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Generalization in Exponential Problems as a Part of Developing Mathematical Abstraction

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ABSTRACT
Generalization has been recognized as a challenge for students as a part of mathematical abstraction. This research is a descriptive study that aims to describe the mathematical abstraction type generalization in exponential problem. The subjects in this research were 3 students of one of the junior high school in Palembang. The students were at grade IX. This study was design research which consists: preliminary design, focus group discussion, trial, interview, and retrospective analysis. The problems are the creative problem solving. Problem 1 show that subject 1 succeed in indicator of perception of generality, expression of generality, symbolic expression of generality and manipulation of generality. Subject 2 only succeed in indicator of perception of generality. Subject 3 succeed in indicator of perception of generality and expression of generality. Problem 2 show that subject 1 and 3 succeed in indicator of perception of generality, symbolic expression of generality, expression of generality, and manipulation of generality. Subject 2 succeed in indicator of perception of generality. Problem 3 show that subject 1 and subject 2 succeed in indicator of perception of generality, expression of generality, and manipulation of generality. Subject 3 can't answer the problem 3.

Keywords*: *Mathematical abstraction, Generalization, Exponential, Developing.

1. INTRODUCTION

Mathematical thinking is defined as activity developing a mathematical point of view, assessing mathematical processes and abstractions, and always having a tendency to apply them [1]. It is possible to categorize these thinking skills in various ways. Karadag categorize them into seven major themes: modeling, reasoning, symbolization, representation, proving, abstraction, and mathematician [2]. One of the first and foremost strategies of mathematical creation is the process of abstraction. Abstraction is the step of isolating essential ingredients in a complex or subtle situation and pinning it down to create definitions and insights [3]. Abstraction as a process in which students vertically reorganize previously constructed mathematics into a new mathematical structure. Students use the outcomes of their previous processes of abstraction in order to make connections and to develop a new and vertical hypothesis or generalization. By this definition, two important aspects of abstraction emerge: abstraction may occur at any level of mathematical activity and abstraction demands high cognitive-mental work [4]. There are five indicator of abstraction; generalization, specialization, observation of patterns, conjecturing and testing conjecture [5][6]. Generalization is the heart and soul of mathematics. Process of generalization is one of the most powerful thinking processes, and to understand its decomposition when we examine a mathematical situation [7]. The ontology of mathematical entities are connected in generalization [8]. Mathematical generalization has four indicators: perception of generality, expression of generality, symbolic expression of generality, and manipulation of generality. The concept of generalization is most commonly understood as a duality between going from particular to general and seeing the particular through the general [2].

The process of generalization is one of the most powerful thinking processes, and to understand its decomposition when we examine a mathematical situation [9]. Generality is so central to all of mathematics that many professionals no longer notice its presence in what is, for them, elementary. But, it is precisely the shifts of attention that experts have integrated into their thinking, which are problematic for novices [10]. Mathematical generalization has been widely recognized as a challenge for many students

[11,12,13]. The main cause of students' difficulties in generalizing can be re-conducted to Duval's argument that mathematical objects are ideal and inaccessible [14]. The student does not seem to be able to overcome particular difficulties, seen as steps in a process of generalization [8]. Students who are comfortable working with specific cases, have difficulty in expressing generality [15]. This study aims to describe the mathematical abstraction type of generalization in exponential problem. How students solve exponential question using mathematical abstraction types generalization.

2. METHOD

2.1 Research design

This study used a design research [16]. The aim of study to describe mathematical abstraction type generalization ability in exponential problems. The study focuses on the generalization aspects. This study was design research which consists: preliminary design, focus group discussion, trial, interview, and retrospective analysis.

2.2 Subject

The research subjects were a class IX SMP Negeri 54 Palembang. The sample of the research took randomly. There were three students in grade 9.

2.3 Instrument

The instrument in this study were three essay problems and interview. This problem designed to explore students' generalization.

2.4 Data Collection and Analysis

The data collection procedure three essay problems and interviews that were recorded using Zoom Education.

3. RESULT AND DISCUSSION

This research followed the stages of design research.

3.1. Preliminary Design

In this stage, the focus of theories abstraction and generalization. The research designed the problem based on the study literature. There were three questions that designed relate to mathematical abstraction type generalization. The three problems have been designed are shown in Table 1.

3.2 Focus group discussion

The three problems were discussed with 2 Mathematics Education Lecturers. Table 2 summarizes the result of the discussion.

Table 1. Designed problems.

Problem number	Problem
1	If $20^{x+2}+20^{x}=401$, then determine the value of x?
2	If $\underbrace{25^{0,25}\times 25^{0,25}\times \ldots \times 25^{0,25}}_{n\ factor}=625$, then determine the value of $(n-4)(n+2)$?

Table 2. Group Discussion

Lecturer	Comment and Advise about Problems
1	Consider the indicator of the problems with curriculum indicator
2	Consider indicator of problems

After the group discussion with the lecturers, the researcher considers the advice. There wasn't any change on the problems.

3.3 Trial

The problems tested on three subjects. The trial shows that the students understand the problems and tried to solve it. There weren't any questions about the problems. The students did the problems in one hour.

3.4 Interview

The interview was after the written test. The interview aims to confirm the student's difficulty and experience during solve problems. It was conducted to clarify which indicator of generalization appears.

3.5 Retrospective Analysis

3.5.1 The analysis of subject's answer in problem number 1

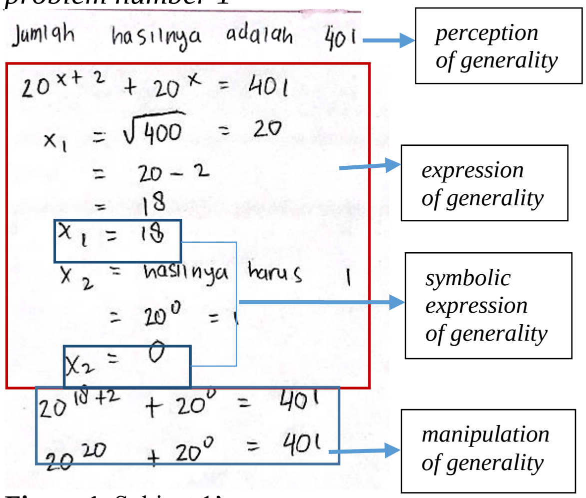

Figure 1. Subject 1's answer

Figure 1 shows generalization indicator of subject 1 in answering problem number 1. Generally, subject 1 solve problem 1. Subject 1 formulate the sum of the number is 401 that show the appearance of a perception of generality. Subject 1 formulate that

$20^{x+2}+20^{x}=401$ then started doing exponential algebraic that show the appearance of the expression of generality. Subject 1 show the appearance of symbolic expression of generality by propose that $x_1=18$ and $x_2=0$. Subject 1 made a calculate that 400=20^{20}. Subject 1 use the root concept rather than exponential concept. Subject misunderstanding that $\sqrt{400}=20$ so $20^{x}=20^{20}$. In interview, subject 1 suspected that 20 must connected to the result 401. Subject 1 guessed the sum of 20 and value of x that give a result 401. But, subject 1 made mistake in calculation of 400=20^{20} by misunderstanding the concept of root of 400. Subject 1 formulate it by mistake that it supposed be 400=20^2. Subject 1 can't formulated that $x=0$.

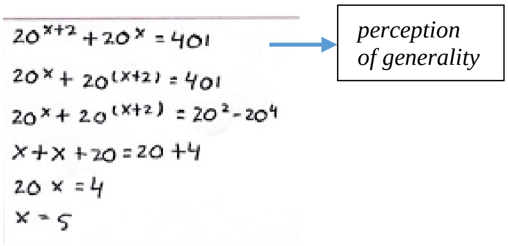

Figure 2. Subject 2's answer

Figure 2 shows generalization indicator of subject 2 in answering problem number 1. Subject 2 formulate the sum of the number is 401 that show the appearance of a perception of generality. Subject 2 succeed to make a perception of generality then failed to make the expression of generality. Subject 2 formulate that $20^{x+2}+20^{x}=401$ then started confuse to connect it with exponential concept. In the interview, subject 2 told that can't figure the pattern and haven't exponential understanding the failed in manipulation generality. Subject 2 also failed to show the appearance of symbolic expression of generality and manipulation of generality.

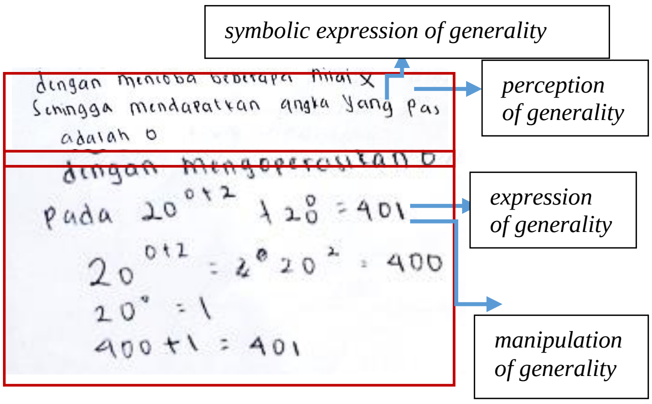

Figure 3. Subject 3's answer

Figure 3 shows generalization indicator of subject 3 in answering problem number 1. Subject 3 formulate $20^{0+2}+20^{0}=401$ that show the appearance of a perception of generality. Subject 3 propose that $20^{0+2}=20^{2}=400$ and $20^{0}=1$. It means subject 3 has exponential understanding to solve the problem. Subject 3 show the appearance of symbolic of generality. Subject 3 didn't formulate the value of x that $x=0$ perfectly. Subject 3 state in the beginning of the answer that value of x is 0. In the interview, Subject 3 formulate that $20^{0+2}+20^{0}=401$ by trial and error. In the end, Subject 3 suspect the pattern of $20^{0}=1$ and $20^{0}=1$. Subject 3 also show the appearance of manipulation of generality by formulate $20^{0+2}+20^{0}=401$. Subject 3 didn't show the process of the generalization thinking in order. But, base in the interview, subject 3 show generalization indicator.

3.5.2 The analysis of subject's answer in problem number 2.

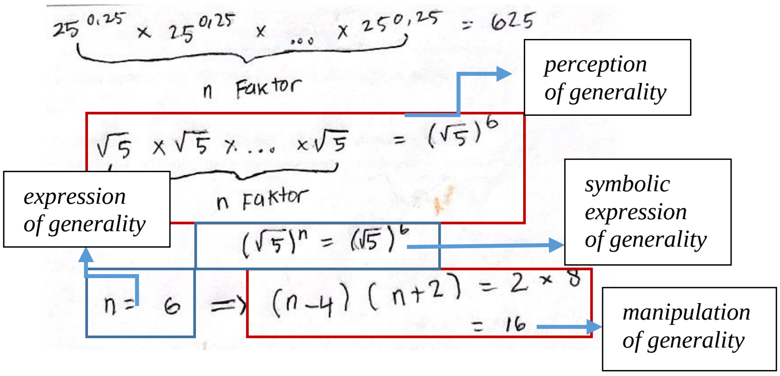

Figure 4. Subject 1's answer problem 2

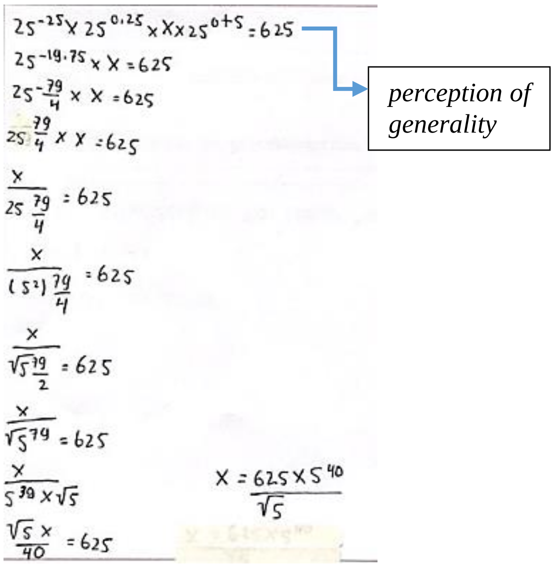

Figure 5. Subject 2's answer problem 2

Figure 4 shows generalization indicator of subject 1 in answering problem number 2. Subject 1 show the appearance of a perception of generality that $\sqrt{5}\times\sqrt{5}\times\ldots\times\sqrt{5}=(\sqrt[6]{5})$. Subject 1 generated the formulate without exponential algebraic that show $25^{0,25}=(5^2)^{\frac{1}{4}}=5^{\frac{1}{2}}$. Subject 1 show the expression of generality by proposed that $(\sqrt{5})^{n}=(\sqrt{5})^{6}$. Subject 3 succeed to conclude that $n=6$ and found the value of $(n-4)(n+2)=16$. Subject 1 show the appearance of symbolic expression of generality and manipulation of generality. In the inverview show that subject 1 has exponential concept to solve problem 2 and generalization indicator.

Figure 5 shows generalization indicator of subject 2 in answering problem number 2. Subject 2 show the appearance of a perception of generality that $\sqrt{5} \times \sqrt{5} \times \ldots \times \sqrt{5} = (\sqrt[6]{5})$ wrongly. Subject 2 can't generated exponential algebraic to solve $25^{0,25}$ and also $625 = 5^6$. In the interview, subject 2 was confused to apply the exponential concept. Subject 2 can't recognize the pattern of exponential of 5. Subject 2 didn't understand the connection between $25^{0,25}$ and n factor.

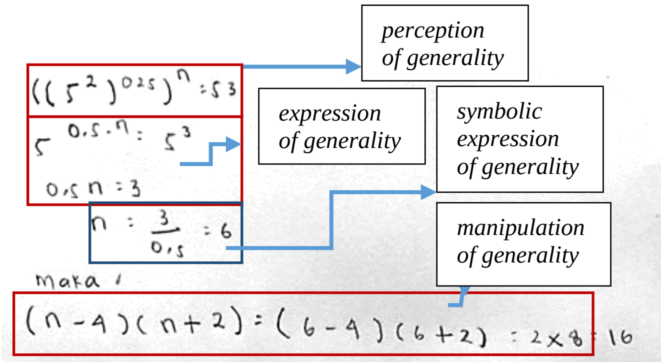

Figure 6. Subject 3's answer problem 2

Figure 6 shows generalization indicator of subject 3 in answering problem number 2. Subject 3 show the appearance of a perception of generality. Subject 3 generated the formulate exponential algebraic that $\left(((5^2))^{0,25}\right)^n = 5^3$. Subject 3 show the expression of generality by proposed that $5^{0,5 \times n} = 5^3 \leftrightarrow 0.5n = 3$. Subject 3 concluded that $n=6$ that show the appearance of symbolic expression of generality. Subject 3 found the value of $(n-4)(n+2)=16$ that show manipulation of generality. In the inverview show that subject 3 has exponential concept and generalization indicator.

Generalizing is the most authentic practice of the mathematics classroom [17]. It is really important for student to have generalization ability in problem solving. The process of generalizing a set of particular instances, and justifying and formalizing the generalization is fundamental to mathematics [17]. The difficulty in generalization shown in problem 1, 2 and 3. Some student can fully have indicator in generalization: perception of generality, expression of generality, symbolic expression of generality, and manipulation of generality. Problem 1 show that subject 1 succeed in indicator of perception of generality, expression of generality, symbolic expression of generality and manipulation of generality. Subject 2 only succeed in indicator of perception of generality. Subject 3 succeed in indicator of perception of generality and expression of generality. Problem 2 show that subject 1 and 3 succeed in indicator of perception of generality, symbolic expression of generality, expression of generality, and manipulation of generality. Subject 2 succeed in indicator of perception of generality. Problem 3 show that subject 1 and subject 2 succeed in indicator of perception of generality, expression of generality, and manipulation of generality. Subject 3 can't answer the problem 3. Base on the interview with subject 3, It's hard to comprehend expression of generality, the symbolic expression of generality, and manipulation of generality. Some researchers even argue that generalizing is a natural way of thinking for students, and suggest that the inclination to notice and discuss regularities and patterns in the number system is the foundation for constructing, testing, and justifying generalizations [18]. Subject 3 can't comprehend the patterns in the exponential problems. During the interview, the interviewer and subject 3 discuss the pattern. Subject 3 able to find the pattern. Subject 2 and 3 have difficulty in symbolic of the generality. In symbolic world generalization happens when the transition from operational procept to potential operation is occurred and this kind of generalization is reconstructive generalization [19]. The types of generalization in symbolic aspect which the main difficulties of students in focusing on this aspect [20,21].

4. CONCLUSION

Based on data analysis that has been done above, it can be concluded that the problems can used to measure the mathematical abstraction type generalization on junior high school student.

REFERENCES

[1] A. H Schoenfeld, Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics Handbook of research on mathematics teaching and learning, pp. 334-370, 1992.

[2] Z. Karadag, Analyzing Student's Mathematical Thinking in Technology Supported Environments [PhD thesis] Toronto, Department of Curriculum, Teaching and Learning Ontario Institute for the Studies in Education of the University of Toronto, 2009.

[3] B. P. Katz, and M. Starbird, An Introduction to Mathematical Thinking Graph, Group, Calculus. America: The Mathematical Association of America, 2013.

[4] R. Hershkowitz, B. B. Schwarz, and T. Dreyfus, Abstraction in Context: Epistemic Actions. *Journal for Research in Mathematics Education, 32*(2), 195-222. 2001.

[5] S. Katagiri, Mathematical Thinking and How to Teach It. Diterjemahkan oleh CRICED, Universitas Tsukuba. Tsukuba: CRICED University of Tsukuba, 2004.

[6] J. Mason, L. Burton & K. Stacey, Thinking Mathematically. Dorchester: Great Britain, 2010.

[7] G. Dumitrascu, Understanding the Process of Generalization in Mathematics through Activity Theory. International Journal of Learning, Teaching and Educational Research, 2017.

[8] G. Santi, and A. Baccaglini-Frank, Forms of generalization in students experiencing mathematical learning difficulties, *PNA, 9*(3), 217-243, 2015.

[9] D. Gabriela, Understanding the Process of Generalization in Mathematics through Activity Theory, *International Journal of Learning, Teaching and Educational Research Vol. 16, No. 12, pp. 46-69,* 2017.

[10] J. Mason, Expressing generality and roots of algebra. In N. Bednarz, C. Kieran, and L. Lee, (Eds.). 1996.

[11] J. Ainley, L. Bills, and K. Wilson, International Journal of Computers for Mathematical Learning, 10, 191-215, 2005.

[12] J. R. Becker, and F. Rineva, Generalization strategies of beginning high school algebra students,. in Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education 4, edited by H. L. Chick and J. L. Vincent, PME, Melbourne, 2005.

[13] K. Stacey, Educational Studies in Mathematics, 20, 147-164, 1989.

[14] Duval, R., Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic issues for learning. In F. Hitt & M. Santos (Eds.), *Proceedings of the Twentieth-First Annual Meeting North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 3-26). Cuernavaca, Mexico: PME. 1999

[15] Z. Rina, and L. Peter, The role of examples in forming and refuting generalizations, 2007.

[16] C. I. P. Rully, Design Research (Teori dan Implementasinya: Suatu Pengantar), Rajawali Pers, 2007.

[17] Strachota, Susanne. Conceptualizing Generalization. Open Mathematical Education Notes. 6, 2016, pp. 41-55.

[18] D. Schifter, S. Monk, S. Russell, and V. Bastable, “Early algebra: What does understanding the laws of arithmetic mean in the elementary grades? In J. J. Kaput, D. W. Carraher & M. Blanton (Eds.), Algebra in the early grades (pp. 389-412). New York: Lawrence Erlbaum, 2008.

[19] D. Tall, “Thinking through three worlds of mathematics.In*Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (4: 281-288)”. Bergen: Bergen University College, Norway, 2004b.

[20] J. A. G. Cruz, A. Martinon, “Levels of Generalization in Linear Patterns. *36th Conference of the International Group for the Psychology of Mathematics Education.* 2: 329-336, 1998.

[21] T. Kabael, “Generalizing Single Variable Functions to Two-variable Functions, Function Machine and APOS”. *Educational Sciences: Theory and Practice*, 11(1), 484-499, 2011.