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The Period of Physical Pendulum Motion with Large Angular Displacement

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ABSTRACT
A physical pendulum experiment now becomes essential in Physics Education. A physical pendulum consists of a rigid object, with a particular shape and size, swung by a pivot point on the object. One of the often-overlooked requirements in measuring practice by this experiment is the small angular displacement. Nowadays, there is little or even no literature explaining the period formula of a physical pendulum with large angular displacement. A typical period formula is used very firmly, requiring this to be related to the approach used in its derivation. Thus, if the pendulum angular displacement is quite large, a correction should be made to the existing formula. This research is aimed to get the formula for determining the period of the physical pendulum experiments with various swinging angles, both small and large angles. After the formula was obtained mathematically, the results were compared to the experimental measurement of the physical pendulum period with various swinging angles. Through the mathematical analysis of calculus and the measurement of the physical pendulum period with the level of time precision measurement to microseconds, it is obtained that the physical pendulum period depends on the swinging angle value of the pendulum but not in a linear relation. This fact emphasizes the formula achieved from mathematical analysis.

Keywords: *Physical pendulum, period, large angle, displacement*

1. INTRODUCTION

A pendulum is a body suspended from a fixed support so that it swings freely back and forth under the influence of gravity. When a pendulum is displaced sideways from its resting, equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position. When released, the restoring force acting on the pendulum's mass causes it to oscillate about the equilibrium position, swinging it back and forth. The mathematics of pendulums is in general, quite complicated. Simplifying assumptions can be made, which in the case of a simple pendulum and physical pendulum, allow the equations of motion to be solved analytically for small-angle oscillations. In everyday practice, a simple pendulum and physical pendulum can be practiced in a basic physics laboratory to determine gravity acceleration and moment of inertia [1], [2].

A simple pendulum experiment is often practiced in schools and the basic physics laboratory in colleges so that the concept of a simple pendulum is well-known to both school students and college students. Meanwhile, the fact that the mass object distribution is not the same, and cannot be ignored by the mass of the string, causes the concept of "the length of the pendulum" in the formula of the pendulum period to become a complicated question [1]. It is also difficult to meet the conditions required for a simple pendulum, especially the string mass and the shape of the object, which is very difficult to ignore [3], [4].

For this time being, in the physics laboratory of engineering departments of universities or colleges, the physical pendulum experiment has not been as famous as the simple pendulum experiment. However, in terms of the application and the accuracy of the result, the physical pendulum experiment is more oriented towards the practical facts than the simple pendulum experiment, which is more likely to be a part of pure science [5], [6]. Another physical pendulum experiment advantage compared with the simple pendulum experiment is the introduction of the concept of real objects in terms of the form and size through the understanding of the moment of inertia [5]. In contrast, the simple pendulum experiment is only applied to objects classified as particles [6]. Therefore, the physical pendulum experiment is preferable to replacing the simple pendulum practicum module.

A physical pendulum is a rigid object, with a particular shape and size, swung by a pivot point on the object. It consists of a rigid body, with a particular shape and size, hanged-up on a particular pivot point, displaced from a balanced position, and then released until it is oscillating. If a simple pendulum fulfills a simple harmonic translational motion, it moves in harmonic rotational motion. It is called to move in simple harmonic rotational motion if it meets the requirements: the pivot is free of friction, the body does not extend elastically, the mass and the moment of inertia are constant, there is no external torque that works besides torque from its weight force, and the angular displacement is relatively small [1]–[6]. It has been proven that this physical pendulum experiment will become an effective and efficient practicum module significantly to determine gravity acceleration and moment of inertia of a rigid body [1], [3], [4], [6].

One of the often-overlooked requirements in measuring practice is the small angular displacement [7], [8]. There is little literature explaining the period formula of a physical pendulum with large angular displacement [9], [10]. Among those few, there are studies on the linearity of oscillations, which have concluded that the period of physics pendulum involving integral elliptic calculations[11]. The results of this study will serve as a comparison to the model outlined in this paper. A typical period formula is used very firmly, requiring this to be related to the approach used in its derivation. Thus, if the angular displacement of the pendulum is quite large, a correction should be made to the existing formula [7], [8], [11].

2. OBJECTIVES

This research is aimed to get the formula for determining the period of the physical pendulum experiments with various swinging angles, both small and large-angular displacements.

3. METHODOLOGY

The methodology used in this research was descriptive analysis. In this paper, the differences in the formula derivation approach between the periods of a small-angle physical pendulum with the period of a large-angle physical pendulum are explained.

After the formula was obtained mathematically, the results were compared with the experimental measurement of the physical pendulum period with various swinging angles. The period measurements were performed using a light barrier backed up by a digital counter with a precision of time measurement in microseconds.

The data was collected at the Applied Physics Laboratory of Politeknik Negeri Bandung with the concept of harmonic motion of the physical pendulum [1]. The data observed from the concept of harmonic motion using a physical pendulum was the period of a homogeneous rod with a certain mass and length and various angular displacements.

4. RESULT AND DISCUSSION

A physical pendulum consists of a rigid body, with a particular shape and size, hanged-up on a particular pivot point, displaced from a balanced position, and then released until it is oscillating. If a simple pendulum fulfills a simple harmonic translational motion, it moves in harmonic rotational motion. It is called to move in simple harmonic rotational motion if it meets the requirements: the pivot is free of friction, the body does not extend elastically, the mass and the moment of inertia are constant, there is no external torque that works besides torque from its weight force, and the angular displacement is relatively small [1]–[6].

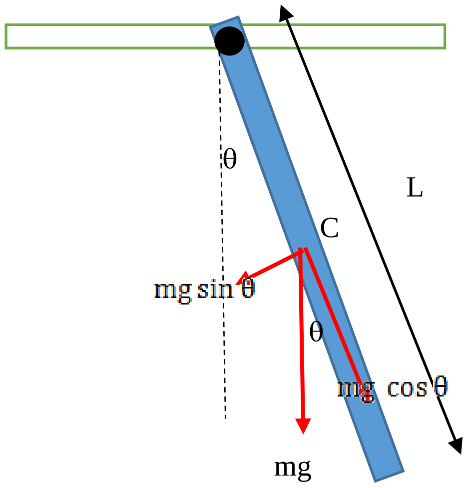

Figure 1 Homogeny rod as a physical pendulum with a pivot at the end of the rod

A homogeneous rod with L in length and m in mass, as seen in Figure 1, swings with a pivot at one end of the trunk. The rod swings with an angular displacement of θ. The torque that drives the rod is

$$\tau = -mg\frac{L}{2}\sin\theta \qquad (1)$$

According to the rotational motion equation, this torque is equal to multiplication between the moment of inertia (I) of the rod and the angular acceleration (α).

$$\tau = I\alpha = I\ \frac{d^2\theta}{dt^2} \qquad (2)$$

If equation (1) is likened to equation (2), then obtained:

$$I\ \frac{d^2\theta}{dt^2} + mg\frac{L}{2}\ sin\theta = 0 \qquad (3)$$

For a small angle of displacement, sin θ ≈ θ, so that

$$I\ \frac{d^2\theta}{dt^2} + mg\frac{L}{2}\ \theta = 0 \qquad (4)$$

or

$$\frac{d^2\theta}{dt^2} = -mg\frac{L}{2I}\,\theta \qquad (5)$$

In this condition, the pendulum swings following harmonic oscillations, which means

$$\frac{d^2\theta}{dt^2} = -\omega^2\,\theta \qquad (6)$$

with ω is an angular frequency, equal to ω=2π/T, with T is the swing period, it is obtained:

$$\omega = \sqrt{mg\frac{L}{2I}} \qquad (7)$$

So the period of a physical pendulum with small angular displacement is

$$T = 2\pi\sqrt{\frac{2I}{mgL}} \qquad (8)$$

Meanwhile, for the large angular displacement, equation (3) is reviewed again. Suppose

$$K = \frac{mgL}{2I} \qquad (9)$$

So, equation (3) can be written as

$$\frac{d^2\theta}{dt^2} = -K\,sin\theta \qquad (10)$$

If equation (10) is multiplied by dθ/dt (where dθ/dt = ω, i.e., angular speed), then it is obtained

$$\frac{d^2\theta}{dt^2}\cdot\frac{d\theta}{dt} = -K\sin\theta\,\frac{d\theta}{dt} \qquad (11)$$

Thus, the motion of the physical pendulum is modeled by this differential equation (11), and it can be written as follows

$$mgL\;\sin\theta\,\frac{d\theta}{dt} + I_p\omega\,\frac{d\omega}{dt} = 0 \qquad (12)$$

or

$$mgL\;\sin\theta\,\frac{d\theta}{dt} + I_p\,\frac{d\theta}{dt}\,\frac{d^2\theta}{dt^2} = 0 \qquad (13)$$

If the left and right segments of the equation (11) are integrated against the time

$$\int\left(\frac{d^2\theta}{dt^2}\right)\left(\frac{d\theta}{dt}\right)dt = -\int K\sin\theta\left(\frac{d\theta}{dt}\right)dt \qquad (14)$$

then it will be obtained

$$\frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 + C_1 = K\cos\theta + C_2 \qquad (15)$$

or

$$\frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 = K\cos\theta + C_3 \qquad (16)$$

then it will be obtained

$$\frac{d\theta}{dt} = \sqrt{2K\cos\theta + 2C_3} \qquad (17)$$

or

$$\frac{dt}{d\theta} = \frac{1}{\sqrt{2K\cos\theta + C_4}} \qquad (18)$$

If this is integrated against the θ angle, the left-field with the t boundary of T /4, is the right field θ from 0 to θ,

$$\int_0^{T/4}\left(\frac{dt}{d\theta}\right)d\theta = \int_0^{\theta}\frac{d\theta}{\sqrt{2K\cos\theta + C_4}} \qquad (19)$$

It will be obtained

$$T/4 = \int_0^{\theta}\frac{d\theta}{\sqrt{2K\cos\theta + C_4}} \qquad (20)$$

or

$$T = 4\int_0^{\theta}\frac{d\theta}{\sqrt{2K\cos\theta + C_4}} \qquad (21)$$

If the integral is calculated, the solution is obtained in a swing period, namely

$$T = \frac{8}{\sqrt{K}}\left(\ln|\sec(\theta/2) + \tan(\theta/2)| + C\right) \qquad (22)$$

or

$$T = \frac{8}{\sqrt{K}}\left(\ln|z| + C\right) \qquad (23)$$

with $z = |\sec(\theta/2) + \tan(\theta/2)|$

Entirely, at this moment the period of a physical pendulum is

$$T = \frac{8}{\sqrt{(mgL/2I)}}\left(\ln|\sec(\theta/2) + \tan(\theta/2)| + C\right) \qquad (24)$$

C states a constant that depends on the initial condition and the structure of the object.

In equation (24), the physical pendulum period depends on the length of rod mass, the rod’s moment of inertia, and the angular displacement. The relation between the periods to the angular displacement stated in equation (24) can be described in the graph, as seen in Figure 2 (small angular displacement) and Figure 4. Meanwhile, the results of experiments in the Physics Laboratory of Politeknik Negeri Bandung using the physical pendulum device are shown in Figure 3 (for small angular displacement) and Figure 5 (for large angular displacement). For the theoretical calculation results, no absolute number is used, but it is still expressed with multiples of 8/√K. Besides, to get an absolute number, it is also necessary to take into account the effect of air friction, which will undoubtedly get more robust if the angular displacement gets larger [12]–[14].

By comparing the graph in Figure 2 (theory) with the graph in Figure 4 (experiment), as well as the graph in Figure 3 (theory) with the graph in Figure 5 (experiment), there is a similar tendency to function of the period to the physical pendulum angular displacement. At a small angular displacement which is smaller than 15°, the swing period increases in line with the enlarged angular displacement with minimal changes, so the equation (8) which states that the period does not depend on the angular displacement should still be considered valid. At a large angular displacement, there is a similar tendency in the calculation of theory, and the results of experiments, i.e., both show that the period is enlarged with the increasing angular displacement.

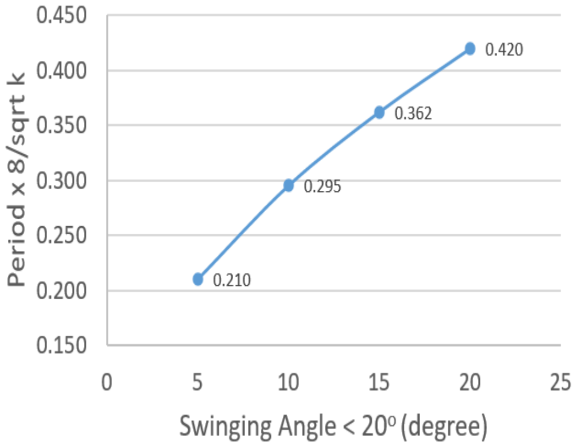

Figure 2. The graph of the relation between swinging angle (degree) to physical pendulum period (s) for small-angle (the result of the calculation)

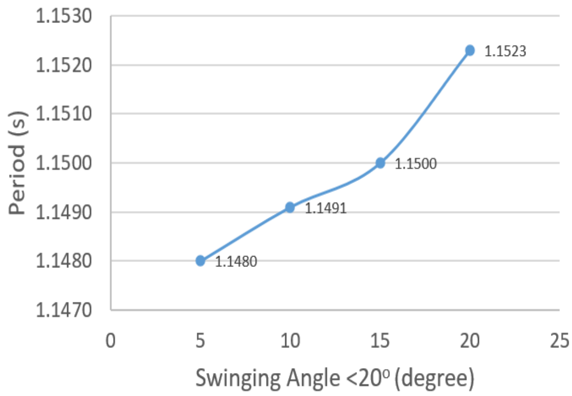

Figure 3. The graph of the relation between swinging angle (degree) to physical pendulum period (s) for small-angle (the result of the experiment)

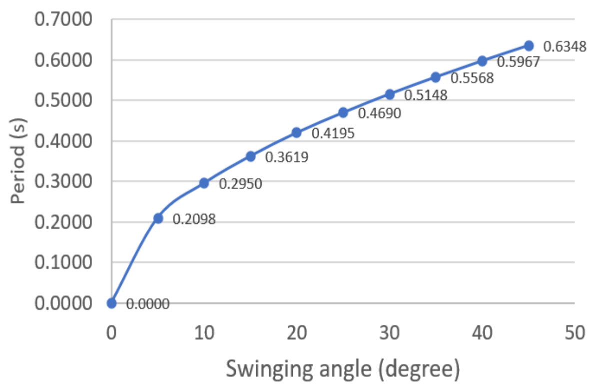

Figure 4. The graph of the relation between swinging angle (degree) to physical pendulum period (s) (the result of the calculation)

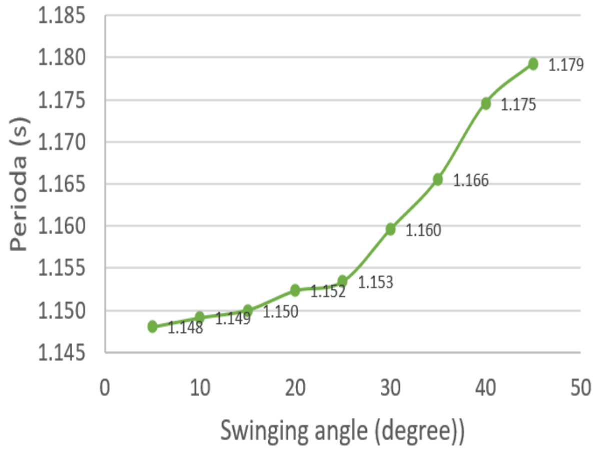

Figure 5. The graph of the relation between swinging angle (degree) to physical pendulum period (s) (the result of the experiment)

If the mathematical calculation results of the models in this paper, i.e., those listed in the equation (24), compared to the results obtained by other researchers, namely the model of the nonlinear pendulum [11], there is a similarity in the tendency that occurs, namely periodic reliance on displacement angles. From the no linear pendulum model, the period of the physical pendulum gets more extensive if the angle of the displacement is enlarged following the integral elliptic form:

$$T = 4\sqrt{\frac{L}{g}}\int \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}} \qquad (25)$$

Therefore, the calculation of the swing period model in this paper further reinforces the assertion that the physical pendulum oscillation is not linear.

Through the mathematical analysis of calculus and the measurement of the physical pendulum period with the level of precision to microseconds, it is obtained that the physical pendulum period depends on the pendulum swinging angle value but not in a linear relation. This fact emphasizes the formula achieved from mathematical analysis as well as other researcher proposals [11].

5. CONCLUSION

At a large swinging angle, the physical pendulum has an inconstant period if the angle of displacement is changed. The relation between the period and displacement angle is not simple, but it can generally be said that if the angle of displacement is enlarged, then the period increases and this increase is not a linear function. Thus, the oscillation motion of the physical pendulum is not linear. As a furtherance of this research, it is necessary to consider the presence of air friction against the motion of this physical pendulum oscillation, especially for large angle displacements.

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