

Predicting Length Game to Help Students Understand Number Line for Decimals

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ABSTRACT

In this study we developed an activity to foster students' understanding on decimal numbers in number line. We tried out Hypothetical Learning Trajectory (HLT) with Realistic Mathematics Education (RME) framework to 4 students grade 5th. For this aim, we conducted design research method. Data of observation, interview, and written work were retrospective analyzed and reflected with the conjectures to evaluate the HLT of measuring lengths activity. We found out that before starting the lesson, students were already familiar with number line for integers, but not for decimal numbers. Then, our design measuring activity supported the teacher to introduce the students number line for decimals. The measuring with the paper strip also provoked students to see the representation of decimal numbers as lengths. The students made line segments on the number line and symbolized it with decimal numbers. In addition, the students could argue which number was closer to a decimal number. This shows that the students could relate to the magnitude of one-digit decimal numbers. The comparing problem also made them able to order decimal numbers on the number line.

Keywords: *Decimal numbers, HLT, Measuring, Number line, RME.*

1. INTRODUCTION

This paper is a part of the study that aimed to develop students' understanding on decimals. In this study we developed an HLT in learning decimals with Realistics Mathematics Education as a framework and length measurement as the context. Previously, we designed activities which concerned more on supporting students' learning decimals with understanding of the meaning of decimals by linking decimals with fractions [1][2]. For that, we used a paper strip as our one-unit measuring tool. The paper strip led the students to decimal numbers when they did measurement activities. We then, in this current study, designed a game and set the situations of measurement, still with the paper strip, to emerge number line as a model of one-digit decimal numbers.

A model is needed for conceptual development [3]. Chazan and Ball [4] claimed that one of the most useful representations of number is a number line. Thomson and Walker, as cited in [5], suggested that the number

line contributes to the development of concepts not only related to the identification and comparison of decimals but also to the ability to perform operations. Therefore, the activity of measuring lengths in this present study are designed intending to support students develop a model of number line for decimal numbers.

In this study, what meant by understanding the meaning of decimals by using the number line is that the number line model is used for students to develop more abstract representation of their thinking of decimals and at the same time create a model of lengths of lines. For example, students can see the representation of 2,4 on the number line and can see that its position is between 2 and 3. They use the number line as tool to see ones, and more precise one-digit decimals. Also, connecting length measurement with the number line model provides both visual and symbolic information for students [6]. Summing up, students might be able to see decimal numbers on number line as it represents length as well as to locate decimal numbers on the number line.

In addition, in this study, the aim of using the number line is for students to learn the density of one-digit decimals and two-digit decimals. Density here means that between any two consecutive whole numbers, there are decimals; and between any two different decimals there are other decimals, and so on [7]. When doing the game proposed in the lesson design, like which is bigger 3,4 or 3,6, the students would order some decimal numbers on number line which consequently makes them see the density of decimals. For this aim, our research question is how predicting length game to help students understand number line for decimals.

2. METHOD

Based on the aim of the study, design research is used as the method.

2.1 Data Collection

Four 5th grade students with varied ability namely student 1, student 2, student 3, student 4 would be the focus in this study. Data were collected from written pre-test, observation and interview, and field notes of a lesson. The lesson design was in the form of Hypothetical Learning Trajectory (HLT) based on Realistic Mathematics Education (RME) framework. We implemented the five tenets of RME [8] to design the activity, namely phenomenological exploration, using models and symbols for progressive mathematization, using students' own constructions and productions, interactivity, intertwinement. In addition, we used framework of RME which refer to guided reinvention, didactical phenomenology, and emergent modelling to develop our design [9]

2.2 Lesson Design

In our HLT, the learning goals are to use number line as a model of one-digit decimals, to identify the density of one-digit decimal numbers on the number line, and to recognize between two consecutive whole numbers there are one-digit decimals. As a starting point, teacher could propose some decimal numbers and ask students to explain the meaning of it. Students also practice how to read the decimal numbers. Teacher reminds students that today the activity is measuring with the paper strip. Then, in the main activities, students play sending SMS game in which they need to guess lengths and be as exact as possible on the guess. To play the game, teacher chooses which student to battle with which student. Each student was given an SMS paper and needed to write their guess of the length of the things on it. The length they write must be on decimal notation as they have learned previous meetings. At the beginning, each student starts as a message sender. Next, each receiver student will meet and verify together the real length of the things by

measuring with the paper strip. The student who predicts closer to or even the same with the real length in term of the length of the paper strip is the winner.

In this lesson, students record their decimal numbers on the table in the worksheet and decide whether their prediction is bigger or smaller. They also need to write the reasons how they choose the winner. Here is the conjecture of students' strategies 3,2 (their guess) is bigger than 2,8 (real measuring) because 3,2 uses 3 paper strips while 2,8 uses 2 paper strips; or 6,7 is less than 6,9 because 7 is less than 9; or 5,2 is less than 5,8

because ,2 is $\frac{2}{10}$ while ,8 is $\frac{8}{10}$. We conjecture that none of the students would compare the numbers with number line.

Teacher would choose students' different strategies to be discussed in whole class. For example, students who predict the edge of the book is 1,4 and 1,7 while real measurement was 2,1. The group of students will do presentation about how they decide the winner, the reasons maybe because group A makes less or bigger guess.

Responding to this, teacher draws a line (which is actually the number line, Figure 1) for students to compare their reasoning. The teacher asks students to point the decimal numbers on the number line, for example their estimation is 1,4 and 1,7 while the exact measurement with paper strip is 2,1. Teacher then may ask other groups their guesses. The discussion would focus on: why you think you have a better guess and how you could compare it by the number line. Through this activity, students see the representation of the length on the number line.



Figure 1. Example of number line between 0-5

It is conjectured that it might be difficult for students to point directly where their decimal number on the number line. Therefore, teacher guides them to count the segment on the number line one by one, for example 1,0; 1,1; 1,2; 1,3; 1,4; 1,5; 1,6; 1,7; 1,8; 1,9 and gives mark on 1,4 and 1,7. Then students with teacher continue 2,0; 2,1; 2,2; 2,3; 2,4; 2,5; 2,6; 2,7; 2,8; 2,9; and 4,0 then give mark on 2,1. If students directly put mark on the number line to represent their decimal numbers on the number line, teacher asks them how they know it such that they would make sequence of the decimal numbers on the number line.

By this number line, teacher would discuss their guesses. Teacher asks students how they compare by using the number line. Here, students might say 'we

need two more decimal number' or 'from 1,4 to 2,1 we need 6 more parts. Furthermore, by drawing on the

number line, students recognize the density of the number line. The HLT is outlined in table 1.

Table 1. HLT of the lesson

| Learning Sequence | Activities | Goals | Conjecture of Students' Strategy |
|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------|
| Lesson 3 Number line as a model of one-digit decimals (SMS game) | Predict lengths and verify the real measure with paper strip, Discussion: 'Who predicts closer (or even the same with) the real length?' | Use number line as a model of one-digit decimals: 1) identify one-digit decimal numbers on number line; 2) recognize the density of one-digit decimal numbers on number line | Ordering decimal numbers, seeking difference |

3. RESULT AND DISCUSSION

3.2 Pre-Test Result and Interview

The goals of the written pre-test are two: to know if students are familiar with the number line and are familiar with fractions and decimal numbers on the number line and hence to make revision of the students' starting point of the initial HLT. The data were obtained from students' work on pre-test also from video-taped interview. The interview was done directly after the written test. It aims to clarify how students reason with the tasks of pre-test and to know whether the questions are clear and understandable for them.

It was noticed that students knew number line. In problem 1 they comprehended positioning consecutive positive integers on the number line. Yet, problem 2 which asked what number labelled with A, B, C, and D in the middle of number 1 and 2, 2 and 3, 3 and 4, 4 and 5, showed the students did not know much about decimals on number line (figure 2). Only student 1 came with a correct answer. He wrote 1,5; 2,5; 3,5; 4,5. When being interviewed, his reasoning was that the alphabets were in the middle, so it should be a half.

Actually with the prior knowledge of fraction students have learned in mathematics class, if not with decimals, it was expected the students would answer with fractions. Student 2 did, but incorrectly, like A was $\frac{1}{5}$, B was $\frac{2}{5}$. The interview showed this misunderstanding, he mentioned 1.5 was one and a half but also said it was $\frac{1}{5}$. Probably he was distracted with the fact he knew a half was ,5, but he wrote with notation $\frac{1}{5}$. Two other students, student 3 and student 4, wrote negative numbers on the number line, consecutively -1, -2, -3, -4. In the interview, these students said they knew on the number line there were positive numbers and negative numbers.

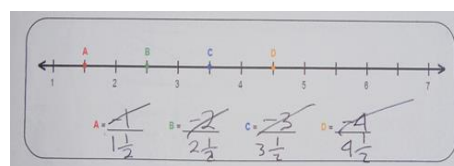


Figure 2. Students' work on pre-test

In problem 3, given number line with 10 partitions between two positive integers, the students needed to figure out the numbers of each segment. Student 2 again used fraction, but incorrectly. He started to use arbitrary numerical relation in making fraction. It seemed that his reason was by counting the strip on the number line (see figure 3). Student 3 and student 4, in problem 3, again wrote negative numbers. And again only student 1 answered correctly, with decimal numbers: 3,4; 3,9; 4,1 and 4,2. However, when we tried to explore more his understanding on decimals in the interview, student 1 did not show he knew much about decimal numbers, he said 'number with comma' and gave example counting 3,0; 3,1; ...; 3,9; 3,10; he also said 3,10 was the same with 4.

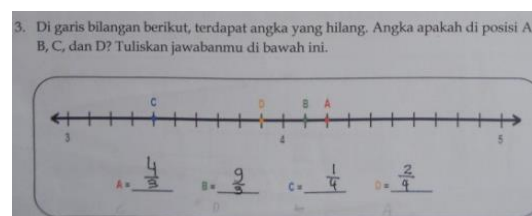


Figure 3. Student 2's incorrect fractions in a pre-test question

From the students' answers in pre-test and interview, the researcher concluded that the students were familiar with number line but not with fractions or decimal numbers on the number line. This was as expected before in the initial HLT. However students' problems with fractions was not expected. Therefore, during

previous lesson, the researcher it was planned that the teacher would intervene the students' understanding of fractions.

3.2 The Implementation of the HLT

We designed an activity named SMS game in which students predict lengths of things in classroom in paper strip unit. Then, students need to compare their predictions with the real measuring with the paper strip. In the discussion, students compare their measurement, and discuss who made closer guess. Then teacher introduces number line as a model of one-digit decimals.

3.2.1 Number Line as a Model of One-Digit Decimals

Description of the Learning

The number line is the most useful representation to connect decimals and fractions, its characteristics also promotes a model for density of real number [9]. Therefore, we planned for the students to play SMS game this meeting. The game aimed to deliver the students using number line model and comparing the decimal numbers on the number line. They would make prediction of length of the things listed on their worksheet, send it to their friends and then together verify whose prediction was closer to the real length. The real length means the length with term of the length of the paper strip.

Student 2 was ill today, so there were only three students. As a starting point, the class began with discussing previous lesson and the students were shown a paper strip and asked to write tenths and its decimals. They showed ability to find relation between fraction and decimal numbers (see figure 4). Additionally, the students could argue their thinking that comma is used to separate parts and ones. However, this time it was noticed the students tended to show that fraction was the whole small box on the paper strip. This made them write fraction number not exactly under line. The teacher told them that the number represented length, so put it under the line.

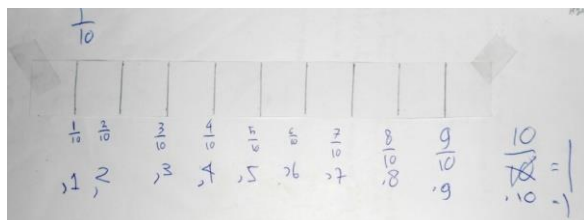


Figure 3. The students made relation of equivalent tenths and decimals

The SMS game was played individually. Student 4, when was asked to predict, measured lengths with her one hand and said the result based on counting how

many fingers needed. She seemed not understand to make prediction regarding the paper strip as the unit. Student 1 and student 3 understood that they needed to predict based on the paper strip. Both of them also used their hands but with an estimation of the length of the paper strip.

In report phase, all the students used decimal numbers to state the lengths. They then verified the lengths together measuring with the paper strip. It was found student 4 wrote cm after her decimal numbers.

As conjectured in the HLT, none of the students used number line to compare. The teacher also did not introduce them number line directly, because they started reasoning with their own words. Some students' idea when comparing and decided who was the winner: "because the numbers are too far", "because it is precise", "4 is a little more from 3,8", "4,0 should be minus 2 parts to make 3,8 because 4,0 3,9 3,8". Student 1 said "we add 7 more parts", student 4 said "because 3,0 is 1 more to make 2,0".

When the students compared 0,8 or 1,3 closer to 2,0? The teacher then drew them a number line from 0 to 2 and asked to put the numbers on it. The students were able to write the decimal numbers on the number line and compare the decimal numbers (see figure 5). From the picture, it was also noticed that they could put the decimal numbers in good order. The problem appeared when student 4 wrote down the decimal numbers between 1 and 2, she counted until 1,10. The teacher intervened her that after 1,9 was 2. She was told it was because we divide the number line until 10 as we did to our paper strip.

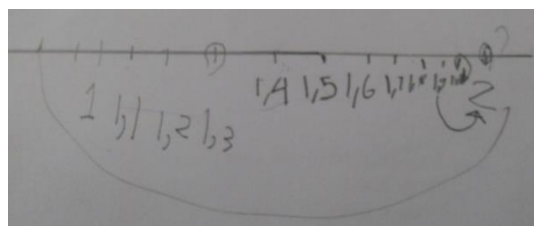


Figure 5. The students compared whether 0,8 or 1,3 was closer to 2,0

Reflection on the Learning

The prediction context through the SMS game supported the teacher to introduce the students about number line. The students were able to write the decimal numbers on the number line when asked by the teacher (the teacher drew the number line) and compare the decimal numbers. They, when comparing their predictions, started to make line segments on the number line given by the teacher and symbolized it with decimal numbers. How could they do this? As seen in pre-test, the students were familiar with number line and so familiar with segments on the number line. But, it

was assumed that it was the comparing problem that provoked them order decimal numbers on the number line.

Our conclusion is so far the students saw clearer which number was closer to a decimal number when they used number line. Indeed, it was shown that students could use simple arithmetic in decimal numbers, like “we add 7 more parts or minus two parts.” The number line helped the students to see clearer which number is closer to a decimal number by ordering the decimal numbers. The problem appeared was when Student 4 wrote down the decimal numbers between 1 and 2, she counted until 1,10. This was not discussed by the teacher but she just said that after 1,9 was 2 because we divide the number line until 10 as we did to our paper strip.

The revision to HLT was made, such as to conjecture how to react in the next teaching experiment(s) if the students write, like Student 4’s case, 1,9 then 1,10 not 1,9 then 2,0. The order of the decimal notations like this should be cleared by the teacher. The revision on the worksheet would be to repair the space where the students could write their answer. All above, however, this meeting was not enough to assure if the students comprehended decimals on the number line. Some problems ordering decimals on the number line would be tried out in next meeting.

4. CONCLUSION

In the context of predicting lengths, it was found one student made prediction but not considered the length of the paper strip as introduced in previous 2 meetings. Therefore, teacher in HLT would emphasize the use of the paper strip as a standard length of a span not an arm. This is hoped the students would get more sense of using standard measurement tool. However, by the game, the students started to make segments on number line to be written with decimal numbers. In this respect, the activity enabled the students to see decimal numbers on number line which also means they saw the magnitude of numbers.

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