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Mathematical Modeling and Forecasting of Student's Academic Performance on Massive Online Courses

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ABSTRACT
Mathematical model for calculating the scores' distributions in massive open online courses is proposed. The model is based on the theory of Markov processes. It allows to calculate the probability to find a student in one of the groups according to the results of passing the tests: unsuccessful students, performing satisfactorily and doing well and excellent. It is shown that in the limit of a sufficiently long history of teaching the course on the educational platform, the distribution of scores for the course becomes asymptotically steady. It is shown also that such asymptotically steady distributions, can be calculated on the base of the model proposed, even for the courses without a long history. Such asymptotically steady distributions can be indicators of the quality of control materials and approaches to student scoring. As an example, several courses of Ural Federal University (UrFU), posted on the National Platform of Open Education have been analyzed. The possibility of using the model to predict the results of control tests based on the data on the current progress of students before passing them is shown.

Keywords: *online courses, probability model, assessing of tests' quality, asymptotically steady scores distribution, forecasting of test results*

1. INTRODUCTION

Online technologies today, has become an integral part of part of the educational system. This creates new challenges and gives rise to new problems that concern all the participants of the educational processes [1-4]. One of them is to ensure a balance between the massiveness of online learning and the benefits of the traditional education models, which mean an individual approach to each student. One of the tasks is to monitor the results of leaning the course materials, reflected in the results of passing the control checkpoints provided for by the course. Such monitoring allows the teachers and heads of educational programs to provide intime reaction on the emerging negative trends.
This in turn increases the role of testing as an important mean for monitoring students' knowledge and strengthens the requirements to the tools of such monitoring. In this paper, we will try to consider some possible algorithms for valuation of these tools. We are going to discuss the algorithms that use the objective and registered in any online course results - students' marks for passed control points. As will be shown below, their analysis based on appropriate probabilistic models can provide useful information to all participants of the educational process.

1.1. Related Work

Since this article will consider the methods of mathematical modeling of students' academic activity, we will briefly outline the results of works using similar approaches. These works can be divided on two groups:

– assessment of the control materials' quality and approaches to knowledge measurement;
– modeling of the academical activity of students

Below, without claiming to be complete, we will consider typical researches in each of them.

1.1.1. Assessment of the control materials' quality and approaches to knowledge measurement

The development of online education, the integration processes in the education gave impetus to the development of testing methods, considered as a universal tool for knowledges' control and measurement. The massive introduction of tests and other control instruments into the educational process, puts forward the tasks of assessing the quality of the control tools themselves. Contemporary, the methods briefly described below are widely used for this purpose.

1.1.1.1. Psychometric researches

Three main approaches are currently occurred for psychometric analysis:

– Classical Test Theory (CTT, see for example Crocker et al. [5]) - the approach that was popular until the beginning of 60s of past century. However, for some types of testing (for example, in psychodiagnostics), its methods are still using now CTT is based on the idea that the individual test score is the sum of the subject's true score and the independent error of measurement. From a few simple, natural assumptions about these components, algorithms for evaluating the validity, reliability and other statistical test's indicators are derived;

– Theory of test items (Item Response Theory – IRT, for example Lord F.M.[6], van der Linden et al. [7], Embretson et al. [8]) is a set of methods that allows to assess the probability of correct answer on the tasks of various difficulty. It is used for the elimination of uninformative questions in the tests and including in tests the questions with scores adequate to their complexity;

– Finally, the generalized theory (G- Theory, see for example Robert L [9] and references there) analyzes the factors (facets) that are the sources of variation in tests' results. These factors can be time, place, organization of testing, etc. The purpose of G-Theory is to assess the variations connected with each factor and with the interactions of factors. This in turn makes it possible to assess the reliability of the test and the reproducibility of its results.

1.1.1.2. The Information Theory Approaches

In the works [10-12], were discussed the approaches to assessing of the tests' materials based on the information theory. In particular, [12] introduced the concept of a test's informativeness coefficient, defined as the ratio of information contained in a message that a test was passed by a student with a certain score (it does not matter which one) to its maximum possible value. With this definition, the control tasks, which are traditionally performed by almost all students, or not performed by anyone, has an informativeness coefficient equal to zero. The maximum value of 1 is obtained with an almost uniform distribution of the scores received by the students over the entire allowable range of their values.

In [10] the informational approach was applied to the valuation of the possibility to predict the results of the final test for the entire course on the base of the results that were obtained in current testing. In particular, the uncertainty reducing of the final test's results according to information about the results of current tests was estimated. A significant decrease of this uncertainty occurred immediately after obtaining the information about the results of the first test in series of control points. Nevertheless, it was shown, that the uncertainty of the final testing cannot be reduced on more than 70-80% of its initial value. This is a natural limitation for the reliability of prediction for the course completion results.

Modeling the academic activity of students

Recently, approaches to the analysis based on the models of students' behavior have been actively developing. Such approaches allow predicting academic activity, learning outcomes and assessing the quality of tests [11,14]

1.1.1.3. Academic activity modelling on the base of the behavioral secularities of students

An analysis of the students' behavioral patterns for online courses was carried out in [13] on the base of log-files storing information about the behavioral activity of students. In particular, the features of their work with different components of the course were studied. The relationship between students' activity and learning outcomes was also analyzed. [10,12] suggest the model of evolution for the probability distribution of students' scores for all tests of the courses based on the theory of Markov processes [14]. This model uses solely the data on academic performance. In this work we are going to expand the approach of [10,12] so, it will be scrutinized below.

1.1.1.4. Probability models of academic activity

The model proposed in [10,12], allows to calculate the probability distribution's evolution of the students' scores, namely the probability to find the definite number of students in each of groups formed in accordance to students' academic performance. Three groups were formed:

1. Unsuccessful students - group 1;
2. Satisfactory students - group 2;
3. Successful students - group 3.

The distribution between groups was carried out according to scores obtained during the tests passed. In the process of studying course and passing the checkpoints, each student performed the transitions from one of the above groups to another. Tracking such transitions for each student, the authors obtained the probabilities of inter-groups transitions for each test (control point) of the course. These probabilities are determined by a matrix that is individual for each control point

$$\hat{P} = \begin{Vmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{Vmatrix} \tag{1}$$

It should be noted that the matrix $\widehat{\mathrm{P}}$ is not symmetric, in addition, its elements satisfy the relation:

$$\sum_{j=1}^{3} P_{ij} = 1 \tag{2}$$

The probability that at the moment *t* in the groups1,2,3 there will be respectively: X_1, X_2, X_3 students – $P(X_1, X_2, X_3|t)$, satisfies the equation [10,12]:

$$\frac{\partial P(X_1, X_2, X_3|t)}{\partial t} = P(X_1, X_2, X_3|t) \cdot \{(1-z) \cdot \sum_{i=1}^{3} P_{ii} - \sum_{i=1}^{3} X_i\} + +z \cdot \sum_{i=1}^{3} (X_i + 1) \cdot P(\dots, X_i + 1 \dots |t) + +(1-z) \sum_{i=1, j\neq i}^{3} P_{ij} \cdot (X_i + 1) \cdot P(\dots, X_i + 1, \dots X_j - 1 \dots |t)\} \tag{3}$$

Here z=z(t) the probability that at the time *t* the student will leave the course Further, following [12], we will set that *z=0*.

1.2. Our Contribution

In this paper, we will continue the study of the probabilistic model [10] on the example of the UrFU courses and apply it to the assessing of the control materials (tests) quality. We also will treat the possibility to apply this model for prediction the results of passing either selected control tasks, or the final test. This probability can be calculated for the whole group of students or for each student individually as well. The knowledge of these probabilities allows to issue recommendations:

– for lectors: to correct the test materials and approaches to knowledge assessing, if necessary, to correct the level of tests complexity, etc.

– for students: to plan efforts in learning the relevant handbooks and other study materials.

1.3. Paper Structure

The work has the following structure. Section 2.1 provides an additional analysis of the model described in Section 1.2.2.2 and proposes an algorithm for its application to assessment of the control materials' quality as well as for learning outcomes' predictions. Section 2.2 describes the results of applying this algorithm to some online courses at UrFU. The results are discussed in Section 2.3. Finally, section 3 contains conclusions.

2. BACKGROUND

2.1 The analysis of the probability model

The solution of the equation (6) formulated above in most practically important problems is not necessary, since usually it is possible to consider only average values and covariances:

$$\overline{X_i} = < X_i > = \int_0^{100} X_i \cdot P(X_1, X_2 X_3|t) \cdot dX_i,$$

$$\sigma_{IJ}{}^2 = \int_0^{100} X_i \cdot X_i \cdot P(X_1, X_2 X_3|t) \cdot dX_i \cdot dX_j - < X_i > \cdot < X_j > \quad (4)$$

integration is carried out over the entire acceptable range of scores from 0 to 100. Multiplying (6) by X_j and integrating over the variables $X_1, X_2 X_3$, one obtains the equation for $\overline{X_i}$:

$$(\partial \overline{X_l})/\partial t = \sum_{k=1}^{3} [\, \widetilde{P_{kl}} \cdot \overline{X_k} - \widetilde{P_{lk}} \cdot \overline{X_l}], \quad (5)$$

Here

$$\widetilde{P_{kl}} = 0 \; for \; k = l$$

$$\widetilde{P_{kl}} = P_{kl}, \text{ defined by (4) for } k \neq l \quad (6)$$

Certainly:

$$< X_1 > + < X_2 > + < X_3 > = N, \quad (7)$$

where N is the total number of students. It can be shown that from (8), (10) it follows:

$$\overline{X_i} \sim N, \quad (8)$$

and the proportionality coefficient is a value of the order of 1. Similarly, based on the definition of σ_{IJ} (7) and equation (6) one can show that:

$$\sigma_{IJ} \sim \sqrt{N}\,, \quad (9)$$

So as $N \to \infty$, the variation coefficients tend to zero:

$$C_V(ii) = \frac{\sigma_{ii}}{\overline{X_i}} \sim \frac{1}{\sqrt{N}} \to 0, \quad (10)$$

This reflects the law of large numbers. Thus, for many students (large N), that is the feature of online courses, the numbers X_i is close to $<X_i>$.

Let us make some estimations. To simplify calculations, we reduce the number of groups to two. First is group number 1, defined in the section 1.1.2.2 (unsuccessful students), second – the group number 4 is the union of the groups 2, 3 entered in 1.1.2.2. Consider the course "Engineering Mechanics".

By direct counting the numbers of students who made the following transitions between the groups: 1 → 1; 1 → 4; 4 → 1; 4 → 4 in the process of passing the final test, we find the matrix $\hat{P}$:

$$\hat{P} = \begin{Vmatrix} 0.32 & 0.402 \\ 0.68 & 0.598 \end{Vmatrix} \quad (11)$$

Solving equations (8) and the equations for the covariance σ_{IJ}, which are not given here due to their bulky form, we find:

$$x_1(t) = N \cdot \frac{P_{41}}{P_+} \cdot [1 - e^{-P_+ \cdot t}], \quad (12)$$

$$\widetilde{\sigma_{11}} = \widetilde{\sigma_{22}} = -\widetilde{\sigma_{12}} = (\frac{P_{14} \cdot P_{41}}{{P_+}^2})^{1/2} \frac{1}{\sqrt{N}} \cdot [1 - e^{-2P_+ \cdot t}], \quad (13)$$

$$x_2(t) = 1 - x_1(t). \quad (14)$$

Here,

$x_{1,2} = \frac{X_{1,2}}{N}$

$\widetilde{\sigma_{11}} = \frac{\sigma_{11}}{N^2}$, etc.

$P_+ = P_{14} + P_{41}$

As $t \to \infty$, he values determined by (14) - (16) tend to stable constant values:

$x_1(t \to \infty) = 0.37$

$x_2(t \to \infty) = 0.63$

$\widetilde{\sigma_{11}} = 0.012$

$C_V(11) = 3{,}1\%$

So, for the courses with a lot of students, coefficients C_V, are very small and the observed number of students who fell into groups 4 and 1 practically coincides with their average values determined by equation (8).

It should be noted that the continuous time t in the above formulas is used for convenience and compact form of the formulas. In fact, basically, we are considering two points: before testing and after. The transition $t \to \infty$ corresponds to a hypothetical situation in which the final testing is carried out an unlimited number of times and each time the same control materials and approaches to assessing test results are used. As a close analogue, we can consider the performance's distribution of all students during lifecycle of the course, conducting for a sufficiently long time without correcting educational, methodological and testing materials.

It can be shown that the stationary states ($\frac{\partial \overrightarrow{x_l}}{\partial t} = 0$) arise as $t \to \infty$ and are stable. Moreover, they can be found as a solution to the equation:

$$\vec{x} = \hat{P}\vec{x}, \tag{15}$$

So, the stationary states – $x_l(t \to \infty)$ are the eigenvectors of the matrix $\hat{P}$ (4) or (14) and correspond to the eigenvalue equal to unity [15]. They describe the distribution established after the above-mentioned multiple passing of the test by students. Below, such distributions are calling asymptotically steady. Therefore, regardless of the initial distribution of students' scores before testing, the result in the limit $t \to \infty$ will be the same (the property of equifinality). This allows us to consider such stationary, asymptotically steady states (18) as a characteristic of the testing process itself (control tasks, approaches to knowledge assessment, etc.).

The algorithm to determine this characteristic includes the following steps:

1. Calculation of the P^matrix using the historical data on the progress of students' scores of the course under analysis.
2. Determination the stationary eigenvectors of the matrix P^that correspond to the eigenvalue equal to unity from (18). These vectors give asymptotically steady distributions of scores between groups formed according to academical performance in section 1.1.2.2. (groups 1,2,3 or 1,4)
3. Quality analysis of the resulting steady-state distribution of student scores (examples are given in the next section).

A knowledge of matrix $\hat{P}$ or the test or other control task allows solving the following forecasting problems:

According to the known distribution of students' scores before the control event – $\overrightarrow{x_b}$ predict distribution – $\overrightarrow{x_a}$ after this event.

The solution is given by the obvious equation:

$$\overrightarrow{x_a} = \hat{P}\overrightarrow{x_b}. \tag{19}$$

We can consider the vector $\overrightarrow{x_b}$ in the previous item as a distribution of individual scores of a student rather than the group. In this case,$\overrightarrow{x_b} = \{x_{1b}, x_{2b}, x_{3b}\}$ is his/her distribution of scores over a series of previous control points and $\overrightarrow{x_a} = \{x_{1a}, x_{2a}, x_{3a}\}$ – the predicted probability of the scores for the upcoming control event.

2.2 Results and discussion

The approbation of the described algorithms was performed for several UrFU courses presented in Table 1.

Table 1 Information about courses and probabilities of transitions between groups 1 - 3, (4)

Course's Id	**Course's name**	**Semester (F - autumn, S - spring)**	**The number of active students**	**Elements of matrix $\hat{P}$ (4) (4)**								
				P_{11}	P_{12}	P_{13}	P_{21}	P_{22}	P_{23}	P_{31}	P_{32}	P_{33}
40 ENGM	Engineering Mechanics	F2019	429	0,25	0,50	0,25	0,21	0,29	0,50	0,17	0,38	0,45
41 ENGM	Engineering Mechanics	S2019	729	0,37	0,23	0,40	0,28	0,32	0,40	0,13	0,24	0,63
42 ENGM	Engineering Mechanics	S2020	310	0,20	0,20	0,60	0,21	0,21	0,59	0,03	0,12	0,85
60 MCS	Natural science view on the world	F2019	478	0,08	0,83	0,08	0,20	0,62	0,18	0,05	0,61	0,33
61 MCS	Natural science view on the world	S2020	250	0,33	0,00	0,67	0,06	0,29	0,65	0,01	0,10	0,89
104 PHILOSOPHY	Philosophy	F2019	858	0,25	0,75	0,00	0,11	0,71	0,18	0,02	0,62	0,36
105 PHILOSOPHY	Philosophy	S2019	1053	0,13	0,19	0,68	0,04	0,47	0,48	0,02	0,40	0,58
106 PHILOSOPHY	Philosophy	S2020	2437	0,00	0,32	0,68	0,01	0,21	0,78	0,00	0,11	0,89
77 RUBSCULT	The culture of Russian business speech	F2019	878	0,00	0,19	0,81	0,04	0,59	0,37	0,01	0,32	0,67
78 RUBSCULT	The culture of Russian business speech	S2019	210	0,13	0,13	0,75	0,00	0,60	0,40	0,06	0,17	0,77
79 RUBSCULT	The culture of Russian business speech	S2020	195	0,00	0,19	0,81	0,00	0,55	0,45	0,01	0,32	0,67
46 HIST_VIEW	History: 5 approaches to historical development	F2019	1995	0,43	0,43	0,14	0,12	0,69	0,19	0,03	0,55	0,43
47 HIST_VIEW	History: 5 approaches to historical development	S2020	134	0,00	0,00	1,00	0,21	0,21	0,57	0,03	0,19	0,79

We considered only those students who tried to pass the final testing - active students. At the same time, we analyzed the transitions from the distribution of the academic performance before the final testing to that one formed after it. We take the final testing, as an example, although a similar analysis can be performed for any control task. To do this, it is necessary to know only the distribution of students' scores by performance groups (1,2,3) before test and after passing it. As you can see the components of the $\hat{P}$ matrices (table 1) for different courses and even for the same course are somewhat different for different launches.

As an integral estimation of the "distance" between $\hat{P}$ matrices of different courses, we will use the so-called cosine distance [15] (in this case, the matrix $\hat{P}$ is represented by the vector $\vec{P} = \{P_{11}, P_{12}, \ldots, P_{33}\}$:

$$d_c(k,m) = \frac{180}{\pi} \cdot \arccos\left(\frac{\sum_{i,j=1}^{3} P_{ij}^k \cdot P_{ij}^m)}{||P^k|| \cdot ||P^m||}\right), \qquad (20)$$

where $||P^k||$ – norm L_2 («length») of vector $\overrightarrow{P^k}$, k, m) courses' identifiers (see Table 1). Table 2 shows the cosine distances between different launches of courses in the same discipline:

Table 2 Cosine distances between matrices $\widehat{P^k}$ for different courses k of the same disciplines

	40 ENGM	**41 ENGM**	**42 ENGM**
40 ENGM		22,56	33,10
41 ENGM			20,13
42 ENGM			
104 PHILOSOPHY		47,64	60,18
105 PHILOSOPHY			25,22
106 PHILOSOPHY			
77 RUBSCULT		10,81	4,36
78 RUBSCULT			10,93
79 RUBSCULT			

As can be seen, even within the same discipline different launches of courses may differ in approaches to final test, which is reflected in noticeable differences in the matrices P (the more are the time intervals between launches the more are the differences). The asymptotically steady probability distributions shown on the Figure 1 also differ. This fact can be an "information for thought" both for teachers and for the heads of educational programs, especially in the cases when results are unsatisfactory, in their opinion.

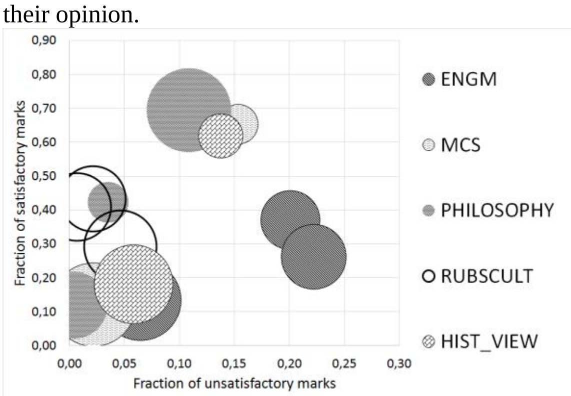

Figure 1 Asymptotically steady distribution of scores for final testing for various courses. The bubbles squares are proportional to the fractions of good and excellent marks (at total) for corresponding courses

It can be noted that the courses in the humanities (Philosophy, RUBSCULT, HIST_VIEW and partly MCS) demonstrate a clear shift in the distributions of scores to the relatively low fractions of unsatisfactory and satisfactory marks. In addition, they have a high percentage of good and excellent marks compared to the technical course (ENGM and others, the results for which are not shown here).

So, it can be predicted that the existing approaches to control testing will lead to the steady distribution of graduates' scores indicating a good command of the humanities, with noticeably more modest results in technical and natural science disciplines. This is a reason to think about changing the approaches to final testing, including their cross-disciplinary coordination.

Note that there is also a difference in the asymptotically steady, stationary distributions of the scores for final tests. To estimate these difference one can use the cosine distance (20) between vectors $\overrightarrow{x^k} = \{x_1^k, x_2^k, x_3^k\}$ Vector $\overrightarrow{x^k}$ describes such distribution - for the course k. The dependence of the cosine distances between vectors $\overrightarrow{x^k}$ for courses within the same disciplines and the distances between the matrices corresponding to these courses – $\widehat{P^k}$ is well described by regression:

$$d_c(\overrightarrow{x^k}, \overrightarrow{x^l}) = 0.983 \cdot d_c(\overrightarrow{P^k}, \overrightarrow{P^l}). \qquad (21)$$

The determination coefficient $R^2 = 0.98$.

For the assessment and comparison of approaches to tests' scoring it is equally possible to use matrices (vectors) $\widehat{P^k}$ or asymptotically steady distributions of scores $\overrightarrow{x^k}$. Indeed, according to (21), both approaches give the same results. However, the usage of asymptotically steady distributions of scores $\overrightarrow{x^k}$ is more convenient and habitually.

Let us now consider the solution of forecasting problems. As was noted above, to do this, the matrix $\widehat{P^k}$ for the corresponding control point and equation (19) can also be used. For illustration, we chose the results of the final testing for ENGM courses. As first example we took 40ENGM course and divided active students into two sets:

– training set (341 listeners, the scores of the representatives of this group were used for the calculation of matrix $\hat{P}$);

– and set for testing (88 listeners, for which the predicted results of final testing were compared with the actual ones).

The following was obtained:
Predicted values:
{0.9%; 13.8%; 75.4%}
Actual values:
{5.6%; 16.9%; 77.5%}

As can be seen, the forecast with a high accuracy (with a 5% confidence level) coincides with the actual values. It is interesting to predict the results of the final testing at new launches of the course on the base of P̂ matrix determined at the previous launch. The results of such predictions for courses 41ENGM, 42ENGM are presented in table 3. The matrix $\hat{P}$ was calculated on the base of course 40ENGM:

Table 3 The results of score distribution predictions for final test of the courses "Engineering Mechanics"

	41 ENGM		**42 ENGM**	
	Predicted value	**Actual value**	**Predicted value**	**Actual value**
x1	19,08%	19,20%	17,99%	5,48%
x2	37,79%	25,79%	37,10%	12,90%
x3	43,14%	55,01%	44,91%	81,61%

It is clearly seen that the more is time interval of the new course launch from that one for which the matrix P̂ was calculated, the worse is the agreement of the predicted results with the actual. This is the reflection of changes in the approaches to the final testing that are increasing with the time interval between sequent launches.

3. CONCLUSIONS

Our results indicate the possibility to apply the proposed algorithms for assessing the quality of test tasks on the base of the analysis of asymptotically steady distributions of students' scores for each control point, including the final test. In addition, the proposed algorithms give reasonable forecasts of the results achieved by students after passing the course checkpoints. This can be used for the intime development of recommendations to all participants of the educational process. This will allow, for example, to prevent the negative trends in academic performance, to ensure contingent saving, etc.

The concept of asymptotically steady distribution of student scores, introduced in the work, can also be used for clustering (grouping) courses by approaches to testing, and forming ratings for each group. In turn, the results of such clustering can be used to automate the procedures of courses' selection by potential students and heads of educational programs.

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