Research on the Teaching Mode of “Linear Algebra” Based on Online Teaching

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ABSTRACT

As a product of informationization, online teaching plays a very important role in talent training and teaching practice, and has gradually become a positive and effective teaching method. In order to ensure a good teaching effect on the premise of completing the teaching task, it is necessary to match the special teaching method with appropriate teaching mode and teaching design. On the basis of investigation and research, this paper establishes a teaching mode of "three-stage integration". The teaching process is divided into pre-class, in-class and after-class according to the teaching design, and corresponding tasks and activities are designed for each link. Finally, the teaching effect is tested with the established assessment and evaluation index system, so as to get the ideas and directions for the next rectification.

Keywords: Online teaching, Teaching mode, Linear algebra

1. INTRODUCTION

At the beginning of 2020, a sudden outbreak of COVID-19 virus disrupted people's normal life and caused a great impact on all aspects of society. Because the school cannot open normally, the state requires all students to study online at home, which makes the teachers feel anxious. If face to face teaching is not possible, what kind of teaching mode should be chosen to ensure the smooth development of teaching? Is the chosen teaching mode acceptable to students and easy to adapt to? How to design the teaching process and link to ensure the quality of teaching? These questions have been on every teacher's mind for countless times.

In order to ensure the smooth development of online teaching, the state and schools at all levels have taken a series of measures to open various online teaching platforms and teaching resources, which provide a guarantee for effective communication between teachers and students.

As a general education course in universities, “Linear Algebra” [1] plays a vital role in the cultivation of students' logical thinking ability, computing and application ability and innovation ability. At present, the research on the reform of teaching mode of “Linear algebra” has been deepened[2-7]. The combination of information technology and traditional teaching mode has been increasing, and some achievements have been made. In this context, the previous teaching mode can no longer cope with the current situation, so we need to study and discuss a suitable teaching mode under the epidemic situation. In this paper, a teaching mode based on online teaching is established and applied to the teaching process of “Linear Algebra”. At the same time, the course group of “Linear Algebra” has formulated the implementation plan of online teaching, which guarantees the quality of online teaching from the aspects of systematic participation in training, multi-aspect integration of resources, scientific classification and pushing, all-round coaching and answering questions, and multi-angle feedback and evaluation.

2. ESTABLISHING THE TEACHING MODE OF "THREE-STAGE INTEGRATION"

2.1. The preparatory stage before class

In order to familiarize teachers with the general steps and basic principles of online teaching, the state has taken a series of measures. The opening of each network teaching platform provides guarantee for teachers to carry out teaching work smoothly. The university organized teachers to conduct independent learning and intensive discussion on the use of various teaching platforms for half a month. After comparing the functions and operations of each teaching platform, the course group chose “Super star” platform for online teaching. During the operation of the learning platform, the course group is actively constructing course resources. In order to make the constructed resources more in line with the academic situation, the teachers of the course group collected the course resources of various platforms, and supplemented the teaching resources with videos and question Banks recorded by themselves. Thus forming the materials needed for online teaching and providing guarantee for the smooth development of online teaching. One week before the formal start of the course, the course group drew up the online teaching implementation plan and the assessment
2.1. Systematic participation in training to improve the level of online teaching

In order to be familiar with the operating process of online teaching in the shortest time, teachers must participate in systematic training organized by the school. The teachers receive training mainly from the aspects of how to build classes, how to answer questions for students in class, and how to analyze students' learning data. The teachers of the class group often discuss the training content together, aiming to explore an online teaching mode suitable for “Linear algebra”. The teaching level of teachers can be greatly improved through training.

2.1.2. Integrating resources in various aspects to provide a guarantee for online teaching

Curriculum resources are the objects of students' learning. In order to carry out online teaching smoothly, teachers should try their best to collect, sort out and build appropriate course resources. In terms of the selection of teaching resources, high-quality network video resources suitable for students' learning level can be selected according to their academic conditions as the main video resources for online teaching. At the same time, Teachers should complete the construction of electronic courseware, electronic teaching plan, task bank, homework bank and test question bank. Considering the operability of the teaching platform, Teachers can choose the online teaching mode of "network teaching platform +QQ group". In the application of resources, the selected teaching videos are edited and integrated according to the requirements of fragmentation, which can make adequate preparation for the teaching work.

2.2. Classroom organization and in-class teaching stage

Before the COVID-19 virus, “Linear Algebra” also tried to use the blended teaching mode of "online + offline" according to the characteristics of knowledge, but it is a new attempt to put all the knowledge online. If the classroom organization is not reasonable, the teaching effect will be greatly reduced and directly affect the smooth progress of subsequent courses. In order to ensure the quality of teaching and let students focus on the completion of learning tasks, Teachers can organize teaching from the following two aspects.

2.2.1. Scientific classification pushes knowledge to ensure orderly online teaching

What kind of resources to push to students needs to consider the acceptance level of students. After all, this is the first attempt of online teaching, and scientific planning must be made on what, how and when to push. Only reasonable arrangement can ensure online teaching in an orderly way. A variety of teaching methods can be used in teaching, including action-oriented, inquisitorial, heuristic, case teaching and other advanced teaching methods. The main implementation process is as follows.

The task ahead-In order to enable students to actively participate in online learning, cases, some relatively simple knowledge points, or some divergent questions will be pushed to students in the form of courseware and videos before class. Notices will be sent to students to make clear requirements for learning tasks and let students complete learning tasks as required.

The student center-During the teaching process, on the one hand, flexible and diverse classroom activities are selected for the early tasks to test the learning effect. On the other hand, the teachers can explain the important and difficult points of this class. The teachers can broadcast or record the teaching for the knowledge that is difficult for students to understand. At the same time, the teachers can choose heuristic, exploratory, group discussion, action-oriented, case teaching and other methods to carry out teaching activities according to different knowledge points and learning situations.

Application of reinforcement-After class, teachers can ask students to construct mind maps, sort out knowledge system, clarify the main line of course learning, strengthen the key points and difficulties in teaching by using homework method, and pay attention to the cultivation of students' application ability.

At first, the teachers push the prepared electronic courseware, e-book and teaching video to the students in stages and steps, and requires the students to complete the learning task within the prescribed time by means of independent learning. When the time is coming, students who have not completed the task can be supervised, and individual students who have difficulties in learning can be given individual tutoring, so as to ensure that no student is left behind. In the process of independent learning, students must be able to timely collect the difficult problems encountered in their learning, and push these problems to students through the form of recording micro lessons, so as to ensure students' smooth learning. After all the students have completed the task, the teacher can choose the appropriate time to push the key and difficult knowledge in the learning materials to the students, and explain these knowledge to the students through the live broadcast. In the process of explanation, some class activities suitable for students can be organized, such as group discussion, topic discussion, test, selection and answer, etc., in order to mobilize the enthusiasm of students to learn. Teachers can choose a variety of teaching methods in the teaching process, such as group
discussion method, task-driven method, inquiry-based method, case method and so on.

2.2.2. All-round coaching and answering questions to ensure that the quality of teaching is not compromised

If the teachers want to ensure the quality of online teaching, the supervision of students plays a very important role. In order to supervise the whole learning process of students, “Linear algebra” can adopt the form of "individual tutoring + concentrated q&A". During the course of the week, students' online learning can be supervised according to the course schedule. For the students who do not participate and do not complete the task on time, the teacher should urge them to complete the task through timely communication. Teachers can record small videos and upload class groups for students to learn according to their own situations. For students who do not submit their homework in time, Teachers must ask the reason through communication in time, and work together with students to solve the problem.

2.3. After-school consolidation and feedback stage

2.3.1. Multi-angle feedback evaluation, and strive for better teaching effect

A very important part of teaching design is after-class consolidation and feedback. The learning effect cannot be known without feedback. After the explanation of all the knowledge points is completed, the teacher should design corresponding homework to test the learning effect of students. It is better to stratify students according to their learning basis and design the homework suitable for students design the homework suitable for the students at the corresponding level according to different levels if the conditions permit. When students finish their homework, teachers should review the homework in time. In the process of reading, teachers should analyze the knowledge points corresponding to the wrong questions of students. If some students make mistakes, teachers can tell these students the wrong reasons separately. If most students make mistakes on a certain topic, it means that the difficulty of the question is not suitable for the students of the class, or the knowledge points involved in the question are not mastered by the students. The results directly reflect the students' actual mastery. For the problems with more mistakes in the homework, you can record a short video of the problem solving methods and steps, and send it to the students together with the solution analysis, so that the students can realize that they do not master the part in the learning process, and consolidate. Through the four steps of "self-study + q&A + concentrated teaching + feedback", most students can master the basic knowledge points. Because the learning of knowledge is a continuous process, if students don't review in time, they will easily forget what they have learned. In order to avoid the suspension of learning, teachers need to summarize the contents that students have learned. Teachers can record videos or make courseware for students to review and summarize. They can choose appropriate time to explain it to students when necessary, so that students can not forget to review the old knowledge while learning new knowledge, so as to "review the old knowledge and learn new".

In order to test the learning effect of students, teachers can set corresponding homework, classroom test questions, and chapter test questions. In the process of answering questions, teachers can also provide some exercises for students to test the teaching effect in a timely manner and reflect on the test results. At the same time, teachers should also carry out intensive discussions on schedule, and study the public problems in the evaluation process in order to find out the reasons and make corresponding curriculum resources after class to solve these problems. Teachers should reflect deeply in the teaching process, and should never ignore or avoid problems. Teachers should adjust the teaching design while practicing and make full use of various functions of the platform to strengthen communication and exchange with students.

2.3.2. Establishing comprehensive evaluation index system

As an important part of teaching design, assessment and evaluation aims to test the effect of teaching and provide teachers with ideas and directions for the next step of teaching reform. As the first attempt of online teaching, , a corresponding comprehensive evaluation index system combined with the above "three-stage integration" teaching mode can be established according to the statistical results of the questionnaire among students and the intensive discussion among teachers in the course group. This index system is based on the above teaching mode, aiming to conduct quantitative assessment on each link of teaching design, and then obtain the final assessment result of each student according to the weight ratio.

3. "THREE-STAGE INTEGRATION" TEACHING MODE APPLICATION EXAMPLE - "LINEAR REPRESENTATION OF VECTOR GROUP"

Before class: Teachers use the class network platform to post learning tasks, provide teaching PPT, videos and other teaching resources, so that students can learn independently; for students’ questions, teachers can answer questions in time through the Internet, QQ, WeChat, etc. Students must clarify their learning tasks and
learn independently through multimedia courseware, videos, etc. At the same time, students can be divided into groups, each group selects a leader to ensure that everyone has a division of labor; the leader is responsible for real-time communication with teachers and group students. Teachers use specific tasks as the carrier to stimulate students' interest in learning, teamwork to learn the related knowledge, and let each student prepare for the classroom. **In class:** Pushing knowledge to students according to the logical structure of knowledge.

**Knowledge point 1: Linear representation of vector group**

**Definition 1:** If a certain relationship between vectors in a vector group can be expressed by the linear operation (multiplication and addition) of the vector group, then this relationship is called the linear relationship of the vector group.

**Definition 2:** Let the vector group \( A: \alpha_1, \alpha_2, \ldots, \alpha_n \) for any set of numbers \( k_1, k_2, \ldots, k_n \), make \( \beta = k_1 \alpha_1 + k_2 \alpha_2 + \cdots + k_n \alpha_n \). If it is true, the vector \( \beta \) is a linear combination of a vector group \( A \). At this time, the vector \( \beta \) can be expressed linearly by the vector group \( A \).

**Definition 3:** Given a vector group \( A: \alpha_1, \alpha_2, \ldots, \alpha_n \) and vector \( \beta \), if there is a group of numbers \( k_1, k_2, \ldots, k_n \), make \( \beta = k_1 \alpha_1 + k_2 \alpha_2 + \cdots + k_n \alpha_n \) If it is true, the vector \( \beta \) is a linear combination of the vector group \( A \). The coefficients \( k_1, k_2, \ldots, k_n \) is called the coefficients of this linear combination.

**Example:** Calculate the linear combination of the vector group \( \alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \).

**Solution:** Let \( \beta = a_1 \alpha_1 + a_2 \alpha_2 + a_3 \alpha_3 \). If \( \beta \) can be expressed linearly with \( \alpha_1, \alpha_2, \alpha_3 \), then \( \beta \) is a linear combination of \( \alpha_1, \alpha_2, \alpha_3 \).

**Exercise:** Can the 3-dimensional vector group \( \alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \) be expressed linearly with \( \alpha_1, \alpha_2, \alpha_3 \)?

**Knowledge point 2: Judgment method of linear representation**

Theorem: Vector \( \beta \) can be expressed linearly by vector group \( \alpha_1, \alpha_2, \ldots, \alpha_n \)

\[ \beta = x_1 \alpha_1 + x_2 \alpha_2 + \cdots + x_n \alpha_n \]

has a solution. \( \iff R(A) = R(B) \)

Among them, \( A = (\alpha_1, \alpha_2, \ldots, \alpha_n), B = (\alpha_1, \alpha_2, \ldots, \alpha_n, \beta) \)

**Thinking:** \( \beta = x_1 \alpha_1 + x_2 \alpha_2 + \cdots + x_n \alpha_n \) has no solution \( \iff R(A) \neq R(B) \).

This part is the focus of this class, and it is difficult to understand. While pushing knowledge to students, use tools such as group chat to guide students to be good at transformation and make things easy. Make full use of heuristic and inquiry-based teaching methods to guide students to understand that the essence of this type of problem is whether there is a solution to the inhomogeneous linear equations, if there is a solution, whether the solution is unique and the problem of the specific expression, so as to divide the vector and the vector. The problem of the relationship is transformed into the problem of the solution of non-subordinate linear equations, which is handled by the discriminant method of the solution of non-homogeneous linear equations. The important transformation method has established two important knowledge points-linear representation and non-homogeneous linear The connection between the solutions of the system of equations.

**Knowledge point 3: Application of vector group linear representation**

Example: Set the vector group \( \alpha_1 = (1,1,2,3)^T, \alpha_2 = (0,2,1,3)^T, \alpha_3 = (3,1,0,1)^T, \beta = (2, –4, –3, –7)^T \), and express \( \beta \) as a linear combination of \( \alpha_1, \alpha_2, \alpha_3 \).

**Solution:**

\[
\begin{pmatrix}
1 & 0 & 3 & 2 \\
1 & 2 & 1 & -4 \\
1 & 1 & 0 & -3 \\
2 & 3 & 1 & -7
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & -2 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

Therefore, \( \beta = -\alpha_1 - 2\alpha_2 + \alpha_3 \).

**Exercise:** when \( t = 7 \), \( \beta \) can be represented by linear \( \alpha_1, \alpha_2, \alpha_3 \) how to express?

Through sample and exercises, students can strengthen the application of knowledge points and improve their application ability. Teachers should ensure that they monitor the learning process of students while pushing knowledge, and should set aside sufficient time for students to answer questions to ensure the effect of learning.

**After class:** Teacher should Assign homework that matches the knowledge points of this lesson. The quantity and quality of the homework should meet the academic conditions, and students of different levels can selectively push different homework.

**Teaching reflection:**

This teaching design takes tasks as the carrier and uses the online teaching platform for online teaching. The teacher adopts heuristic, inquiry and other teaching methods, with application as the main line, closely surrounding the teaching goal, and the student-centered completion of theoretical and applied teaching.

1. The concept of student center runs through the entire teaching process.
Before class, the task is pushed as the carrier, and the group cooperation method is used to let students learn independently with tasks and problems; in teaching, the knowledge points are pushed to the students according to the logical structure of the knowledge points, and the key and difficult knowledge are analyzed by live broadcast or recording. Explain, and take the methods of centralized answering and decentralized answering to solve difficult problems raised by students. So as to achieve the expected effect of online teaching.

2. Student-oriented, highlight application skills
The whole teaching process gives full play to the role of "student subject", and students acquire knowledge through autonomous learning and active exploration.

4. CONCLUSION
The establishment of online teaching mode is a process of continuous exploration and improvement. All work must aim at the mastery of students' knowledge and the cultivation of their abilities. Online teaching should be further studied and deepened in the future to make it a mode that can stand the test and serve for some emergencies. This article has created the teaching process of "case first, student center", and use the inquiry-based and case study methods to enable students to study independently or in groups. According to the characteristics of "Linear algebra" courses and students, there is a "simplified" learning content, a "complete" learning process, a "complete" learning resource, and a "complete" learning evaluation, which highlights the cultivation of students' application ability.

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