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Pranav Quasi Gamma Distribution: Properties and Applications

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ABSTRACT

We have developed Pranav Quasi Gamma Distribution (PQGD) as a mixture of Pranav distribution (θ) and Quasi Gamma distribution ($2, \theta$). We obtained various necessary statistical characteristics of PQGD. The flexibility of proposed model is clear from graph of hazard function. The reliability measures of proposed model are also obtained. Sample estimates of unknown parameters are obtained by making use of maximum likelihood estimation method. We have also carried out the simulation study for comparing our model with its related models. We then tested the significance of mixing parameter. Finally, applications to real-life data sets is presented to examine the significance of newly introduced model.

1. INTRODUCTION

Continuous efforts have been made by researchers for many years to bring more and more flexibility in fitting probability models to real-life data. Flexibility can be introduced by generalizing the classical probability models or by mixing the two probability models. Need of mixture models arise when the population or distribution from which the data is obtained is a genuine mixture of k distinct populations or distributions and our aim is to estimate the proportions $\left(p_1, p_2, \dots p_k\right)$ in which these k distinct populations in which these occur. As we deal mostly with the data obtained from two or more populations mixed in different proportions, so mixture models find greater applicability in fitting models to data. Mixture models also extract more variation from the data. Data analysts use mixture models to the complex data for better interpretation of results. Stacy [1] obtained generalized form of gamma model using power transformation of gamma distribution. Nadarajah *et al.* [2] obtained another generalized form of gamma model and applied it to various real-life situations. Shukla [3] obtained Pranav distribution by mixing gamma and exponential models in appropriate proportions and obtained its properties. Ghitany *et al.* [4] formulated Lindley distribution by mixture technique and studied its important properties. Shanker *et al.* [5] introduced a Quasi Gamma distribution and obtained its vital properties. Shanker and Shukla [6] obtained Ishita distribution by using mixture technique. Hassan *et al.* [7] obtained Lindley-Quasi Xgamma distribution and studied its important properties. Hassan, Wani and Shafi [8] introduced Poisson Pranav distribution and obtained its various mathematical properties along with obtaining applications of the proposed model. Hassan, Wani and Para [9] formulated three parameter Quasi Lindley distribution by using weighting technique and obtained various properties of that model. Shafi et al. [10] obtained properties and applications of Sanna distribution.

A continuous r. v X will have a mixture distribution if its p.d.f $f(x)$ is obtained as a mixture of k distinct populations having density functions $f_1(x), f_2(x), \dots f_k(x)$ and with mixing proportions $p_1, p_2, \dots p_k$ respectively. Mathematically

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \dots + p_k f_k(x)$$

where

$$0 \leq p_i \leq 1$$

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$$\sum_{i=1}^{k} p_i = 1$$

We have used mixture technique to obtain Pranav Quasi Gamma distribution (PQGD) in this paper.

2. PRANAV QUASI GAMMA DISTRIBUTION

A nonnegative r.v X would follow a PQGD if it will have p.d.f $f(x)$ which can be obtained as a mixture of Pranav (θ) having p.d.f $f_1(x)$ & Quasi Gamma distribution $(2, \theta)$ having p.d.f $f_2(x)$, in which θ is a scale parameter. Mathematically

$$f(x) = (1-p) f_2(x) + p f_1(x) \tag{1}$$

In Equation (1) p is a mixing parameter and

$$f_1(x) = \frac{\theta^4}{(\theta^4+6)} \left(\theta + x^3\right) e^{-\theta x} \quad x > 0, \theta > 0 \tag{2}$$

(2) is a p.d.f of Pranav (θ) distribution with the corresponding c.d.f $F_1(x)$ given below

$$F_1(x) = \left\{1 - \left[\frac{\left(3\theta x + 6 + \theta^2 x^2\right)\theta x}{(\theta^4+6)} + 1\right] e^{-\theta x}\right\} \tag{3}$$

And p.d.f of Quasi Gamma distribution $(2, \theta)$ is given in (4)

$$f_2(x) = 2\theta^2 e^{-\theta x^2} x^3 \quad \theta > 0, x > 0 \tag{4}$$

And corresponding c.d.f $F_2(x)$ of QGD is given below

$$F_2(x) = \left(-\Gamma\left(2, \theta x^2\right) + 1\right) \tag{5}$$

Substituting values of $f_1(x)$ & $f_2(x)$ in (1) we obtain p.d.f $f(x)$ of PQGD as given below

$$f(x) = \left[\frac{p\theta^4}{(\theta^4+6)} \left(\theta + x^3\right) e^{-\theta x} + 2(1-p)\theta^2 e^{-\theta x^2} x^3\right] \quad x > 0, \theta > 0, 0 \le p \le 1 \tag{6}$$

The graphs of p.d.f of PQGD are given in Figure 1a and 1b. These graphs show that for different parameter values indicating positively skewed nature of proposed model.

And the c.d.f of PQGD is found by using (3) and (5) and is given below

$$F(x) = \left[p\left(1 - \left[\frac{\left(\theta^2 x^2 + 3\theta x + 6\right)\theta x}{(\theta^4+6)} + 1\right] e^{-\theta x}\right) + (1-p)\left(1 - \Gamma\left(2, \theta x^2\right)\right)\right] \tag{7}$$

The above graphs represents the cumulative distribution function of PQGD.

3. RELIABILITY ANALYSIS

This different reliability measures are obtained in this particular area of paper. Expressions for survival function, failure rate and reverse failure rate of proposed PQGD are obtained in (8), (9), (10) respectively.

$$R(x) = 1 - \left\{p\left(1 - \left[\frac{\left(3\theta x + 6 + \theta^2 x^2\right)\theta x}{(\theta^4+6)} + 1\right] e^{-\theta x}\right) + (1-p)\left(1 - \Gamma\left(2, \theta x^2\right)\right)\right\} \tag{8}$$

$$h(x) = \frac{\left[p\theta^4\left(\theta + x^3\right) e^{-\theta x} + 2\left(\theta^4+6\right)\left(1-p\right)\theta^2 e^{-\theta x^2} x^3\right]}{(\theta^4+6) - \left\{p\left((\theta^4+6) - [(\theta^4+6) + \theta x(\theta^2x^2 + 3\theta x + 6)] e^{-\theta x}\right) + (1-p)(\theta^4+6)(1 - \Gamma(2, \theta x^2))\right\}} \tag{9}$$

$$R.H.R = h_r(x) = \frac{\left[p\theta^4\left(\theta + x^3\right) e^{-\theta x} + 2(\theta^4+6)\left(1-p\right)\theta^2 e^{-\theta x^2} x^3\right]}{\left\{p\left((\theta^4+6) - [(\theta^4+6) + \theta x(\theta^2x^2 + 3\theta x + 6)] e^{-\theta x}\right) + (1-p)(\theta^4+6)(1 - \Gamma(2, \theta x^2))\right\}} \tag{10}$$

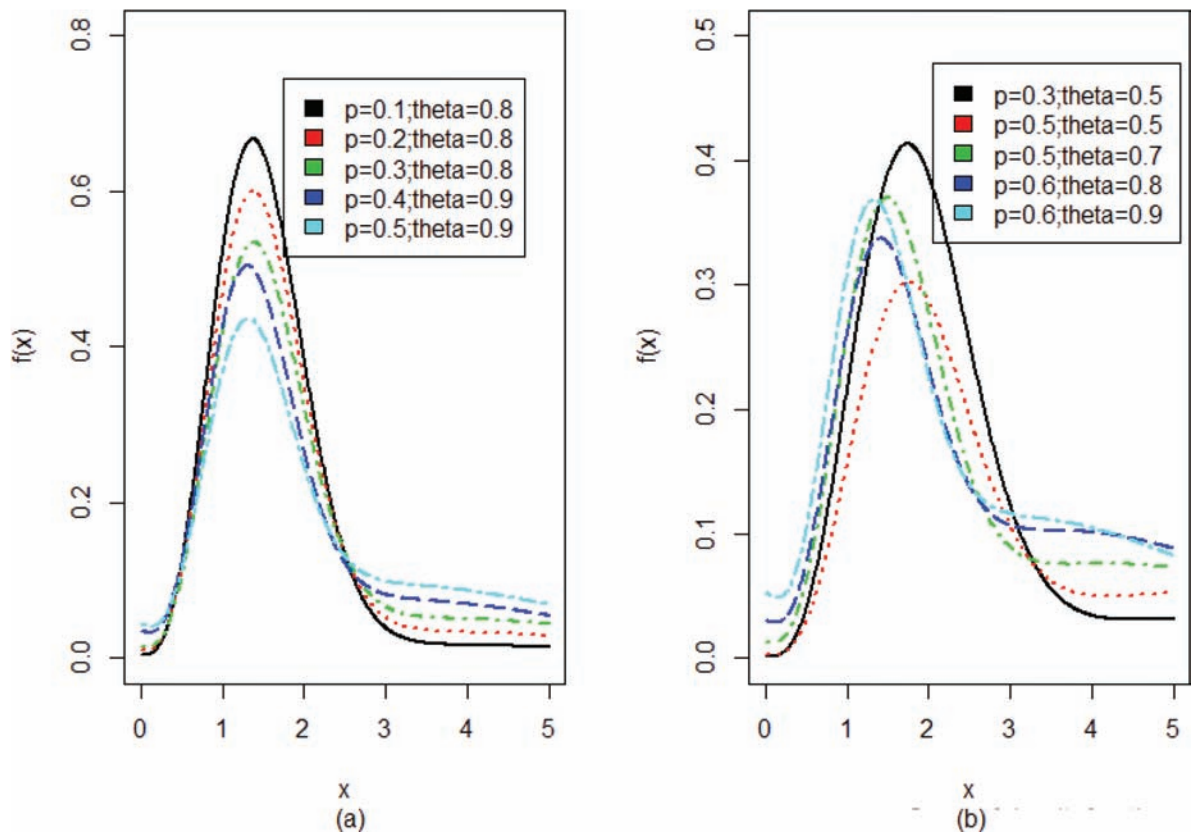

Figure 1 | Graph of density function.

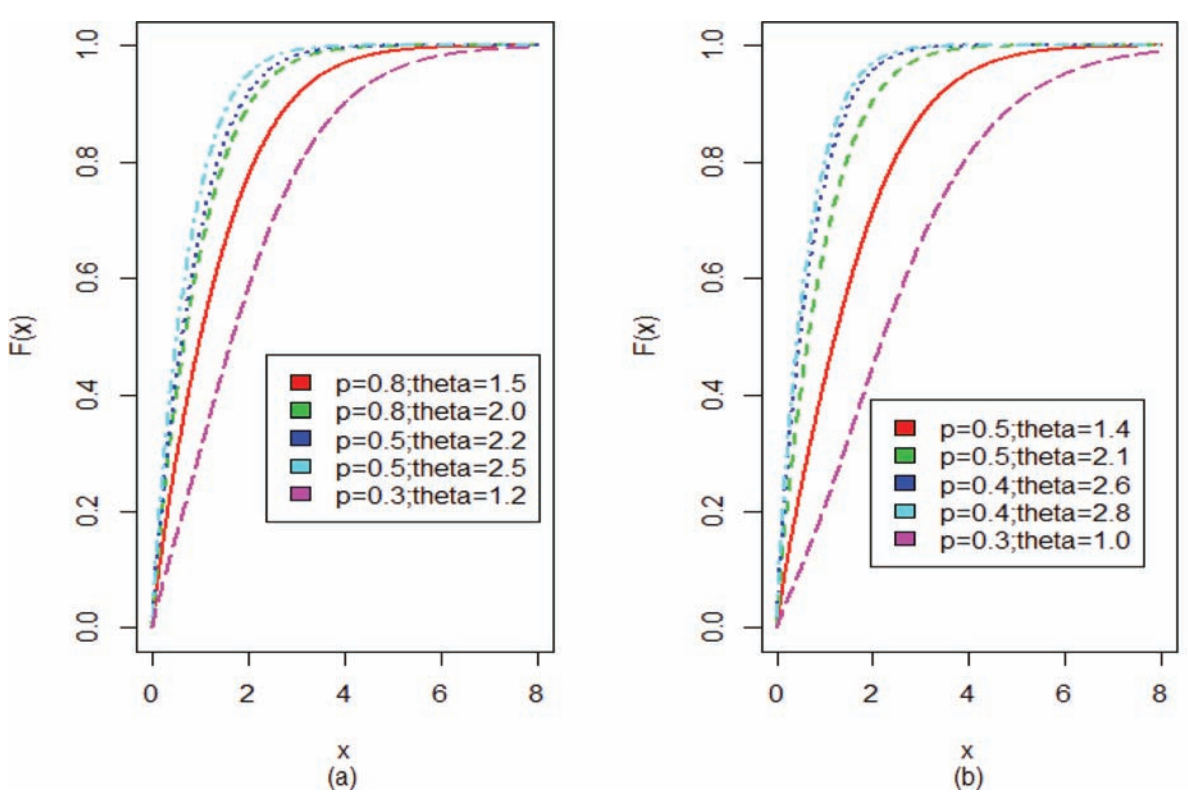

Figure 2 | Graph of distribution function.

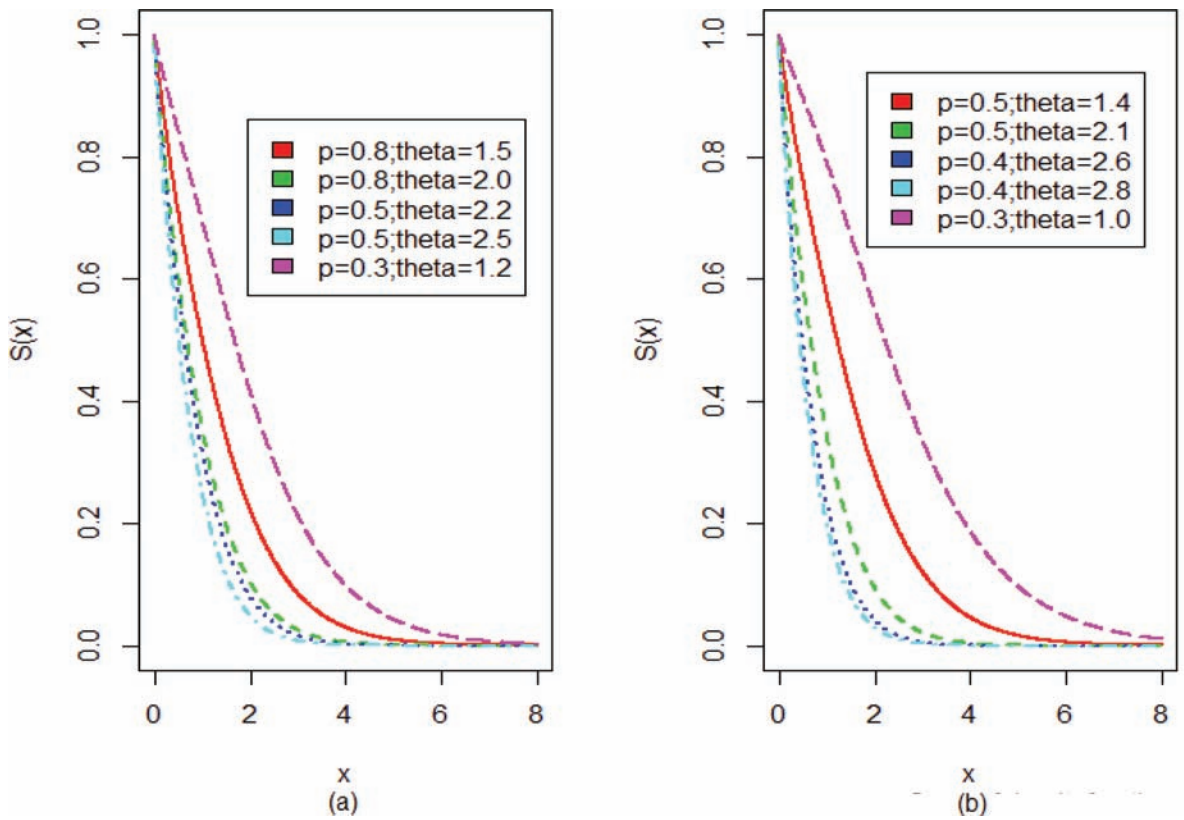

Figure 3 Graph of survival function.

Figure 3a and 3b represent the survival function of PQGD. Figure 4 represents the hazard rate of PQGD which shows the flexibility of proposed model as the graph is inverted bathtub shaped. The hazard rate is of monotonic increasing as well as monotonic decreasing nature which shows the flexibility of proposed model.

4. STATISTICAL PROPERTIES

Moments, mean deviation about mean, median characterize probability model among other properties. Here we have obtained these statistical properties for our proposed Pranav Quasi Gamma model.

4.1. Moments

Assuming X being a r.v having PQGD (θ, p). We now know that kth moment about origin of PQGD is given as below

$$\mu_k' = E\left(X^k\right) = \int_0^\infty x^k f\left(x, \theta, p\right) dx$$
$$= \int_0^\infty x^k \left[\frac{p\theta^4}{(\theta^4+6)}(\theta + x^3)e^{-\theta x} + 2(1-p)\theta^2 e^{-\theta x^2} x^3\right] dx$$

$$\mu_k' = \frac{k!}{\theta^k(2+\theta\alpha)^2}\left[\frac{pk!\,(k+3)(\theta^4+(k+1)(k+2)}{\theta^k(\theta^4+6)} + \frac{(1-p)\left(\Gamma\left(\frac{4+k}{2}\right)\right)}{\theta^{\frac{k}{2}}}\right] \tag{11}$$

Put $k = 1$ in Equation (11) we get

$$\mu_1' = \left[\frac{p\left(\theta^4+24\right)}{\theta\left(\theta^4+6\right)} + \frac{(1-p)\left(\Gamma\left(\frac{5}{2}\right)\right)}{\theta^{\frac{1}{2}}}\right]$$

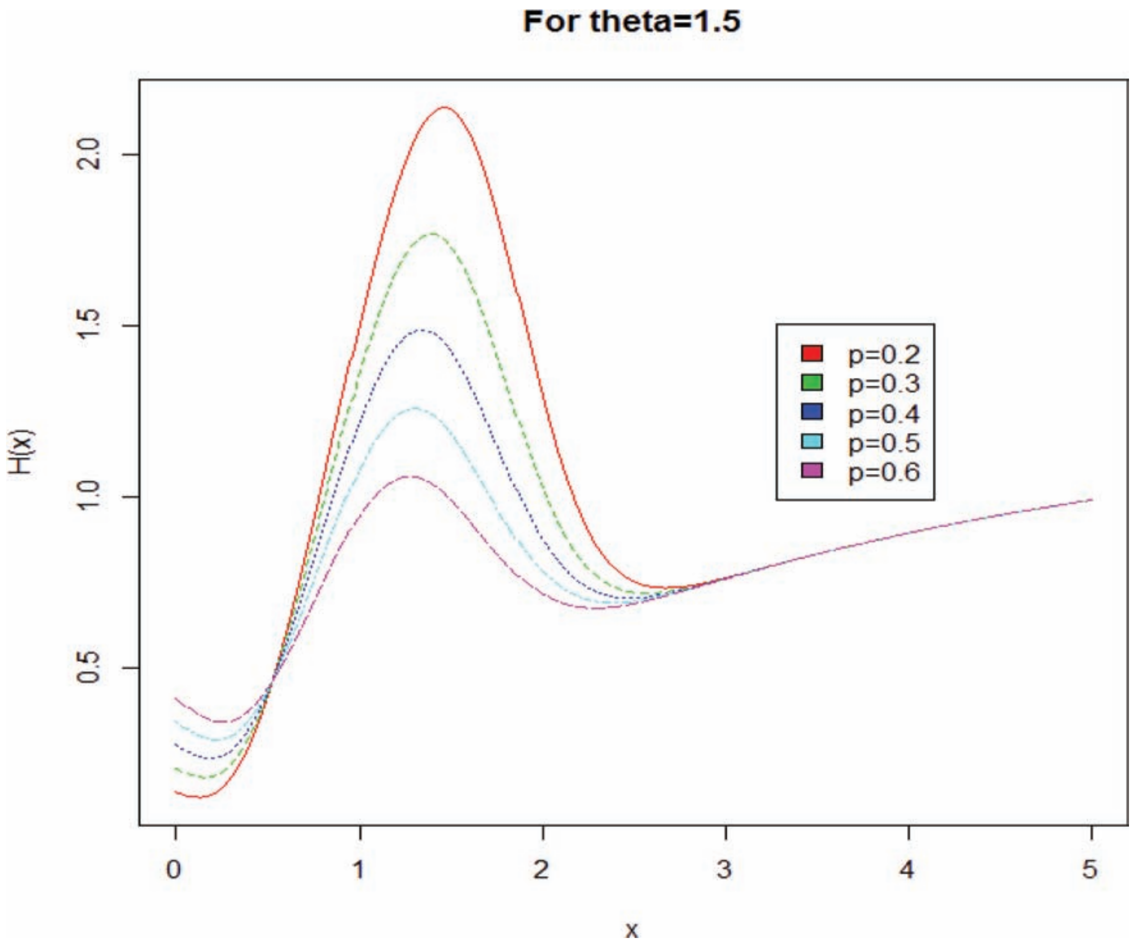

Figure 4 Graph of hazard function.

which is mean of the PQGD

Put $k = 2$ in Equation (11) we get

$$\mu_2' = \left[\frac{2p\left(\theta^4 + 60\right)}{\theta^2\left(\theta^4 + 6\right)} + \frac{(1-p)\left(\Gamma(3)\right)}{\theta}\right]$$

And variance of PQGD is

$$V(x) = \left[\frac{p\left(\theta^8 + 84\theta^4 + 144\right)}{\theta^2\left(\theta^4 + 6\right)^2} + \frac{(1-p)\left(2 - \left(\Gamma\left(\frac{5}{2}\right)\right)^2\right)}{\theta}\right]$$

4.2. Average Deviation About Median and Arithmetic Mean of PQGD

We have derived the expressions for average deviation about median & arithmetic mean of PQGD in this area of paper.

Theorem 1: *If r.v X follows PQGD* (θ, p), *then average deviation about median* $(\delta_2(x))$ *and arithmetic mean* $(\delta_1(x))$

$$\delta_1(X) = \left[\begin{array}{l} 2\mu\left\{p\left(1 - \left(1 + \frac{\theta\mu\left((\theta\mu)^2 + 3\theta\mu + 6\right)}{(\theta^4 + 6)}\right)e^{-\theta\mu}\right) + (1-p)\left(1 - (\Gamma(2, \theta\mu^2)\right)\right\} \\ -2\left\{\frac{p}{\theta\left(\theta^4 + 6\right)}\left(\theta^4\gamma(2, \mu) + \gamma(5, \mu)\right) + \frac{(1-p)}{\theta^{\frac{1}{2}}}\gamma\left(\frac{5}{2}, \mu\right)\right\} \end{array}\right]$$

And

$$\delta_2(X) = \left[\mu - 2\left\{\frac{p}{\theta\left(\theta^4+6\right)}\left(\theta^4\gamma(2,M)+\gamma(5,M)\right) + \frac{(1-p)}{\theta^{\frac{1}{2}}}\gamma\left(\frac{5}{2},M\right)\right\}\right]$$

respectively.

Proof: Average deviation about median ($\delta_2(x)$) & arithmetic mean ($\delta_1(x)$) are well defined as

$$\delta_1(X) = \left(\int_0^\infty |x-\mu| f(x)dx\right)$$

$$\& \ \delta_2(X) = \left(\int_0^\infty |x-M| f(x)dx\right)$$

respectively.

where μ and M are arithmetic mean and median of PQGD respectively. The measures $\delta_1(X)$ & $\delta_2(X)$ are obtained by making use of the simplified relations given below.

$$\delta_1(X) = \int_0^\mu (\mu - x) f(x)dx + \int_\mu^\infty (x-\mu) f(x)dx$$

$$\delta_1(X) = 2\mu F(\mu) - 2\int_0^\mu x f(x)dx \tag{12}$$

and

$$\delta_2(X) = \int_M^\infty (x-M) f(x)dx + \int_0^M (M-x) f(x)dx$$

$$\delta_2(X) = \mu - 2\int_0^M x f(x)dx \tag{13}$$

Using (6) we obtain

$$\int_0^\mu x f(x)dx = \left[\frac{p}{\theta\left(\theta^4+6\right)}\left(\theta^4\gamma(2,\mu)+\gamma(5,\mu)\right) + \frac{(1-p)}{\theta^{\frac{1}{2}}}\gamma\left(\frac{5}{2},\mu\right)\right] \tag{14}$$

$$\int_0^M x f(x)dx = \left[\frac{p}{\theta\left(\theta^4+6\right)}\left(\theta^4\gamma(2,M)+\gamma(5,M)\right) + \frac{(1-p)}{\theta^{\frac{1}{2}}}\gamma\left(\frac{5}{2},M\right)\right] \tag{15}$$

where $\gamma(s,x) = \int_0^x t^{s-1}e^{-t}dt$ is an incomplete gamma function.

$$\gamma(2,\mu) = \int_0^\mu t^{2-1}e^{-t}dt$$

$$\gamma\left(\frac{5}{2},\mu\right) = \int_0^\mu t^{\frac{5}{2}-1}e^{-t}dt$$

and $\gamma(5,\mu)=\int_0^{\mu} t^{5-1}e^{-t}dt$

Using expressions (12), (13), (14), and (15) and expression for c.d.f (7) we obtain average deviation about median ($\delta_2(x)$)&and arithmetic mean ($\delta_1(x)$)

$$\delta_1(X)=\left[\begin{array}{l} 2\mu\left\{p\left(1-\left(1+\dfrac{\theta\mu((\theta\mu)^2+3\theta\mu+6)}{(\theta^4+6)}\right)e^{-\theta\mu}\right)+(1-p)(1-\left(\Gamma\left(2,\theta\mu^2\right)\right)\right\} \\ -2\left\{\dfrac{p}{\theta\left(\theta^4+6\right)}\left(\theta^4\gamma(2,\mu)+\gamma(5,\mu)\right)+\dfrac{(1-p)}{\theta^{\frac{1}{2}}}\gamma\left(\dfrac{5}{2},\mu\right)\right\} \end{array}\right]$$

$$\delta_2(X)=\left[\mu-2\left\{\dfrac{p}{\theta\left(\theta^4+6\right)}\left(\theta^4\gamma(2,M)+\gamma(5,M)\right)+\dfrac{(1-p)}{\theta^{\frac{1}{2}}}\gamma\left(\dfrac{5}{2},M\right)\right\}\right]$$

5. ORDER STATISTICS OF PQGD

Assuming $X_{(1)},X_{(2)},X_{(3)}....,X_{(n)}$ being an order statistics for the random sample $x_1,x_2,x_3,....x_n$ obtained from PQGD having c.d.f $F\left(x,\theta,p\right)$ and p.d.f $f\left(x,\theta,p\right)$, then the p.d.f of vth order statistics $X_{(v)}$ is given by: $f_v\left(x,\theta,p\right)=\frac{n!}{(v-1)!(n-v)!}f\left(x,\theta,p\right)\left[F\left(x,\theta,p\right)\right]^{v-1}\left[1-F\left(x,\theta,p\right)\right]^{n-v}\quad v=1,2,\ldots n$

Using the Equations (6) and (7), the probability density function of vth order statistics of PQGD is specified as

$$f_{(v)}\left(x,\theta,p\right)=\left[\begin{array}{l} \dfrac{n!}{(v-1)!\,(n-v)!}\left[\dfrac{p\theta^4}{(\theta^4+6)}\left(\theta+x^3\right)e^{-\theta x}+2(1-p)\theta^2e^{-\theta x^2}x^3\right] \\ \left[p\left(1-\left[1+\dfrac{\theta\left(\theta^2x^2+3\theta x+6\right)}{(\theta^4+6)}\right]e^{-\theta x}\right)+(1-p)\left(1-\Gamma\left(2,\theta x^2\right)\right)\right]^{v-1} \\ \left[1-p\left(1-\left[1+\dfrac{\theta\left(\theta^2x^2+3\theta x+6\right)}{(\theta^4+6)}\right]e^{-\theta x}\right)+(1-p)\left(1-\Gamma\left(2,\theta x^2\right)\right)\right]^{n-v}. \end{array}\right]$$

Then, the p.d.f of first order statistic $X_{(1)}$ of of PQGD is specified as

$$f_{(1)}\left(x,\theta,p\right)=\left[\begin{array}{l} n\left[\dfrac{p\theta^4}{(\theta^4+6)}\left(\theta+x^3\right)e^{-\theta x}+2(1-p)\theta^2e^{-\theta x^2}x^3\right] \\ \left[1-p\left(1-\left[1+\dfrac{\theta\left(\theta^2x^2+3\theta x+6\right)}{(\theta^4+6)}\right]e^{-\theta x}\right)+(1-p)\left(1-\Gamma\left(2,\theta x^2\right)\right)\right]^{n-1}. \end{array}\right]$$

and the p.d.f of nth order statistic $X_{(n)}$ of of PQGD is specified as

$$f_{(n)}\left(x,\theta,p\right)=\left[\begin{array}{l} n\dfrac{p\theta^4}{(\theta^4+6)}\left(\theta+x^3\right)e^{-\theta x}+2(1-p)\theta^2e^{-\theta x^2}x^3 \\ \left[p\left(1-\left[1+\dfrac{\theta\left(\theta^2x^2+3\theta x+6\right)}{(\theta^4+6)}\right]e^{-\theta x}\right)+(1-p)\left(1-\Gamma\left(2,\theta x^2\right)\right)\right]^{n-1}. \end{array}\right]$$

6. ESTIMATION OF PARAMETERS OF PQGD

Assuming $X_1,X_2,X_3.....X_n$ being a randomly selected sample of size n obtained from PQGD having density function given by (2.6), then the likelihood function of PQGD is given as

$$L\left(x|\theta,p\right)=\prod_{i=1}^{n}\left[\dfrac{p\theta^4}{(\theta^4+6)}\left(\theta+x_i^3\right)e^{-\theta x_i}+2(1-p)\theta^2e^{-\theta x_i^2}x_i^3\right]$$

The log-likelihood function becomes

$$\log L = \left\{ 2n\log\theta - n\log\left(\theta^4+6\right) + \sum_{i=1}^{n}\left[\log\left(p\theta^2\left(\theta+x_i^3\right)e^{-\theta x_i} + 2\left(\theta^4+6\right)(1-p)e^{-\theta x_i^2}x_i^3\right)\right]\right\} \tag{16}$$

By partially differentiating (16) w. r. to θ and p and then equating the outcome to zero, we get the resulting normal equations specified below

$$\frac{\partial \log L}{\partial\theta} = \left[\frac{2n}{\theta} - \frac{4n\theta^3}{(\theta^4+6)} + \sum_{i=1}^{n}\left\{\frac{p\left(\left(3\theta^2+2\theta x_i^3\right)e^{-\theta x_i} - \left(\theta^3+\theta^2 x_i^3\right)x_i e^{-\theta x_i}\right) + 2(1-p)x_i^3\left(4\theta^3 e^{-\theta x_i^2} - x_i^2 e^{-\theta x_i^2}\left(\theta^4+6\right)\right)}{\left(p\theta^2\left(\theta+x_i^3\right)e^{-\theta x_i} + 2\left(\theta^4+6\right)(1-p)e^{-\theta x_i^2}x_i^3\right)}\right\}\right] = 0 \tag{17}$$

$$\frac{\partial \log L}{\partial p} = \left[\sum_{i=1}^{n}\left\{\frac{\left(\theta^2\left(\theta+x_i^3\right)e^{-\theta x_i} - 2\left(\theta^4+6\right)e^{-\theta x_i^2}x_i^3\right)}{\left(p\theta^2\left(\theta+x_i^3\right)e^{-\theta x_i} + 2\left(\theta^4+6\right)(1-p)e^{-\theta x_i^2}x_i^3\right)}\right\}\right] = 0 \tag{18}$$

MLEs of θ, p cannot be obtained by solving above complex equations as these equations are not in closed form. So we solve above equations by using iteration method through R software.

7. SIMULATION STUDY

We have generated a data of 50 observations through R software by using inverse c.d.f technique from proposed model and we have obtained loss of information values AIC, BIC, AICC, and HQIC values for our model and its related models. We have also obtained the Shannon's entropy of our model and its related models. For testing the significance of mixing parameter p we used likelihood ratio (LR) statistic. In Table 1 estimates of parameters of fitted models along with model functions are given.

In order to test the statistical significance of mixing parameter p for proposed PQGD we computed LR statistic by testing $H_0 : p = 0$ against $H_1 : p \neq 0$ using LR statistic $\omega = 2\left\{L\left(\hat{\Theta}\right) - L\left(\hat{\Theta}_0\right)\right\} = 23.01$, where $\hat{\Theta}$ and $\hat{\Theta}_0$ are MLEs under H_1 and H_0. LR statistic $\omega \sim \left(\chi^2_{(1)}(\alpha = 0.01) = 6.635\right)$ as $n \to \infty$, d.f being the difference of dimensionality. From Table 2 $\omega = 23.01 > 6.635$ at 1% significance level, hence we rejected H_0 and conclude that mixing parameter p plays statistically a significant role.

Table 1 | ML estimates with standard errors in parenthesis, model function of proposed model, and its related models for simulated data of 50 observations.

Distribution	**Parameter Estimates**	**Model Function**
Quasi Gamma (QGD)	$\hat{\theta} = 3.176(0.317)$	$2\theta^2 e^{-\theta x^2}x^3$
Pranav (PD)	$\hat{\theta} = 2.217(0.156)$	$\frac{\theta^4}{(\theta^4+6)}\left(\theta+x^3\right)e^{-\theta x}$
Pranav Quasi Gamma (PQGD)	$\hat{\theta} = 2.978(0.309)$ $\hat{p} = 0.1222(0.068)$	$\frac{p\theta^4}{(\theta^4+6)}\left(\theta+x^3\right)e^{-\theta x} + 2(1-p)\theta^2 e^{-\theta x^2}x^3$

Table 2 | Model comparison and likelihood ratio statistic of proposed model and its related models.

Distribution	**− log *L***	**AIC**	**BIC**	**AICC**	**HQIC**	**Shanon Entropy *H(X)***	**Likelihood Ratio**
PQGD	11.67180	27.3436	31.167	27.598	28.799	0.233	23.01
QGD	23.1788	48.3576	50.2696	48.440	48.44094	0.46	
PD	41.09660	84.1932	86.105	84.276	84.921	0.82	

Loss of information criteria's like AIC, BIC, AICC, and HQIC are computed along with measure of average uncertainty that is Shannon entropy *H(X)* for comparison of models fitted to data.

$$\text{AIC} = 2\text{v} - 2\log L \qquad \text{AICC} = \text{AIC} + \frac{2v(v+1)}{f-v-1}$$
$$\text{BIC} = \text{v}\log f - 2\log L \qquad \text{HQIC} = 2\text{v}\ \log\left(\log\left(f\right)\right) + 2\log L$$
$$H(X) = -\frac{\log L}{f}$$

where v gives count of parameters in the statistical model, f represents the sample size, and $-2\log L$ shows the maximized value of the log-likelihood function. From Table 2, it is observed that the PQGD possesses the lesser AIC, AICC, BIC, and HQIC and *H(X)*values as compared to QGD and PD for simulated data. Hence we can conclude that the PQGD gives much better fit as compared to QGD and PD for simulated data.

8. SPECIAL CASES

Case I: By putting $p = 0$, then PQGD (6) reduces to Quasi Gamma distribution with p.d.f as

$$f_2(x) = 2\theta^2 e^{-\theta x^2} x^3 \quad x > 0, \theta > 0$$

Case II: By putting $p = 1$, PQGD (6) reduces to Pranav distribution with p.d.f as

$$.f_1(x) = \frac{\theta^4}{(\theta^4 + 6)}\left(\theta + x^3\right)e^{-\theta x} \quad x > 0, \theta > 0$$

9. APPLICATIONS OF PQGD

We fitted PQGD and its related distributions to two real-life data sets and showed that our proposed model fits well to these data sets as compared to its related models.

Data Set 1: The data set given in Table 3 has been taken from Kotz and Johnson [11] and represents the survival times (in years) after diagnosis of 43 patients with some kind of leukemia.

Data set 2: This data set given in Table 4 represents the Kevlar 49/epoxy strands failure times data (pressure at 90%) is taken from Makubate *et al.* [12].

Table 3 | Survival times (in years) after diagnosis of 43 patients with a certain kind of leukemia.

0.019	**0.129**	**0.159**	**0.203**	**0.485**	**0.636**	**0.748**	**0.781**
0.869	1.175	1.206	1.219	1.219	1.282	1.356	1.362
1.458	1.564	1.586	1.592	1.781	1.923	1.959	2.134
2.413	2.466	2.548	2.652	2.951	3.038	3.6	3.655
3.745	4.203	4.690	4.888	5.143	5.167	5.603	5.633
6.192	6.655	6.874					

Table 4 | Kevlar 49/epoxy strands failure times data (pressure at 90%).

0.01	**0.01**	**0.02**	**0.02**	**0.02**	**0.03**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**
0.07	0.08	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.13	0.18
0.19	0.20	0.23	0.24	0.24	0.29	0.34	0.35	0.36	0.38	0.40
0.42	0.43	0.52	0.54	0.56	0.60	0.60	0.63	0.65	0.67	0.68
0.72	0.72	0.72	0.73	0.79	0.79	0.80	0.80	0.83	0.85	0.90
0.92	0.95	0.99	1.00	1.01	1.02	1.03	1.05	1.10	1.10	1.11
1.15	1.18	1.20	1.29	1.31	1.33	1.34	1.40	1.43	1.45	1.50
1.51	1.52	1.53	1.54	1.54	1.55	1.58	1.60	1.63	1.64	1.80
1.80	1.81	2.02	2.05	2.14	2.17	2.33	3.03	3.03	3.34	4.20
4.69	7.89									

R software version 3.5.3 [13] is used for analyzing the data. We have fitted QGD, PD, GD, WD, and PQGD to the data sets 1 and 2. The summary statistics of data sets 1 and 2 are displayed in Tables 5 and 6, MLEs of the parameters with standard errors in parenthesis, model functions are displayed in Table 7 and log-likelihood values, LR statistic, AIC, AICC, BIC, HQIC, and Shannon's entropy are displayed in Tables 8 and 9 respectively.

In order to test the statistical significance of mixing parameter p for proposed PQGD we computed LR statistic by testing $H_0 : p = 0$ against $H_1 : p \neq 0$ using LR statistic $\omega_1 = 2\left\{L\left(\hat{\Theta}\right) - L\left(\hat{\Theta}_0\right)\right\} = 101.188$ for data set 1 and $\omega_2 = 2\left\{L\left(\hat{\Theta}\right) - L\left(\hat{\Theta}_0\right)\right\} = 506.18$ for data set 2 where $\hat{\Theta}$ and $\hat{\Theta}_0$ are MLEs under H_1 and H_0. LR statistic $\omega \sim \left(\chi^2_{(1)}(\alpha = 0.01) = 6.635\right)$ as $n \to \infty$, d.f being the difference of dimensionality. From Table 8 $\omega_1 = 101.188 > 6.635$ and from Table 9 $\omega_2 = 506.18 > 6.635$ at 1% significance level, hence we rejected H_0 and conclude that mixing parameter p plays statistically a significant role for both the data sets.

Table 5 Summary statistics of data set 1.

Number of Observations	Minimum	First Quartile	Median	Mean	Third Quartile	Maximum
43	0.019	1.212	1.923	2.534	3.700	6.874

Table 6 Summary statistics of data set 2.

Number of Observations	Minimum	First Quartile	Median	Mean	Third Quartile	Maximum
101	0.010	0.240	0.800	1.025	1.450	7.890

Table 7 ML estimates with standard errors in parenthesis, model function of proposed model, and its related models for data set 1 and 2.

Distribution	Parameter Estimates		Model Function
	Data Set 1	Data Set 2	
Quasi Gamma Distribution (QGD)	$\hat{\theta} = 0.199032\,(0.021465)$	$\hat{\theta} = 0.8730(0.0614)$	$2\theta^2 e^{-\theta x^2} x^3$
Pranav Distribution (PD)	$\hat{\theta} = 1.244312\,(0.075493)$	$\hat{\theta} = 1.8976(0..0821)$	$\frac{\theta^4}{(\theta^4+6)}\left(\theta + x^3\right) e^{-\theta x}$
Pranav Quasi Gamma Distribution (PQGD)	$\hat{\theta} = 1.143384\,(0.093793)$ $\hat{p} = 0.76297\,(0.11982019)$	$\hat{\theta} = 1.8784(0.0878)$ $\hat{p} = 0.7782(0.0870)$	$\frac{p\theta^4}{(\theta^4+6)}\left(\theta + x^3\right) e^{-\theta x} + 2(1-p)\theta^2 e^{-\theta x^2} x^3$
Gamma Distribution (GD)	$\hat{\alpha} = 1.3017130\,(0.2527684)$ $\hat{\beta} = 1.946648\,(0.4588967)$	$\hat{\alpha} = 0.8718\,(0.10669)$ $\hat{\beta} = 1.17547\,(0.19074)$	$\frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma\alpha}$
Weibull distribution (WD)	$\hat{\lambda} = 2.702256\,(0.3482947)$ $\hat{\beta} = 1.2397431\,(0.1532526)$	$\hat{\lambda} = 0.98994\,(0.111770)$ $\hat{\beta} = 0.92588\,(0.072598)$	$\frac{\beta}{\lambda}\left(\frac{x}{\lambda}\right)^{\beta-1} e^{-(x/\lambda)^\beta}$

Table 8 Model comparison and Likelihood ratio statistic of proposed model and its related models for data set 1.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	Shanon Entropy *H(X)*	Likelihood Ratio
PQGD	80.07714	164.1543	167.6767	164.4543	165.4532	1.862	101.188
QGD	130.6714	263.3428	265.104	263.4404	263.9923	3.038	
PD	82.17808	166.3562	168.1174	166.4537	167.0056	1.911	
GD	82.12227	168.2445	171.7669	168.5445	169.5435	1.909	
WD	81.61015	167.2203	170.7427	167.5203	168.5192	1.897	

Table 9 Model comparison and Likelihood ratio statistic of proposed model and its related models for data set 2.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	Shanon Entropy *H(X)*	Likelihood Ratio
PQGD	102.8728	209.7456	214.9758	209.868	211.8629	1.018	506.18
QGD	355.9660	713.9321	716.5472	713.9725	714.9907	3.52	
PD	106.3711	214.7422	217.3573	214.7826	215.8008	1.05	
GD	102.9827	209.9655	215.2048	210.2421	212.2049	1.020	
WD	102.9768	209.9536	215.1839	210.0761	212.1071	1.019	

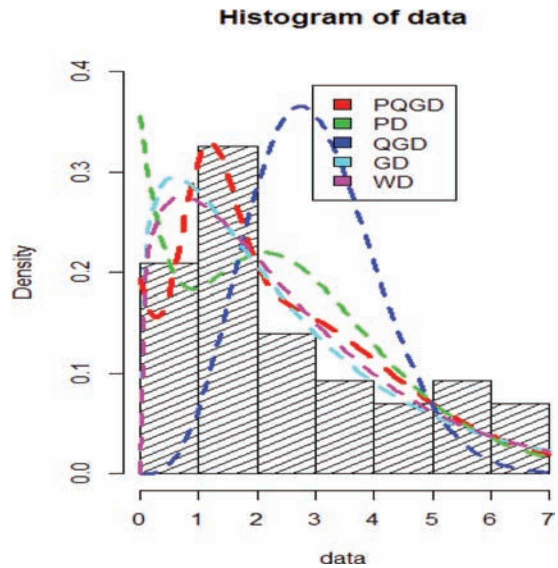

Figure 5 Curve fitting of data set 1.

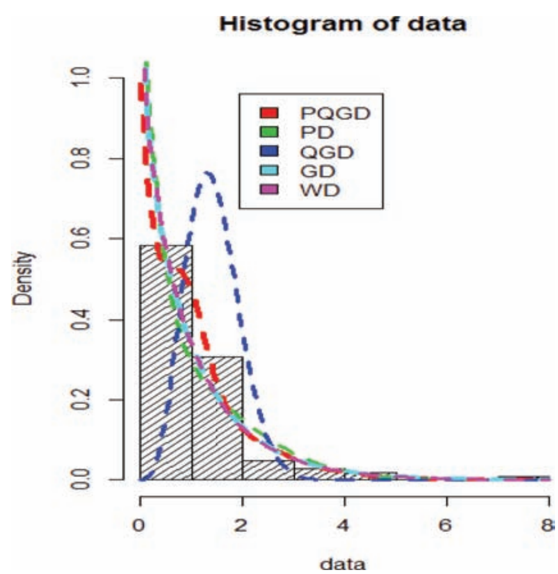

Figure 6 Curve fitting of data set 1.

Loss of information criteria's like AIC, BIC, AICC, and HQIC are computed along with measure of average uncertainty that is Shannon entropy *H(X)* for comparison of models fitted to data.

$$\text{AIC} = 2\text{v} - 2\log L \qquad \text{AICC} = \text{AIC} + \frac{2v(v+1)}{f-v-1}$$

$$\text{BIC} = \text{v}\log f - 2\log L \qquad \text{HQIC} = 2\text{v}\ \log\left(\log\left(f\right)\right) + 2\log L$$

$$H(X) = -\frac{\log L}{f}$$

where v gives count of parameters in the statistical model, f represents the sample size, and $-2\log L$ shows the maximized value of the log-likelihood function. From Tables 8 and 9, it is observed that the PQGD possesses the lesser AIC, AICC, BIC, HQIC, and *H(X)*values as compared to QGD, GD, WD, and PD for both the data sets 1 and 2. Hence we can conclude that the PQGD leads to a better fit than the QGD, GD, WD, and PD for data sets 1 and 2.

10. CONCLUSION

We incorporated PQGD as a mixture of Pranav distribution and Quasi Gamma distribution. We obtained crucial properties of our proposed model. We carried out the simulation study and showed superiority of our model over its related models. We also obtained the estimates of our proposed model by using maximum likelihood method of estimation. Significance of mixing parameter has been tested. Finally we fitted our model and its related models to two real-life data sets and concluded that our model gives better fit to these data sets as compared to its related models.

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CONFLICTS OF INTEREST

All the authors have no conflict of interest.

AUTHORS' CONTRIBUTIONS

All the authors contributed equally.

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