New Approaches to the Analytical Methods on the Teaching of Reintegration

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ABSTRACT
In current textbooks, when discussing the solution of reintegration, geometric figures are generally used to find the integration area to determine the upper and lower limits of the integration and to calculate the reintegration into repeated integration on a simple area. This method of calculating reintegration with the help of geometric figures is intuitive and simple, and is therefore widely used. As a supplement to the above method, this article will provide an analytical method of reintegration. Using this method to calculate reintegration, it is effective for those complex integration calculations.
***Keywords**: Reintegration, analytical method, upper integral point, lower integral point, cumulative integral*

1. INTRODUCTION

In the face of the new challenges of higher education reform for talent training in the 21st century, how to reform and innovate university mathematics education has become a major theme of current innovative education. To this end, starting from the reform of university mathematics teaching methods, this article discusses in-depth the new ideas in analytical methods in higher mathematics teaching.

2. ANALYSIS AND CALCULATION OF DOUBLE INTEGRAL

Example 1. Calculate $\iint\limits_D f(x,y)dxdy$, where D Is surrounded by $y=x, y=4x, xy=1, xy=2$.

Solution: Find the curve intersection first

(1) $\begin{cases} y=x \\ xy=1 \end{cases} \Rightarrow (1,1)$,

(2) $\begin{cases} y=x \\ xy=2 \end{cases} \Rightarrow (\sqrt{2},\sqrt{2})$,

(3) $\begin{cases} y=4x \\ xy=1 \end{cases} \Rightarrow \left(\frac{1}{2},2\right)$

(4) $\begin{cases} y=4x \\ xy=2 \end{cases} \Rightarrow \left(\frac{\sqrt{2}}{2},2\sqrt{2}\right)$

(5) $\begin{cases} y=x \\ y=4x \end{cases} \Rightarrow (0,0)$,

(6) $\begin{cases} xy=1 \\ xy=2 \end{cases} \Rightarrow$ No Intersection .

Obviously $(0,0)$ does not belong to the intersection of the closed area D. Otherwise D will be enclosed by $y=x, y=4x, xy=1$ or $y=x,\ y=4x, xy=2$, which is not consistent with the requirements of this question.

$\therefore$ The intersection of D

is $(0,0),(\sqrt{2},\sqrt{2}),\left(\frac{1}{2},2\right),\left(\frac{\sqrt{2}}{2},2\sqrt{2}\right)$

Then determine the projection area of the boundary curve D on the axis x:

$\because$ The abscissas of the intersections of $xy=1$ and $y=x, y=4x$ are 1 and $\frac{1}{2}$ respectively,.

$\therefore$ The projection interval of the boundary curve $xy=1$ of D onto the axis x is $\left[\frac{1}{2},1\right]$.

Accordingly, the projection intervals of the boundary curve $xy=2$, $y=x, y=4x$ of D onto the axis x are

$\left[\frac{\sqrt{2}}{2},\sqrt{2}\right],\left[1,\sqrt{2}\right],\left[\frac{1}{2},\frac{\sqrt{2}}{2}\right]$ respectively.

Then project 4 intersection points onto the axis x and divide into the three integration intervals in order of their size

$$\left[\frac{1}{2},\frac{\sqrt{2}}{2}\right],\left[\frac{\sqrt{2}}{2},1\right],\left[1,\sqrt{2}\right].$$

Finally, find the corresponding curve on each integration interval and determine the integration limit:

On $\left[\frac{1}{2},\frac{\sqrt{2}}{2}\right]$, there are $y=4x$ and $xy=1$.

Take $x_0=0.6\in\left[\frac{1}{2},\frac{\sqrt{2}}{2}\right]$ and substitute it with the above two curves, we have $y_1=2.4, y_2=1.67$

$\therefore y=4x$ is the upper limit, $y=\frac{1}{x}$ is the lower limit

Therefore, $\iint_{D_1} f(x,y)dxdy=\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}dx\int_{\frac{1}{x}}^{4x}f(x,y)dy$

Likewise, on $\left[\frac{\sqrt{2}}{2},1\right]$, there are $xy=1$ and $xy=2$,and moreover

$$\iint_{D_2} f(x,y)dxdy=\int_{\frac{\sqrt{2}}{2}}^{1}dx\int_{\frac{1}{x}}^{\frac{2}{x}}f(x,y)dy$$

On $\left[1,\sqrt{2}\right]$, there are $xy=2$ and $y=x$, and moreover

$$\iint_{D_3} f(x,y)dxdy=\int_{1}^{\sqrt{2}}dx\int_{x}^{\frac{2}{x}}f(x,y)dy$$

Thus, the original point is

$$\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}dx\int_{\frac{1}{x}}^{4x}f(x,y)dy+\int_{\frac{\sqrt{2}}{2}}^{1}dx\int_{\frac{1}{x}}^{\frac{2}{x}}f(x,y)dy+\int_{1}^{\sqrt{2}}dx\int_{x}^{\frac{2}{x}}f(x,y)dy$$

3. SUMMARY OF THE METHOD OF DOUBLE INTEGRAL ANALYSIS CONCERNING THE FINITE INTEGRAL REGION

(1) Identify all the intersection points of the boundary curve of the integration area and determine the intersection points belonging to the area D;
(2) The projection interval on the coordinate axis (x axis or y axis) of the boundary curve is determined by the intersection point;
(3) Project the intersection point of D onto the coordinate axis (x axis or y axis) and divide the integration interval according to its order of size (the interval where the projections overlap should be counted as two integration intervals);
(4) Determine the integration limit on each integration interval, and find the corresponding two curves on each interval and determine the upper and lower limits;
(5) If all the focal points of the obtained boundary curve are projected onto the axis x, then each integrated area obtained from this is called an X-shaped area. Otherwise, it is called a Y-shaped area;
(6) In osrder to obtain the double integral accurately and simply, the principle of selecting the type of integration area is to consider both the shape of the area D and the characteristics of the integrand $f(x,y)$ [1-4].

4. ANALYSIS AND CALCULATION OF TRIPLE INTEGRAL

Now set $f(x,y,z)$ on the area Ω continuously, there are
$\iiint_{\Omega} f(x,y,z)dxdydz=\iint_{D_{xy}}dxdy\int_{z_1(x,y)}^{z_2(x,y)}f(x,y,z)dz$ Where D_{xy} is the projection of the area Ω onto the plane xoy.

Since we have already discussed the double integral earlier, the key to using the analytical method to calculate the triple integral is to find out the integral region Ω and the projection D_{xy} on the surface xoy.

In this regard, we can use the plane $z=z_0$ parallel to the plane xoy to intercept the region Ω. When z_0 changes, the largest one of the infinitely many cross sections is the projection of the region Ω on the plane xoy.

5. STEPS AND EXAMPLES OF ANALYSIS AND CALCULATION OF TRIPLE INTEGRAL IN FINITE INTEGRAL AREA [5-10]

(1) Take $z=z_0$ and the surface equations surrounding the area Ω to find the intersection equation;
(2) Derivate the pair of intersection equations from z_0 and find the value of z_0;
(3) Substitute the value of z_0 with the intersection equation to obtain its maximum cross-section, namely D_{xy};
(4) Determine the limit of integration within this integration domain.

Example 1. Calculate $\iiint_{\Omega} f(x,y,z)dxdydz$, where Ω is enclosed by the area of $x^2+y^2+z^2=z$.

Solution:

(1) Find the intersection equation

$$\begin{cases} x^2+y^2+z^2=z \\ z=z_0 \end{cases} \Rightarrow x^2+y^2+{z_0}^2=z_0$$

(2) Derive $x^2 + y^2 + z_0{}^2 = z_0$ from z_0, we have

$$2z_0 = 1 \Rightarrow z_0 = \frac{1}{2}$$

(3) Substitute $x^2 + y^2 + z_0{}^2 = z_0$ with $z_0 = \frac{1}{2}$, we have

$x^2 + y^2 = \frac{1}{4}$, namely D_{xy}

(4) Take $x_0 = \frac{1}{4}, y_0 = \frac{1}{4} \in D_{xy}$ and substitute it with

$$z = \frac{1 \pm \sqrt{1 - 4(x^2 + y^2)}}{2}$$

We have $z_1 = \dfrac{1 + \sqrt{\frac{1}{2}}}{2}, z_2 = \dfrac{1 - \sqrt{\frac{1}{2}}}{2}$

$\therefore$ $z = \dfrac{1 + \sqrt{1 - 4(x^2 + y^2)}}{2}$ is the upper limit, $z = \dfrac{1 - \sqrt{1 - 4(x^2 + y^2)}}{2}$ is the lower limit

$\therefore$

$$\iiint\limits_{\Omega} f(x, y, z)dxdydz = \iint\limits_{D_{xy}} dxdy \int_{\frac{1-\sqrt{1-4(x^2+y^2)}}{2}}^{\frac{1+\sqrt{1-4(x^2+y^2)}}{2}} f(x, y, z)dz$$

Where D_{xy} is the enclosed area by $x^2 + y^2 = \frac{1}{4}$

Example 1. Calculate the volume enclosed by a plane $x = 0, x = 1, y = 0, y = 2, y = 4z, x^2 + y^2 = 6 - z$

Solution:

$\because$ The boundary of this question is the surfaces of $y = 4z$ and $x^2 + y^2 = 6 - z$ are intercepted by $x = 0, x = 1, y = 0, y = 2$

$\therefore$ The projection of the integration area D_{xy} on the plane xoy is: $0 \le x \le 1, 0 \le y \le 2$

Take $p_0 : (0,1) \in D_{xy}$ randomly, substitute with $y = 4z$ and $x^2 + y^2 = 6 - z$ respectively,

We have $z_1 = \frac{y}{4} = \frac{1}{4}, z_2 = 6 - x^2 - y^2 = 5$,

$\therefore$ $z = \frac{y}{4}$ is the lower limit, $z = 6 - x^2 - y^2$ is the upper limit.

Therefore

$$V = \iiint\limits_{\Omega} dxdydz = \iint\limits_{D_{xy}} dxdy \int_{\frac{y}{4}}^{6-x^2-y^2} dz = \int_0^1 dx \int_0^1 dy \int_{\frac{y}{4}}^{6-x^2-y^2} dz = \frac{49}{6}$$

6. CONCLUSION

With the rapid development of modern science and technology, the information age has increasingly imposed higher requirements on the comprehensive innovation quality of college students. As an important fundamental curriculum in university science and engineering, finance and economics, and humanities education, university mathematics teaching should closely integrate the development of modern science and technology, continue to explore teaching methods, and strive to cultivate and develop students' innovative thinking and independent, motivated learning ability. How to cultivate students to become high-quality talents with a comprehensive innovative spirit has become an urgent problem that every college math teacher needs to tackle. To this end, in our teaching, while thoroughly explaining the basic concepts and paying close attention to basic training, we must make flexible use of the dialectical thinking, grasp some representative typical problems and the sort of confusing and ambiguous problems, and carry out in-depth analysis. In this way, we can open the students’ minds to learn the basic dialectic methods of analyzing problems.

Revolving around the learning characteristics of university mathematics, in teaching, we should also pay special attention to guiding students to think and analyze problems dialectically and from multiple perspectives and to inspire them to classify, summarize, apply, open up, and practically solve the issues drawing on the theoretical frameworks and thinking methods unique to university mathematics. We should enthusiastically encourage students to apply their knowledge of college mathematics deeply, actively participate in extracurricular scientific and technological activities, and choose some topics within their ability to conduct research. Students need to be provided enough room for independent thinking and self-exploration, so as to further promote students’ self-motivation and initiative in those teaching activities.

To reform university mathematics education is to make full use of modern teaching methods and multimedia platforms with the aim to continue inspiring students' comprehensive innovation consciousness and innovative ability. We need to pay close attention to the reform of the university mathematics curriculum and strive for the so-called “Six Breakthroughs,” that is, the prospective, innovative, groundbreaking, scientific, rigorous, and practical breakthroughs. We should organically integrate the development of the discipline and the latest knowledge into the teaching of university mathematics courses, continuously expand students’ horizons and knowledge, and cultivate a full range of qualities among students in their innovation and comprehensive abilities of analyzing and solving practical problems.

The extremely wide application of mathematics powerfully illustrates the famous saying “the 21st century is the information age whereas the information age is the mathematics age.” Therefore, this requires that university mathematics education must closely combine theory with practice, and constantly reform and innovate to adapt to the new requirements of the development of the information society for university mathematics teaching. Through college mathematics innovation education, we will cultivate more innovative and compound talents for the country.

Practice of teaching proves that advanced mathematics education is a complex system of engineering[11-14]. In the curriculum of college mathematics, we must focus closely on the theme of "innovative education," breakthrough the traditional teaching model, and continuously explore and deepen effective, groundbreaking innovative education in the reform of college mathematics education. We would also need to continuously reform teaching methods to further stimulate students' motivation and creativity in learning. We strive to cultivate more comprehensive talents for higher education in China.

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