+a_5x_5)$$

（3）For A_2, A_6, $A_7$and A_8, the solution is as follows:

$$m_3=g(c_2x_2+c_6x_6+c_7x_7+c_8x_8)+(1-g)(a_2x_2+a_6x_6+a_7x_7+a_8x_8)$$

Establish model 2:

$$\max=m_1+m_2+m_3$$

$$\sum_{i=1}^{8}b_ix_i\le 180000;$$

$$x_i\in N\ ,(i=1,2,...,8);$$

$$e,f,g\in\{0,1\}.$$

Using Lingo programming, we can get the following conclusion: The investment shares of project $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ are as follows: $x_1=0, x_2=3, x_3=7, x_4=5, x_5=6, x_6=4, x_7=5, x_8=5$. At this time, the maximum profit is 435.135 million yuan.

Method 2: Enumeration method

Considering the mutual influence of profit between investment projects, the number of investment will make the profit of each project change. Based on the meaning of the title, there are three investment conditions, so in view of the knowledge of permutation and combination, there are seven situations in which the maximum profit can be obtained. Now we list one of the models: (only invest in project 1 and project 3 at the same time)

Model 3 is obtained:

$$\max=1005x_1+1018.5x_3+1056x_2+1256x_4+1160x_5+714x_6+1840x_7+1575x_8;$$

s.t. $x_1x_3\ge 1;$

$$x_4x_5=0;$$

$$x_2x_6x_7x_8=0;$$

$$x_ib_i\le d_i,(i=1,2,...,8);$$

$$\sum_{i=1}^{8}b_ix_i\le 180000;$$

$$x_i\in N^*,(i=1,2,...,8)$$

The other six models can be obtained by using Lingo. Therefore, we compare the seven results of the software operation. When the investment share of the project $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ is:

$$x_1=3, x_2=0, x_3=7, x_4=5, x_5=6, x_6=4, x_7=5, x_8=5$$

respectively, the maximum profit is 433.605million yuan.

After comparing the maximum profit of the two methods, we find that the result of method one is better than that of method two, so we can get a better project investment plan.

2.3. Model establishment and solution of problem 3

As for problem three, after adding the factor of investment risk and considering the conditions of problem 1 and problem 2, we need to make the risk as small as possible under the condition of maximizing the profit. Obviously, this is a double-objective problem. Therefore, we adopt the double objective planning model[6], and assume that the total risk of the investment project is and the maximum profit is, two objective functions are obtained, objective function 1:

$$Q = \max b_i x_i p_i;$$

Objective function 2: $Z = \max a_i x_i$ In this case, the problem becomes a double objective optimization model. In order to convert it into a single objective optimization model, we use the hierarchical method to solve the problem, considering the minimum risk and the maximum profit, model 4 can be obtained:

$$\min Q = \max b_i x_i p_i;$$

$$\text{s.t.} \quad \sum_{i=1}^{8} b_i x_i \le 180000;$$

$$\sum_{i=1}^{8} a_i x_i \ge Z_1;$$

$$b_i x_i \le d_i;$$

$$x_i \in N^*, (i = 1,2,...,8.)$$

(Note: Z_1 is the minimum value of total profit in problem1. We take it as $Z_1 = 42404$ yuan.)

Lingo software is used to solve the model mentioned above, the investment share of the eight projects is

$$x_1 = 6, x_2 = 1, x_3 = 0, x_4 = 5, x_5 = 6, x_6 = 6, \quad x_7 = 5, x_8 = 5$$

respectively, and the maximum profit is 120.6 million yuan.

2.3.Model establishment and solution of problem 4

For problem four, due to the need to retain appropriate amount of cash to reduce the risk that customers can not cash in, considering the four situations obtained after consulting experts, we decided to establish a model and solve the problem by using the difference of total profit between simultaneous investment and non-simultaneous investment and the conditions of realizing the minimum investment, the minimum risk and the maximum profit at the same time[7] . In order to further simplify the model, we take the fixed total investment and risk loss as constraints, and establish model 5

$$\max Z = \sum_{i=1}^{8} a_i x_i + \Delta Z;$$

$$\min Q = \max b_i x_i p_i;$$

$$\min G = \sum_{i=1}^{8} x_i b_i;$$

$$\text{s.t.} \quad \max b_i x_i p_i \le Q \quad (5)$$

$$\sum_{i=1}^{8} x_i b_i \le G$$

$$G < 180000$$

$$b_i x_i \le d_i;$$

$$\Delta Z = (-134x_1 + 291x_3) + (-220x_4 + 116x_5) + (279x_2 + 126x_6 - 230x_7 - 225x_8)$$

Lingo software is used to solve the model mentioned above, the investment share of the project $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ is

$$x_1 = 0, x_2 = 2, x_3 = 7, x_4 = 5, x_5 = 5, x_6 = 6, x_7 = 5, x_8 = 5$$

respectively. At this time, the maximum profit is 427.255 million yuan.

3. CONCLUSION

The advantages of models:
(1) For problem 1, we use (0-1) variable to simplify the complex problem, and finally use mathematical software to solve the problem.
(2) For problem 2, due to the influence of the profit between various projects, we continue to use (0-1)

variables and enumeration method to solve the problem, and through comparison, we can get a relatively better solution to the problem.
(3) For problem 3, after adding the factors of investment risk, we consider using the double objective planning model to solve the problem successfully.
(4) For problem 4, when dealing with the problem of keeping an appropriate amount of cash to prevent customers from cashing in, we get the maximum profit by solving the minimum investment and total risk, and avoid the discussion of the amount of cash to be retained, which simplifies the model and makes the solution more convenient.
The disadvantages of the models:
The model is too idealistic and does not consider the influence of all external factors, so some errors may exist. In the practical investment problems, the subscription fee, selling fee and redemption fee of the investment project cannot be ignored, and the investors' preferences should be carefully considered.

ACKNOWLEDGMENTS

This work was supported by the Scientific Research Fund of Guangxi College of Education （No. A2017001）.

REFERENCES

[1] Qiong He , Research on the relationship between investment style and market P / E ratio of open-end funds , Business News. 11(2018) 57-58.

[2]J. H. Xiao, Enron Xu, Does fund size change affect investment behaviour? -- Empirical Evidence from open-end funds, Securities Market Guide. 7 (2017) 59-69.

[3]Ying Li, An empirical study on the relationship among capital flow, investment behaviour and performance of open-end funds, Northeast University of Finance and economics, 2019.

[4]Q. Y Jiang, J.X Xie, Jun Ye, Mathematical Model (Third Edition) , Higher Education Press, Beijing, 2003.

[5] P. Xu; N. Na; S. Gao; C. Geng, Determination of sodium alginate in algae by near-infrared spectroscopy, Desalination and Water Treatment, 168(2019)117-122.

[6] P. Xu,Y, Su, Design and Implementation of landscape system for East and West Huashi Street in Beijing based on virtual reality technology, Applied Mechanics and Materials, 263(2012)1849-1852.

[7]J.X Xie, Yi Xue, Optimization model and LINDO/LINGO software, Tsinghua University Press, Beijing, 2005.

[8]Information on http://wenku.baidu.com/view/fdc7f92f647d27284b735156.html.

[9]Information on http://wenku.baidu.com/view/8ccff56a561252d380eb6e23.html.

[10] P. Xu, N. Na, A. M. Mohamad, Investigation the application of pristine graphdiyne (GDY) and boron-doped graphdiyne (BGDY) as an electronic sensor for detection of anticancer drug, Computational and Theoretical Chemistry,1190(2020):112996.