

# Hypothetical Learning Trajectory for First-Order Ordinary Differential Equations

Yarman<sup>1,\*</sup>, Ahmad Fauzan<sup>2</sup>, Armiati<sup>3</sup>, Lufri<sup>4</sup>

<sup>1,2,3,4</sup> Doctoral Program Of Education, Universitas Negeri Padang

\* Corresponding author. Email: [yarman@fmipa.unp.ac.id](mailto:yarman@fmipa.unp.ac.id)

## ABSTRACT

This research is a design research with the aim of designing student learning trajectories in first-order ordinary differential equations learning. There are three phases to be carried out in design research, namely the Preliminary Phase; Prototyping Phase; and Assessment Phase. However, the focus of the present research is only until the second stage, especially the expert validation stage, because the main objective of this research is to develop a learning trajectory for first-order ordinary differential equations. The results of this study are in the form of a hypothetical learning trajectory (HLT) designed using a realistic mathematics education (RME) approach. Based on the characteristics of RME, learning will begin with contextual problems. One of the contextual problems used in designing this HLT is the problem of population growth in Indonesia. In this case, students are invited to start by making a list of the factors that can affect the increase and decrease in population according to their understanding in everyday life. With the help of using iceberg, the learning process will move from horizontal mathematical to vertical mathematics which ends in the discovery of the first order differential equation model of the contextual problem

**Keywords:** *Ordinary Differential Equation, hypothetical learning trajectory, design research, iceberg, horizontal mathematics, and vertical mathematics.*

## 1. INTRODUCTION

Ordinary differential equation (ODE) is a branch of mathematics grouped under applied mathematics. This ODE has many applications in real life. Apart from having applications in the field of mathematics itself, it also has a fairly broad application in various other disciplines. For examples, its application in biology, physics, chemistry, engineering, medicine, economics, and so on [1].

The role of ODE is also very important in modern technology to solve various real life problems. However, ironically, researchers encounter that there are still many problems arise on ODE learning in several universities. Among these problems are (1) even though the students succeeded in getting the algebraic solution of ODE, they did not understand the interpretation of the solution. (2) students who have passed the ODE course cannot apply their knowledge to the related advanced level courses. (3) the structure of ODE textbooks used is more abstract and procedural. This is not in accordance with the ODE

Observing the problems that occur, the researcher argues that the RME approach has the potential to overcome these problems, because RME aims to change

course as an applied subject that contributes a lot in real life [2].

Furthermore, from the experience while developing ODE courses, teaching tends to follow the flow of material presentation as in text books. Learning begins with the activity of conveying information accompanied by sample questions, which is then continued by asking students to work on several questions [3]. The available text books do not contain a learning flow that can lead students to reinvention their own concepts. The existing textbooks are mostly abstract and procedural in nature, and prioritize analytical calculations only.

Another problem, the frequent teaching method used is chalk and talk. As a result, most students think that the mathematical concepts studied in ODE are very difficult to understand because they are not related to real life. This is contradictory to mathematics as human activity and learning mathematics as doing mathematics [4].

mathematics education in such a way that most students will be able to do and enjoy mathematics (in this case ODE), solve math problems, and develop knowledge

and math skills [5]. ODE learning with the RME approach is focused on how ODE material is taught and how students learn ODE in the classroom in a meaningful way. To achieve this goal, it is necessary to develop an HLT based on the RME approach

2. METHOD

This research is a design research that aims to design HLT for first-order ODE learning. HLT contains three components, namely: learning objectives, learning activities, as well as a hypothetical learning process or an alleged learning process [6]. In designing this HLT, the RME approach was used. For this reason, the contextual problem used in designing HLT is a problem of population growth. Guided by iceberg, several learning trajectories were made in the hope that students could find their own first-order ODE model from these contextual problems.

3. RESULT AND DISCUSSION

Making iceberg is the first step in making HLT using the RME principle. Iceberg contains three main principles in RME, namely: (1) progressive mathematizing, (2) didactic phenomenology, and (3) making your own model [7]. The problem to be addressed is how students can construct a first-order ODE model from the population growth problem by utilizing the HLT to be designed.. Population growth is a change in population over time, and can be calculated as the change in the number of individuals in a population using "per unit time" for measurement. The term population growth refers to all species, but in specific it usually refers to humans, and is often used informally as the demographic value of population growth, and is used to refer to world population growth.

To get a mathematical model of a real problem in the form of first-order ODE, it is very important to know what variables are associated with the model. Furthermore, in order to support students in constructing their own model, contextual problems about population growth is raised in the learning. Students are directed to construct their own "first-order ODE model of population growth problems" through several learning activities. Learning activities are designed using the RME approach which begins with the Iceberg form of the given problems. Below is a form of solution in designing icebergs in the first-order ODE model of population growth problems.

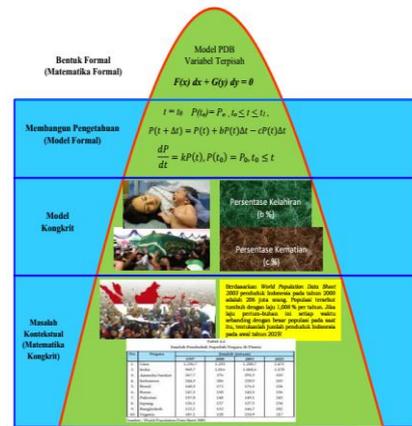


Figure 1. Iceberg Differential Equation Model of the Population Growth Problem

Based on the iceberg, there are three forms of activities carried out by students during the learning process, namely (1) listing the factors that cause population growth, (2) stating the relationship between population growth factors in mathematical sentences, and (3) constructing a first-order differential equation model from population growth problem.

In the first activity, students were asked to list the factors that can increase and decrease the population. f the results of group discussions conducted by students do not cover all the expected factors, such as: birth, health care during pregnancy, attitudes towards the availability of contraceptives, attitudes towards abortion, and immigration, as well as factors that can reduce population, such as: death, natural disasters, war, environmental health, pollution, infectious disease outbreaks, availability of medicines, and emigration, then the lecturer will ask several questions to direct student answers. These questions include the following: 1) Are you sure that all of the factors that you have written have covered all the factors causing the increase in population? 2) Are there any of the factors that you wrote down that you doubt its correctness? After the two questions were asked, the lecturer asked students to re-examine and revise the results of the discussion. However, if the results of their group discussion have resulted in complete factors as expected, the lecturer asks students to continue their activities to work on the next assignment, namely determining one main factor that can increase the population and one main factor that can reduce the population.

From the factors that affect population growth as discussed in points 1) and 2) above, students are asked to continue the discussion in their group to determine one main factor causing the increase in population (namely birth) and one main factor causing the decline of total population (namely deaths). If in the lecturers' observation during the student discussion there were groups that did not correctly determine the one main factor requested (namely the birth factor as the main

factor in increasing the population, and the death factor as the main factor in reducing the population), the lecturer asked several questions to lead students to the answer. For instance, the questions posed are: 1) Which factor has a more dominant influence than the other factors that can increase population? 2) Which factor has the more dominant influence than the other factors that can reduce population? However, if the results of their group discussion have produced the main factors as expected, the lecturer asks students to continue their activities to do the next task, namely determining the relationship between population growth and the two main factors that cause population growth.

In determining the relationship between population growth and the two main factors causing population growth, if there is a group of students who have not fully stated the relationship, namely: the size of population growth is equal to the number of births minus the number of deaths, the lecturer will lead their answers to be more complete by proposing the following questions: 1) How are these factors related to the total population? 2) How do you involve the percentage of population growth, both the percentage of births and the percentage of deaths? The expected answer is: The total population is equal to the total population at the start of the observation plus the percentage of births times the number of the initial population minus the percentage of deaths times the initial population. If students have not arrived at an answer like this, the lecturer directs them to arrive at the expected answer. However, if students have arrived at an answer like this, then the lecturer appoints one of them to present the answer on the class discussion and the other students give their responses. The results of the presentations that have been given input by other groups with lecturers' directions are then agreed as conclusion-1.

In the second activity, the student's estimated answer is

$$PB = PA b (PA) t - c (PA) t \quad (1)$$

$$PB = PA (b - c) (PA) t$$

$$PB = PA k (PA) t$$

with  $k$  (constant for growth), number of new population ( $PB$ ), number of population at the beginning of observation ( $PA$ ), number of births ( $L$ ), number of deaths ( $M$ ), percentage of births ( $b\%$ ), percentage of deaths is  $c\%$ , and time is expressed as  $t$ . If students have not arrived at an answer like this, the lecturer directs them to arrive at the expected answer. However, if students have arrived at an answer like this, then the lecturer appoints one of the students to represent his group to present the answer in class discussion and students from other groups give their responses. The results of the presentations that have been given input

by other groups with lecturers' directions are then agreed as conclusion-2.

After obtaining the relationship between population growth factors in mathematical sentences in informal form as seen in conclusion-2, then in the third activity a first-order ODE model will be constructed from the problem of population growth. The model that will be produced later is a formal model of the population growth problem.

Based on the 2 conclusions above, students are directed to find the answer

$$P(t + \Delta t) = P(t) + bP(t) \Delta t - cP(t) \Delta t \quad (2)$$

If students give answers like this, then the lecturer directs the students by reminding them that: "The mathematical model that has been obtained is not yet a differential equation, so use the derivative principles related to getting a model of the differential equation."

The next possible answers from students are: Based on the answers to part b) above they write their answers

$$P(t+\Delta t)=P(t) + bP(t) \Delta t - cP(t) \Delta t \text{ or}$$

$$P(t+\Delta t) - P(t) = bP(t) \Delta t - cP(t) \Delta t$$

If both sides are divided by  $\Delta t$ , will be obtained

$$(P(t+\Delta t)-P(t))/\Delta t= bP(t)-cP(t)$$

or

$$\Delta P/\Delta t= bP(t)-cP(t)$$

$$= (b - c)P(t)$$

$$= kP(t)$$

By using the instantaneous rate of change to approximate the average rate of change, the following model is obtained

$$dP/dt = kP(t), P(t_0) = P_0, \text{ to } t < t_1 \quad (3)$$

where  $k$  (constant for growth) is a positive constant. If students write answers like this they are asked to group the differential equation model obtained into one of the existing first-order ordinary differential equations.

The students' possible responses, they classify the final form of the differential equation model obtained in section c) above into the form of separate variable differential equations as follows:

$$dP / dt = kP (t) \quad (4)$$

$$dp = kP (t) dt$$

$$kP (t) dt-dp = 0$$

If  $F(t) = kP(t)$  and  $G(t) = -1$ , the final equation can be written as

$$F(t) dt + G(t) dp = 0 \quad (5)$$

This form is the same as the general form of the separate variable differential equation:  $F(x) dx + G(y) dy = 0$

Table 1 is the result of the HLT which is designed and contains learning activities and objectives.

**Table 1. Hypothetical Learning Process**

Activity Stages	Estimated Student Answers	Lecturer Anticipation
1. Students are asked to list the factors which can affect population growth	1. Possibilities of students:	1. The lecturer asks probing questions :
<b>Objectives</b> Students can register all the factors that cause population growth and determine the main (dominant) factor.	<b>Students write down the factors that can cause population growth to increase as follows:</b>	
	a. Students do not completely write down the factors that can increase the growth of the population (in the form of births: health care during pregnancy, attitudes towards the availability of contraceptives, attitudes towards abortion, immigration, etc.). Apart from that, some of the factors listed are also factors that can reduce population growth.	a.1 Are you sure that all of the factors that have been written have covered all of the factors causing the increase in population? a.2 Are there any factors that can reduce the population?
	b. Students write only the factors that cause an increase in population (in the form of births: health care during pregnancy, attitudes towards the availability of contraceptives, attitudes towards abortion, immigration, etc.)	b.1 Are there any more factors to add to all of the factors that you have listed? b.2 If there is still something to write down, if there is no more, please continue to the next assignment!
	<b>Students write down the factors that can cause population growth to decline as follows: Students write down the factors that can cause population growth to decline as follows:</b>	
	c. Students do not completely write down the factors that can reduce population growth (in the form of deaths: natural disasters, war, environmental health, pollution, infectious disease outbreaks, availability of drugs, emigration, etc.). Apart from that, some of the factors listed are also factors that can increase population growth.	c.1 Are you sure that all the factors that have been written have covered all of the factors causing the decline in population? c.2 Are there any factors that can increase the population of the factors that you wrote down?
d. Students write down the factors that cause a reduction in population only (in the form of deaths: natural disasters, war, environmental health, pollution, infectious disease outbreaks, availability of medicines, emigration, etc.)	d.1 Are there any more factors to add to all of the factors that you have listed? d.2 If there is still something to write down, if there is no more, please continue to the next assignment!	

Furthermore, this HLT was validated by 4 experts consisting of 1 math expert, 1 mathematics education expert, 1 educational technology expert, and 1 Indonesian language expert who assessed the content and language aspects, you can see the results in Table 2.

**Table 2. Expert Validation Results**

Aspect	No Item	Validator Score				V-value	Category
		V1	V2	V3	V4		
CONTENT	1	3	4	4	4	0.94	very high
	2	4	4	4	4	1.00	very high
	3	4	4	4	4	1.00	very high
	4	3	4	3	3	0.80	high
	5	4	4	4	4	1.00	very high
	6	4	3	3	3	0.80	high
	7	3	4	4	4	0.94	very high
	8	3	4	4	4	0.88	very high
	9	3	4	3	4	0.88	very high
	10	4	4	4	4	1.00	very high
	11	4	3	4	3	0.88	very high
	12	4	4	4	4	1.00	very high
LANGUAGE	1	4	4	4	4	1.00	very high
	2	3	4	4	4	0.94	very high
	3	3	4	3	3	0.80	high
	4	3	4	4	4	0.94	very high
<b>Average Value of Validity</b>						<b>0.92</b>	<b>very high</b>

**4. CONCLUSION**

Based on the results of the expert validation stage, it appears that HLT is designed to make it easier for students to achieve learning goals. In stages students are guided to rediscover formal mathematical concepts. Students are facilitated to carry out horizontal to vertical mathematics processes based on the RME approach

**AUTHORS' CONTRIBUTIONS**

**ACKNOWLEDGMENTS**

The author would like to thank all those who have supported the completion of this paper, hopefully this can contribute to the mathematics learning process in particular.

**REFERENCES**

- [1] Arslan, S. Traditional instruction of differential equations and conceptual learning. Article in Teaching Mathematics and Its Applications, 29:94-107. doi:10.1093/teamat/hrq001 Advance Acces publication 15 February 2010.
- [2] Czocher, Jennifer & Baker, Greg. Contextual Learning in Math Education for Engineers. Proceeding International Conference on Engineering Education ICEE-2010 July 19-22, 2010, Gliwice, Poland.
- [3] Widjaja, W; Dolk, M.; Fauzan, A. The role of contexts and teacher's questioning to enhance students' thinking. In Journal of Science and Mathematics Education in Southeast Asia. 2010. Vol 33(2), p 168-186.
- [4] Fauzan, A., Plomp, T., & Gravemeijer, K. P. E. The development of an RME-based geometrycourse for Indonesian Primary schools. In T. Plomp, & N. Nieveen (Eds.), Educational design research - Part B. Illustrative cases. 2013. (pp. 159-178). Enschede, The Netherlands: SLO : Netherlands institute for curriculum development..
- [5] Sembiring, R.K. Pendidikan Matematika Realistik Indonesia (PMRI): Perkembangan &

- Tantangannya, artikel dalam IndoMS Journal of Mathematics Educatio. 2010. Vol. 1(1), pp. 11-16
- [6] Gravemeijer, K.P.E. Local Instructional Theories as Means of Support for Teachers in Reform Mathematical Education. *Mathematical Thinking and Learning*. 6(2), 125-127. Copyright 2004. Lawrence Erlbaum Associates. Inc.
- [7] Gravemeijer, KPE. 1994 *Developing Realistic Mathematics Education* (Utrecht: Freudenthal Institute)
- [8] Plomp, T., & Nieveen, N. (Eds.). 2013. *Educational design research: An Introduction*. Enschede, the Netherlands: SLO.
- [9] Kwon, O.N. Conceptualizing the Realistic Mathematics Education Approach in the Teaching and Learning of Ordinary Differential Equations. Presented at International Conference on the Teaching of Mathematics (at the Undergraduate Level) 2002.
- [10] Tessmer, M. *Planning and Conducting Formative Evaluations, Improving the Quality of Education and Training*. 2005. London: Routledge.
- [11] Khotimah, R. P & Masduki. (2016). Improving Teaching Quality and Problem Solving Ability Through Contextual Teaching and Learning in Differential Equations: *Journal of Research and Advances in Mathematics Education*. SSN: 2503-3697 (Print) Vol. 1, No. 1, 1-13, January 2016. <https://files.eric.ed.gov/fulltext/ED565784>