ATLANTIS PRESS
Advances in Economics, Business and Management Research, volume 159
Fifth International Conference on Economic and Business Management (FEBM 2020)

The Effect of Auction Duration on Seller's Expected Revenue

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ABSTRACT
The research object of this paper is online auction with storage cost, and the relationship between seller's expected revenue and the auction duration has been discussed. Based on intuition that as the duration of auction increases, the increment in seller's expected revenue due to the increment in the number of bidders is first greater than the cost and then less than the cost, and two conclusions are drawn under the independent private value model: 1. If the auction duration is too long, the seller's expected revenue will become negative. 2. There exists a positive value, and when the auction duration T exceeds this value, the seller's expected revenue is monotonously decreasing about the auction duration .
Keywords: *Expected revenue, independent private value model, auction duration*

1. INTRODUCTION

As an effective way of allocating resources, auction has a very long history. As early as the Babylonian period, there's someone won his wife by auction. But the research on traditional auctions theory is only a few decades old, and an important study was published by Vickrey [1] in 1961. Vickrey portrayed the framework for auction research and laid the foundation for future auction in this paper. Since then, many scholars have joined the ranks of auction theory research and have made tremendous contributions to the development of auction theory. The outstanding contributions include the research by Mcafee and Mcmillan [2], Milgrom and Weber [3], etc. Theories about the traditional auctions are becoming more and more mature.
With the development of the Internet, auction has changed a lot. Online auction has become a dynamic part of e-commerce, and this brings new features to the auction, such as low participation costs, the uncertain number of participations, and different people participate in auctions in different places, etc. These new features have led to a lot of differences between the bidding based on online auction and the bidding based on traditional auction. Bapna, Goes and Gupta [4] had analysed the strategies of online bidders and found that not everyone will bid according to the rational principle of maximizing their profits and divided the bidders into three categories. Ockenfels [5] noted that there is a great deal of late bidding on eBay and found that the strategies of bidders had a lot to do with the ending rules of online auctions. Shared, Peter and Joseph [6] found that the bidding experience also affects the bidder's behaviour by analysing the transaction data on the eBay website. They thought that the experienced bidders are more active either toward the close of auction or toward the start of the auction and the impact of experience on the extent of multiple bidding is monotonic across the auction interval.
In addition to the behaviour of the bidders, the strategies of the sellers are also one of the research hotspots. How should the sellers choose the practical strategies to improve their own revenue is the main research content. Wood [7] researched the results of the transaction of collecting coins on eBay and found that: the transaction price of the items auctioned on the weekend is two percentage higher than the transaction price of normal time; the transaction price of items with pictures is nearly ten percentage higher than that without pictures; the longer the auction duration, the more bidders it attracts. Bajari [8] also used eBay as a research object and found that that the expected revenue of sellers in online auction increases with the number of bidders. Guanquan Ni [9] had proved that setting a reasonable price can increase the expected revenue of the seller. Lucking-Reiley [10] found that the longer the auction duration, the higher the transaction price of the goods by analysing the statistics on coin auctions on eBay.
However, for some auction items with storage costs or perishable items, it's irrational for the sellers to set a too long auction duration. Pinker [11] found that excessive auction time will bring some extra costs to buyers and sellers. So how to set the auction duration is important for sellers. There is little research on the optimal duration in online auctions and we consider the impact of the auction duration from the perspective of seller's expected revenue.

1.2. Our Contribution

In this paper, we consider the impact of auction duration and calculate the seller's expected revenue. We find some relationship between the auction duration and seller's expected revenue and prove it, which has reference significance for setting the auction duration in real life.

1.3. Paper Structure

The rest of the paper is organized as follows. Section 2 introduces the basic assumptions of the model used in this paper. Section 3 introduces the relationship between seller's expected revenue and auction duration. Section 4 introduces the shortcoming of this paper and the future research direction.

2. THE MODEL

Our model is based on the standard eBay auction model. In order to simplify the problem, our research is conducted under the assumption of independent private value, and we don't consider the impact of minimum bid increment. The bidders' valuation of the auction items will not be affected by other bidders' valuations under the independent private value model. The basic assumptions of our model are as follows:

1. The auction duration is denoted as T and the auction has a bidding agency mechanism. The number of arrivals in the whole auction process is subject to the Poisson process with the parameter λ.
2. The bidders' valuation of the auction items obeys the independent and identical distribution on (v_1, v_2), which is denoted as $F(x)$ with a continuous density function $f(x)$.
3. The starting price of the auction is denoted as q and satisfies the relationship $v_1 < q < v_2$. The storage cost of the auction item per unit time is denoted as C.
4. The current price during the auction is the second highest submitted reservation price among the n arrived people. There is one exception to this: The current price is q if the second highest price is lower than q. And we consider the second highest price is 0 when the number of arrivals is 1. The bidder with the highest bid wins the auction at the end and if there're more than one person at the same highest bidding level, then the person who bids first wins the auction.

According to the research of Li Du [12], it is a dominant strategy for bidders to submit their real valuation of the auction items as soon as they arrive under the model of independent private value. With this conclusion, we can calculate the seller's expected revenue when facing a group of rational bidders.

3. THE RELATIONSHIP BETWEEN THE AUCTION DURATION AND SELLER'S EXPECTED REVENUE

3.1. Seller's Expected Revenue

There are three different results in our auction model:

1. The auction item is not successfully sold. Two situations will lead to this result, one is that no one arrives at the auction and the other is all the bidders' valuations are lower than the starting price q. The probability of this result is $\sum_{n=0}^{\infty} \frac{(\lambda T)^n}{n!} e^{-\lambda T} F(q)^n = e^{-\lambda \overline{F}(q) T}$, where $\overline{F}(q) = 1 - F(q)$.
2. The second result is that the auction item is sold at price q. The situation that the second highest price submitted by arrived bidders is lower than q and the highest price is not less than q will lead to this result. The probability of this result is $\lambda T \overline{F}(q) e^{-\lambda \overline{F}(q) T}$.
3. The third result is that the auction item is sold at price x, where $q < x < v_2$. And the probability density of x is $n(n-1)F(x)^{n-2} f(x)[1 - F(x)]$.

and we can get the seller's expected revenue E_s:

$$
\begin{aligned}
E_s &= \sum_{n=2}^{\infty} \frac{(\lambda T)^n}{n!} e^{-\lambda T} \int_q^{v_2} n(n-1) x F(x)^{n-2} f(x) \\
&\quad [1 - F(x)] dx + \lambda q T \overline{F}(q) e^{-\lambda \overline{F}(q) T} - CT \\
&= \sum_{n=1}^{\infty} \frac{(\lambda T)^n}{n!} e^{-\lambda T} l_1(n) + \lambda q T \overline{F}(q) e^{-\lambda \overline{F}(q) T} - CT \qquad (1)
\end{aligned}
$$

Where $l_1(n) = \int_q^{v_2} n(n-1)\, x F(x)^{n-2} f(x)[1 - F(x)] dx$.

3.2. The Effect of the Auction Duration

In the above process, we have calculated the seller's expected revenue. But we cannot directly perceive the relationship between E_s and T from the expression. We notice that the storage cost CT is proportional to the T, so will there be a negative expected revenue when T is too large? Based on this idea we get the following conclusion.

Conclusion 1: The seller's expected revenue will become negative when the auction duration T is too long.

Proof:

$$
\begin{aligned}
&\because \int_q^{v_2} x F(x)^{n-2} f(x)[1 - F(x)] dx \\
&= \int_q^{v_2} x F(x)^{n-2} [1 - F(x)] dF(x) \\
&= -q F(q)^{n-1} [1 - F(q)] - \int_q^{v_2} F(x)^{n-1} dx \\
&\quad + \frac{n-1}{n} \int_q^{v_2} F(x)^n dx + \frac{v_2}{n} - \frac{q}{n} F(q)^n \\
&\quad -(n-2) \int_q^{v_2} x F(x)^{n-2} f(x)[1 - F(x)] dx \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
\therefore l_1(n) &= n(n-1) \int_q^{v_2} x F(x)^{n-2} f(x)[1 - F(x)] dx \\
&= v_2 - (n-1) q [1 - F(q)] F(q)^{n-1} - q F(q)^{n-1} \\
&\quad - n \int_q^{v_2} F(x)^{n-1} dx + (n-1) \int_q^{v_2} F(x)^n dx \quad (3)
\end{aligned}
$$

$\because 0 < F(q) < 1, 0 \le F(x) \le 1$

$\therefore l_1(n) < v_2$

$\because n[1 - F(q)] F(q)^{n-1} < q$

$$\therefore E_s \le \sum_{n=1}^{\infty} \frac{(\lambda T)^n}{n!} e^{-\lambda T}[v_2 + q] - CT \le v_2 + q - CT \quad (4)$$

$$\therefore E_s < 0, while\ T > \frac{v_2 + q}{C}$$

We have proved that the seller's expected revenue will become negative when the auction duration T is too long and it's easy to understand this. Due to the upper limit of the bidder's valuation and any reasonable bidder will not submit a bid that exceeds this limit, the seller's expected revenue will not exceed the highest bid and will become negative with the storage cost which is proportional to auction duration. We guess that there exists a value T^* and the seller's marginal expected revenue is less than the storage cost while the auction duration exceeds T^*. Based on this idea we get the conclusion 2.

Conclusion 2: There exists a positive value T^*, and the seller's expected revenue is monotonously decreasing about the auction duration T while $T > T^*$.

Proof:

$$\because \frac{dE_s}{dT} = \sum_{n=1}^{\infty} \frac{(\lambda T)^n}{n!} \lambda e^{-\lambda T}[l_1(n+1) - l_1(n)] + \lambda q \overline{F}(q)[1 - \lambda T \overline{F}(q)] e^{-\lambda \overline{F}(q) T} - C \quad (5)$$

$$l_1(n+1) - l_1(n) = nqF(q)^{n-1}\overline{F}(q)^2 + n\int_q^{v_2} F(x)^{n-1}[1 - F(x)]^2\,dx \quad (6)$$

$$\therefore \frac{dE_s}{dT} = \sum_{k=0}^{\infty} \frac{\lambda(\lambda T)^{k+1}}{k!} e^{-\lambda T} \int_q^{v_2} F(x)^k [1 - F(x)]^2\,dx + \lambda q \overline{F}(q) e^{-\lambda \overline{F}(q) T} - C \quad (7)$$

Let $g(k) = \int_q^{v_2} F(x)^k[1 - F(x)]^2\,dx$, we can get the following equation through the mean value theorem for definite integrals.

$$\exists \varepsilon_k \in (q, v_2), s.t.\ g(k) = F(\varepsilon_k)^k [1 - F(\varepsilon_k)]^2 (v_2 - q)$$

and we can get a bounded sequence $\{\varepsilon_k\}_{k=1}^{\infty}$ through the equation, and the sequence exists least upper bound ε^* by the supremum and infimum principle. We can get that $g(k) \le F(\varepsilon^*)^k (v_2 - q)$ by the nature of the distribution function. And we can get the following inequality:

$$\frac{dE_s}{dT} \le \sum_{k=0}^{\infty} \frac{\lambda(\lambda T)^{k+1}}{k!} e^{-\lambda T} F(\varepsilon^*)^k (v_2 - q) + \lambda q \overline{F}(q) e^{-\lambda \overline{F}(q) T} - C \le \lambda q \overline{F}(q) e^{-\lambda \overline{F}(q) T} - C + \lambda^2 T (v_2 - q) e^{-\lambda \overline{F}(\varepsilon^*) T} \quad (8)$$

Let $h(T) = \lambda q \overline{F}(q) e^{-\lambda \overline{F}(q) T} - C + \lambda^2 T (v_2 - q) e^{-\lambda \overline{F}(\varepsilon^*) T}$, next we will demonstrate that there exists T^*, and $h(T)$ is negative while $T > T^*$.

$$\because \frac{dh(T)}{dT} = [1 - \lambda \overline{F}(\varepsilon^*) T] \lambda^2 (v_2 - q) e^{-\lambda \overline{F}(\varepsilon^*) T} - \lambda^2 q \overline{F}(q)^2 e^{-\lambda \overline{F}(q) T} \quad (9)$$

$$\therefore \frac{dh(T)}{dT} < 0, while\ T > \frac{1}{\lambda \overline{F}(\varepsilon^*)}$$

$$\because \lim_{T \to \infty} h(T) = -C < 0$$

$$\therefore \exists T^* > 0, h(T) < 0\ and\ \frac{dh(T)}{dT} < 0, while\ T > T^*$$

$$\because \frac{dE_s}{dT} \le h(T)$$

$$\therefore \frac{dE_s}{dT} < 0, while\ T > T^*$$

We have completed the proof of conclusion 2 and this conclusion tells us the seller's expected would not always increase with the auction duration.

Although we don't get the expression of the optimal auction duration, our conclusions are still helpful. For example, setting an auction duration more than $(v_2 + q)/C$ is not a good idea. Especially for companies, which can estimate the consumers' valuations of the auction item from large transaction data, it's helpful to estimate the interval of the optimal auction duration.

3.3. A Simulation Example

In the previous section, we obtained two conclusions without specifying the specific distribution function of the bidder's valuation. In the next, we will give the specific parameters in the auction to calculate the seller's expected revenue. We assume that the bidder's valuation of the auction item is subjected to a uniform distribution on (0,50), the number of arrivals in the whole auction process is subject to the Poisson with the parameter $\lambda = 2$, and the starting price q is 10. The seller's expected revenue is shown in Figure 1.

As we can see in the Figure 1, the seller's revenue reaches the maximum value around $T = 5$, and is monotonously decreasing about T while $T > 5$. The results of the Figure 1 is in line with our conclusions. And it shows us that it's important for a rational seller to set the auction duration T to maximize his expected revenue.

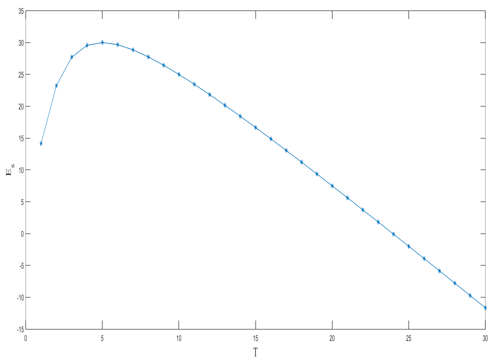

Figure 1 The seller's expected revenue curve

4. FUTURE WORK

In online auctions, there are many factors that can affect the seller's revenue and the duration of the auction is a factor that can't be ignored. The seller must consider not only the psychology that the buyer may wish to obtain the auction product earlier, but also the risk of low revenue due to too few participants in the auction. Especially for auction item with storage costs or perishables, setting a reasonable duration is more important. In this paper we have proved that the seller's revenue is monotonously decreasing with the duration T while T exceeds a value and may become negative under the assumption of the independent private value model. However, in this paper, we did not get the expression of the optimal auction duration and the impact of the perishability was not considered. This is a question that needs further study and we look forward to providing greater help for the development of online auction theory in the future.

ACKNOWLEDGMENT

This work was supported by Soft Science Project for Research on the Design of Bidding and Auction Mechanism and Optimal Decision Making under the Internet Environment of Sichuan, China (2019JDR0014).

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