Conjecture in Completing Creative Problem-Solving Question as a Part of Development

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ABSTRACT
This research is a descriptive study that aims to describe the mathematical abstraction type conjecture in triangle and square. The subjects in this research were 3 Grade VIII students of Junior High School. This study was a design research which consisted of preliminary design, focus group discussion, trial, interview, and retrospective analysis. The data collection techniques used were test with problem solving type and interviews. Test data and interview data were analyzed by using qualitative descriptive methods, by telling everything that was obtained both from student answers and from interviews. From the results of data analysis, it was concluded that all students understood the problem. However, understanding the problem does not guarantee that students can put forward and propose conjectures. Even though the conjecture has the possibility of being true and of being false, good conjecture must be supported by the underlying theories and concepts. Students who do not have basic concepts are often unable to make conjectures and prove these conjectures, thus it can be concluded that students' ability to make conjectures is influenced by their previous knowledge. The stages in making conjecture consists of understanding the problem, exploring the problem, formulating conjecture, justifying conjecture, and proving conjecture.

Keywords: *Mathematical abstraction, Conjecture, Square, Triangle*

1. INTRODUCTION

Karadag categorized mathematical thinking into seven major themes: modeling, reasoning, symbolization, representation, proving, abstraction, and mathematization [1]. One of the part in mathematical thinking is abstraction. Abstraction as a process in which students vertically reorganize previously constructed mathematics into a new mathematical structure [2]. There are five indicators of abstraction; generalization, specialization, observation of patterns, conjecturing and testing conjecture [3,4]. Abstraction as a process of cognition for all subjects in constructing mathematical conjecture was done by determining common processes for all subjects and eliminating the condition of the problem used [5]. A conjecture is a statement that can be true or false, appears reasonable, “has not been convincingly justified and yet it is not known to be contradicted by any example, nor is it known to have any consequences which are false [6].

Conjecture is a statement about all possible cases based on empirical fact with an element of doubt. One way to construct mathematical knowledge with constructing conjecture which is a way to know information that’s in a problem and knowledge that has been possessed before [7]. Formulating conjecture means making statement about all possible cases, based on empirical facts, but with an element of doubt [8]. True or false of the conjecture can be proven by common sense process with logic, after it has been proven, it becomes a valid question [9]. Conjecture has five indicators: understanding the problem, exploring the problem, formulating a conjecture, justifying the conjecture, proving the conjecture [10,11]. In addition, constructing mathematical conjecture and developing proofs are two fundamental aspects of professional mathematical work [12]. NCTM stated that a program in mathematics instruction should enable all students to recognize reasoning and proof as fundamental aspects of mathematics, make and investigate mathematical conjectures, develop and evaluate mathematical arguments and proofs, and select and use various types of reasoning and methods of proof [13]. Research on conjecturing in problem solving has been widely practiced [8, 14, 15].

The low level of abstraction in the conjecture stage is due to the fact that students are less accustomed to work on Higher Order Thinking Skills (HOTs) questions, students tend to work on routine problems [16]. Based on the research conducted by Adelia [17]. It was found that students were able to propose conjectures but were unable to correctly do the trials, were unable to provide reasons for some of the solutions given and were unable to draw conclusions correctly. And also, according to a research conducted by Bergqvist explains that students tend using abstraction conjecture common sense but still couldn’t prove which one is true or false and not providing the conclusion according their expectations which they made by the lack of practice. An alternative to guide conjecture abstraction process is to use creative problem solving. Creative problem solving is a method to solve a problem that’s creative and innovative [18]. There’s also creative problem solving phase, which visioning or objective finding, fact finding, problem finding, idea finding, solution finding dan acceptance finding [19]. This study aims to describe the mathematical abstraction type of conjecture in triangle and square. How students solve triangle and square by using mathematical abstraction type conjecture.

2. METHOD

2.1 Research design

This study used a design research [21]. The aim of this study was to describe mathematical abstraction type conjecture ability in square and triangle. The study focused on the conjecture aspects. This study was design research which consisted of preliminary design, focus group discussion, trial, interview, and retrospective analysis.

2.2 Subject

The research subjects were 3 Grade VIII students of SMP Negeri 07 Jambi. The sample of the research was chosen randomly. The characteristics of the subjects were categorized as heterogeneous consisting of a mixture of high-ability, medium-ability, and low-ability student. The three subject also consisted of good, medium and low mathematical abilities. Selection of sample was based on the willingness of students to become respondents and the availability of facilities and infrastructure that support learning activities online.

2.3 Instrument

The instruments used in this study were test and an interview. The written test sheet consisted of two essay questions with a duration of 40 minutes’ completion. This question was designed to explore students' conjecture. The written and interviews were recorded by using Zoom.

2.4 Data Collection and Analysis

The data analysis used was descriptive qualitative technique. The data from the test results were presented in screenshots and then given comments on how the student was doing the conjecture. After the researcher observed the students’ answers, then an interview was conducted to dig up information about the students' reasons for giving answers as they had written on the test.

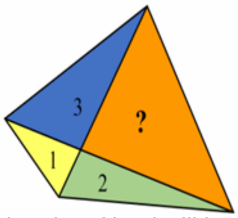
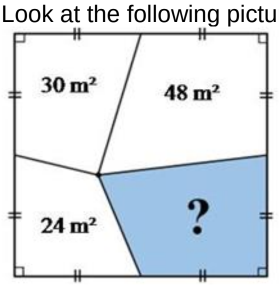
3. RESULTS AND DISCUSSION

This research followed the stages of design research. The results of this study focused on the students' ability to do conjectures when solving problems. The conjecture ability of students was observed based on the answers that have been written by the students and the results of the interviews. In this result, the students' ability to relate a theory to make allegations based on facts was explained to obtain and prove the association of these allegations in a formal evidence. In making conjectures students must think broadly and flexibly about the ideas in solving problems and try to understand mathematical problems. From the results of data analysis of the test, information was obtained that the three subjects understood the problem given. This seems to be the result of fact-finding. Students can identify the information contained in the problem.

3.1. Preliminary Design

In this stage, the focus was on the theories of abstraction and conjecture. The problem in this research was designed based on the literature of the study. There were two questions that were designed related to mathematical abstraction type conjecture. Here are two problems that have been designed.

Table 1. Designed problems.

Problem number	Problem
1	Look at the following picture!  The plot of land will be planted with various flowers, including 1 square meter roses, 2-meter square jasmine flowers, 3 meter square tulips, and the rest will be planted with Lily flowers. Determine the remaining area to be planted with Lily flowers.
2	Look at the following picture!  Mr. Ahmad has an inheritance of $a\ m^2$ plot of land. Mr. Ahmad wants to distribute the inheritance of the land to his four children in the following parts; The first child gets an area of 24 square meters, the second child gets 30 square meters, the third child gets an area of 48 square meters, and the fourth child gets the rest. Can you guys help Mr. Ahmad to calculate the remaining land area?

3.2 Focus Group Discussion

The two problems were discussed with 2 Mathematics Education Lecturers. The following table summarizes the result of the discussion.

Table 2. Group Discussion

Lecturer	Comment and Advise about Problems
1	• Consider the indicator of question • Correct the EYD in the questions.
2	• Correct the sentences in the questions so they don't have multiple interpretation • Add the allocated time to solve the problem. • Correct in EYD

After the group discussion with the lecturers, the researcher considered the advice. There was not any change on the problems.

3.3 Trial

The problems were tested on three subjects. The trial showed that the students understood the problems and tried to solve it. There was no question about the problems.

3.4 Interview

The interview was conducted after the written test. The interview used were semi-structured. Firstly, the researcher only had key questions, but as the interview progressed, the researcher can develop questions from these key questions. The interview aimed to confirm the student's difficulty and experience during problems solving. It was conducted to clarify which indicator of conjecture appeared.

3.5 Retrospective analysis

3.5.1 The analysis of subject's answer in problem number 1.

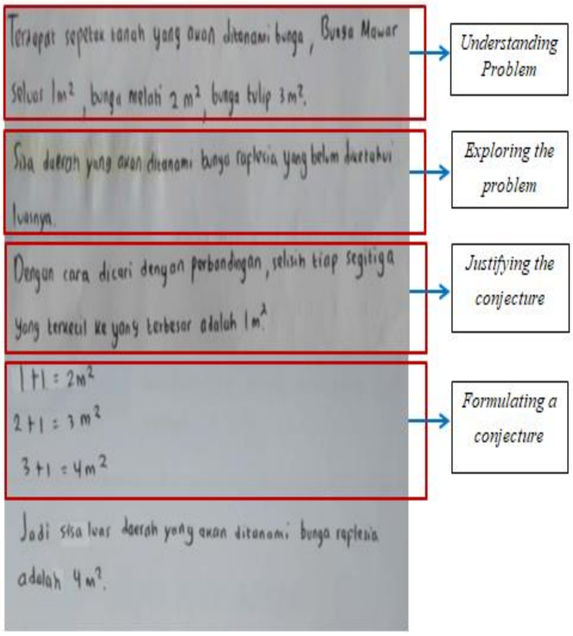


Figure 1. Subject ZB's answer

Figure 1 shows conjecture indicator of subject ZB in answering problem number 1. ZB seemed to understand the problem given, was able to know all the information in the problem, was able to argue the conjecture by seeing the regularity in solving the problem, namely that the area of each triangle had a difference of one. Furthermore, with this assumption, Subject ZB wrote that if it is known that the area of the first triangle is 1, then the area of the second triangle is 1 + 1, the area of the third triangle is 2 + 1 and the area for calculating the area of the fourth triangle can be done in the same way as 3 + 1. At first glance, it seemed that the conjectures made by the students look correct, but the conjectures that the students made were not supported by theory or concept.

In problem 1 it is clear that the problem presented is about triangles so the concept used should be triangles. In finding the area of a triangle, one must be able to identify which is the base and which is the height of the triangle so that it can be concluded that the conjecture made by the students is wrong and and the way of proving the conjecture is also wrong.

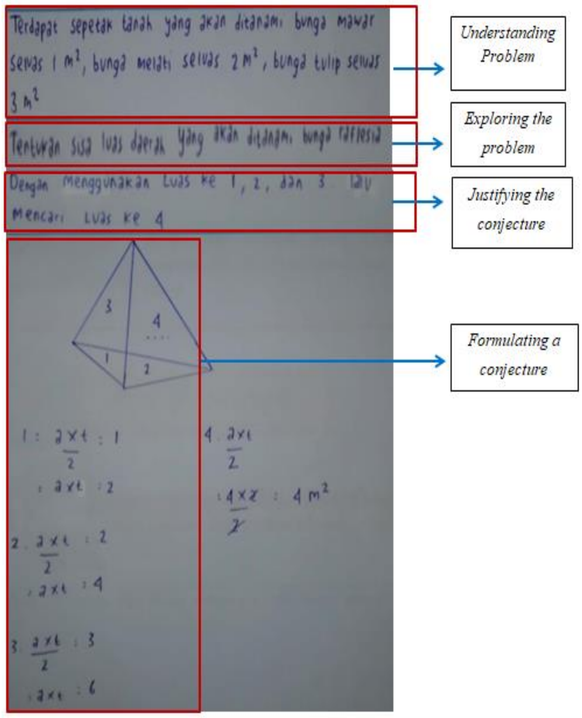


Figure 2. Subject ZH's answer

Figure 2 shows conjecture indicator of subject ZH in answering problem number 1. Subject ZH seemed to understand that the problem presented was a problem regarding the area of a triangle so that the concept used was the concept of the area of a triangle. Therefore, ZH suspected that to solve the given problem, it was necessary to examine and find the size of each triangle by using the concept of a triangle, conceptually the conjecture made by ZH was logical and correct. To prove the conjecture that was made, ZH started by re-sketching the problem given. Next, ZH wrote the formula for the area of a triangle four times for each region and it was as if there were four equations. However, ZH cannot connect the four equations. So that it caused the solution made to be wrong.

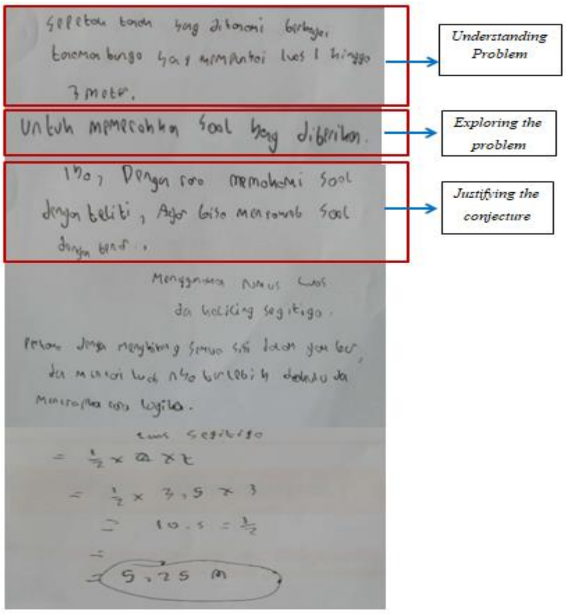


Figure 3. Subject TQ's answer

Figure 3 shows conjecture indicator of subject TQ in answering problem number 1. In solving this problem, TQ seemed to understand the problem and was able to explore it. This can be seen from the fact-finding results presented in the problem. This can be seen from the following interviews with TQ:

P	:	*What do you understand from this exercise?*
TQ	:	*So this land belongs to Mrs. Mira. This plot of land will be planted with various flowers. Roses are 1 square meter wide, Jasmines are 2 square meters, Tulips are 3 square meters wide, so our job is to find the area of land to be planted with Raflessia flowers.*
P	:	*Good, that means you understand. Is there a sentence that you don't understand on this problem?*
TQ	:	*No, miss.*

Furthermore, when asked to explain what must be done to solve problem 1. TQ stated that he used the formula for the area and perimeter of the triangle. Based on the results of the interview, it was also obtained information that TQ was not sure about the answers written. Next, from the written solutions and the results of the interviews, information was also obtained that in order to obtain a solution to problem 1, TQ assumed that the height of the fourth triangle was 3 and to determine the length of the base of the triangle, TQ measured by using a ruler. The result of measurement by using a ruler showed that the length of the base of the triangle was 3.5 cm. then using the triangle formula, TQ got that the area of the triangle was 5.25 cm. This resulted in an incorrect answer to problem 1. It can be concluded that the given conjecture was wrong and the way of proving the conjecture is also wrong.

3.5.2 The analysis of subject's answer in problem number 2.

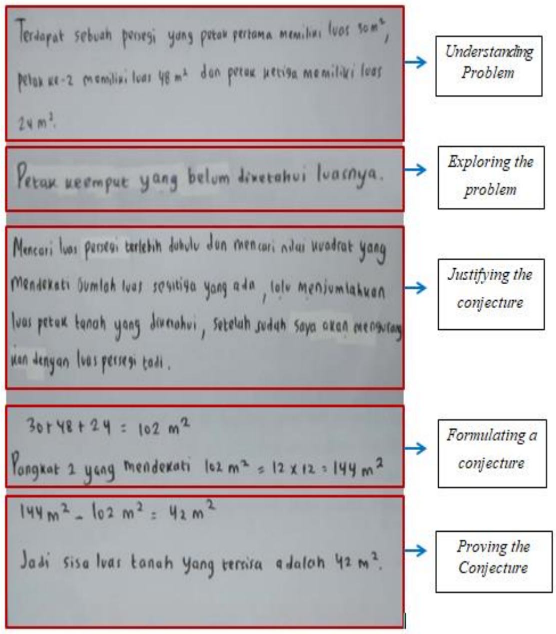


Figure 4. Subject ZB's answer problem 2

Figure 4 shows conjecture indicator of subject ZB in answering problem number 2. In solving problem 2, ZB understood the existing problem, he was able to explore the given problem, and was able to argue the conjecture. This can be seen when ZB has an idea to use a simple conjecture. ZB only related the concept that the area of a square was the square value of a number. To find the length of the side of the number, ZB used the concept that the area of a square was the product of the side times the side. Due to the concept of squares s^2, this was in line with the concept of squares.

Furthermore, to prove whether the conjecture was correct or not, ZB calculated all known areas. From the calculation results, it was found that the area of the mentioned areas was $102m^2$. Next, ZB chose a quadratic number that was close to 102 was 144. The reason for choosing 144 as area was because in the figure it can be seen that the sides of each square can be divided in half. So the easiest number to divide by two was number 12. After ZB was sure that the area of the square was 144, ZB calculated the unknown area of the square by finding the discrepancy of 144 and 102, so it can be concluded that the area of the square was 42 square meters.

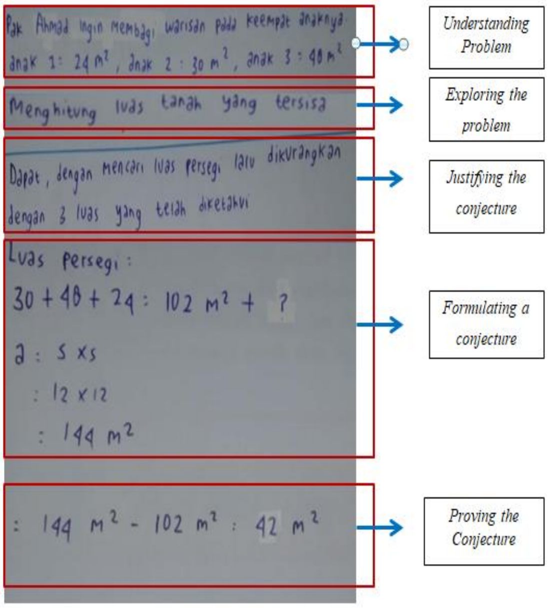


Figure 5. Subject ZH's answer problem 2

Figure 5 shows conjecture indicator of subject 2 in answering problem number 2. In solving the problem, ZH has the same idea as ZB. ZH used a simple conjecture. ZH only related the concept of area square which was the square value of a number. To find the length of the number, ZB used the concept of the area of a square which was the product of the side times the side. Due to the square concept s^2, this was in line with the concept of squares. Furthermore, to prove whether the conjecture was correct or not, ZH did the same thing as ZB, which was by calculating all known areas. From the calculation results, it was found that the area of the mentioned areas was 102 m2. Then, ZH chose a square number that was close to 102 was 144. The reason for choosing 144 as area was because in the figure it can be seen that the sides of each square can be divided in half. So the easiest number to divide by two was number 12. The number 12 will produce an integer number if divided by 2. After ZH was sure that the area of the square was 144, ZH calculated the unknown area of the square by finding the discrepancy of 144 and 102, thus it can be concluded that the area of the square was $42m^2$. Therefore, it can be concluded that students can formulate conjectures and prove the conjectures correctly.

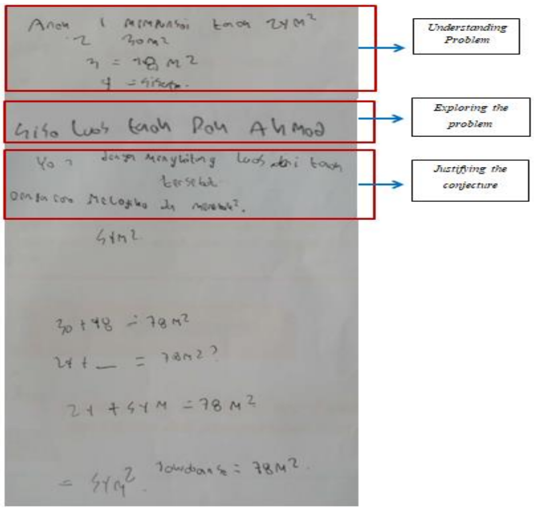


Figure 6. Subject TQ's answer problem 2

Figure 6 shows conjecture indicator of subject 3 in answering problem number 2. In solving problems, TQ understood the problems given, was able to explore the problems given, was able to argue the conjecture. In problem 2 it appeared that the guesswork made by TQ was also unclear. TQ only stated that to solve problem 2 one must use the concept of square area because in the problem it was stated that it was known to be a square. However, TQ cannot add the concept that should be used to solve the problem, as well as how the problem should be solved. From the allegations written by TQ, an interview was then conducted to find out why TQ thought that way and what was the connection between the allegations and the solutions he wrote down. From the results of the interview, it was found that the allegations made by TQ previously were not used. TQ used logic in solving problem 2. In solving the problem, TQ looked for the sum of the two known squares and then assumed that the other two squares have the same area as the first two squares.

In mathematical development, conjecture as formalization of conjecture [21]. It is very important for students to have conjecture ability in case of solving the problems. The advantages of the conjecture process are to elaborate the concept, to play the important rule of understandings, and to support the learning process [22]. Students difficulties in conjecture process can be seen in exercise 1 and 2. Problem 1 show that subject ZB succeeded in the indicators understanding problem, exploring the problem, justifying the conjecture and formulate conjectures but the conjectures that made ware mistaken and prove the conjectures also mistaken. ZH succeeded in the indicators understanding problem, exploring the problem, justify assumptions and formulate conjectures but the conjecture that was wrong that proved the conjecture also wrong. TQ is successful in understanding the problem and exploring the problem. Problem 2 shows that subject ZB succeeds in indicator understanding problem, exploring the problem, justifying the conjecture can formulate a conjecture but the conjecture is made correct and proves the conjecture is also correct. Subject ZH has the same idea as Subject ZB so it is concluded that the conjecture is made correct and proves the conjecture is also true. Subject TQ succeeded in indicator understanding the problem exploring the problem. Based on study, it showed that there were still doing the conjecture without knowing the right concept. Therefore, when they were asked to prove the conjecture, some students seemed doing many mistakes in making completing [23].

Students who can make conjecture mathematical were based on five indicators are understanding problem, exploring the problem, formulating conjecture, justifying the conjecture, and proving the conjecture [10,11]. they were understanding the problem based on the information given, exploring the problem by knowing the basic concepts of the problem presented, formulating conjectures by linking all the information on the problem with basic concepts and knowledge, justifying the conjecture stage, explaining the reasons for the conjecture, generalizing the conjecture and being aware of the deficiency or mistake underlying the formulation of conjectures or their reasons and the proving the conjecture stage were being aware that the truth of conjecture must be proved, and choosing the type of proof according to the constructed conjecture. Proving the conjecture was done by showing figures or doing some algebra manipulation, connecting relevant mathematical knowledge. After the conjecture is probed then it can be valid statement [9].

4. CONCLUSION

The process of student cognition in constructing mathematical conjecture was explained in five different stages, they were understanding the problem, exploring the problem, formulating conjecture, justifying conjecture, and proving the conjecture. From the result of analysis, it was concluded that all students understood the problem. However, understanding this problem did not guarantee that student can put forward and propose conjectures. Even though the conjecture has the possibility of being true or false. Good conjecture must be supported by underlying theories and concepts. Student who do not have basic concepts are often unable to make conjectures and prove these conjecture. So it can be concluded that students' ability to make conjectures is influenced by their previous knowledge.

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