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Mathematical Representation Ability on Quadratic Function Through Proof Based Learning

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ABSTRACT
This study aims at finding out students' mathematical representation ability through proof-based learning. The research is descriptive qualitative research at Madrasah Aliyah Negeri 2 Palembang with 32 students. The instructional process and collecting data were conducted online class because of COVID-19, and the test aimed to describe students' mathematical representation ability. The results show that students' visual representation in intermediate category (60.34), symbolic representation in very low category (30.60) , and verbal representation in low category (41.95). Overall students' mathematical representation ability in low category (43.82). Online learning cause lack of scaffolding when teaching proof. Visual representation can be improve by learning quadratic function. Constant employment of proof-based learning can improve symbolic and verbal representations.

Keywords: *Mathematical representation ability, Proof-based learning, Visual, Symbolic.*

1. INTRODUCTION

The study aims to assess students' representation ability through proof based learning. Mathematics is a subject that all school must teach. Math is needed for developing science and technology [1]. For Achieving it, logic's and critical thinking must be emphasized in learning mathematics. Minister of education also emphasized the ability to reason in mathematics [2]. The teacher's explanation that is difficult to understand makes students less interested in mathematics [3]. Technology in 21th century makes mathematics looks for students boring because in their tough are only given the questions that just count numbers.

Mathematics is an understanding of logic's in framework of thinking based on reason [4,5] Therefore what is important is the meaning and usefulness of mathematics, not only solving the questions. Teachers must guide the student what are the benefits they get by learning mathematics. NCTM stated that representation is one of the five abilities that students must have, namely communication, connection, problem solving, reasoning, and representation [6,7]. Representation ability is an important ability and students need guidance to develop it [8-11]. The reason why NCTM emphasizes representation to be on of the five abilities that must be possessed so the students are able to create and use representation for specific purposes, and communicate representation into a mathematical model [6,7, 8-12].

Representation is a symbol or a special form image, written symbol, object, code, or picture [8,9]. For example, 7 is a special symbol that represent seven object determined from counting. Cartesian graph, function, and solutions of algebraic equation are abstracts representation in mathematics Representation should emphasize the ability to express mathematical ideas such diagrams, graph, mathematical symbols, mathematical models, and arguments to solve problems [8,9,14-16].

Representation ability in this research are divided into three major parts, namely visual representation, symbolic representation, and verbal representation. According to [10, 14, 17], he stated that students had low representation ability, especially symbolic representation, due to the lack of knowledge and expirience. The study of [13-15,18] found that student's representation ability was low in symbolic representation and word representation. The importance

of understanding mathematics should have the same progress as mathematical representation ability.

The study of [1] used contextual learning to teach representation. There is study used inquiry learning to improve representation ability [5,19,20]. In some other studies used problem solving [8], also geogebra assisted learning [21], and RME [20]. Based on the studies mentioned, there has been no specific studies used proof-based learning to improve mathematical representation ability.

Prove and proving are important role in mathematics and must be taught in school (Heritage, Sari). By learning proof, students will sharpen their logic and argue [22]. Proof has important role to learn mathmatics and also a tool for understanding mathematics [23-24]. By learning mathematics, students will sharpen their logics and reasoning [22-25]. Proof-based learning is one of the ways to teach proof and proving [23,26,27]. In proof-based learning, teacher will teach how to prove and students will also experience how to prove. Also argumentation in proofing is one of the way for learning representation.

2. METHOD

They study categorized as a descriptive qualitative research. The research took place in the science class at Madrasah Aliyah Negeri 2 Palembang, South Sumatera, Indonesia. The subjects consisted of 32 students from tenth-grade, with online learning because of COVID-19. There was 2 weeks of learning quadratic function with proof-based learning conducted online. The souce of learning was youtube videos and google meet. Before field test, all the instrument validated by 3 validators with qualification as follows: 1 hold doctoral degree and 2 hold magister degree. Validator stated that the instruments were valid with minor revision. The following week, a test was conducted with the indicators of mathematical representation ability.

For research purpose, the study assessed students' mathematical representation ability and will be analyzed later. The assessment focused on 3 abilities, namely visual representation, symbolic representation, and verbal representation. The data will be grouped based on mathematical representation ability. Students' performance from the worksheet would be analized to desribe their mathematical representation ability through proof-based learning.

3. RESULT AND DISCUSSION

At the first meeting, proof-based learning [23,26,27] was conducted with online learning with the topic of quadratic function. The instruments made in form of student worksheets, google meet, and learning videos on youtube. In the worksheets, the materials were types of quadratic function, peak point of quadratic function proof, and questions about proof of quadratic function by given peak point and an intersect point. The videos on youtube were the material that explain about worksheets. And later on, the learning was using google meet for discussion with students.

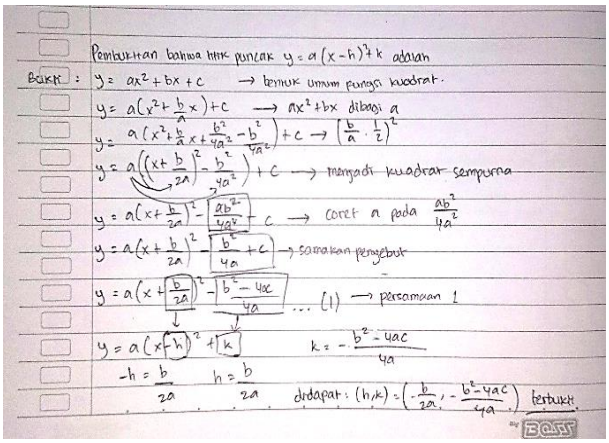


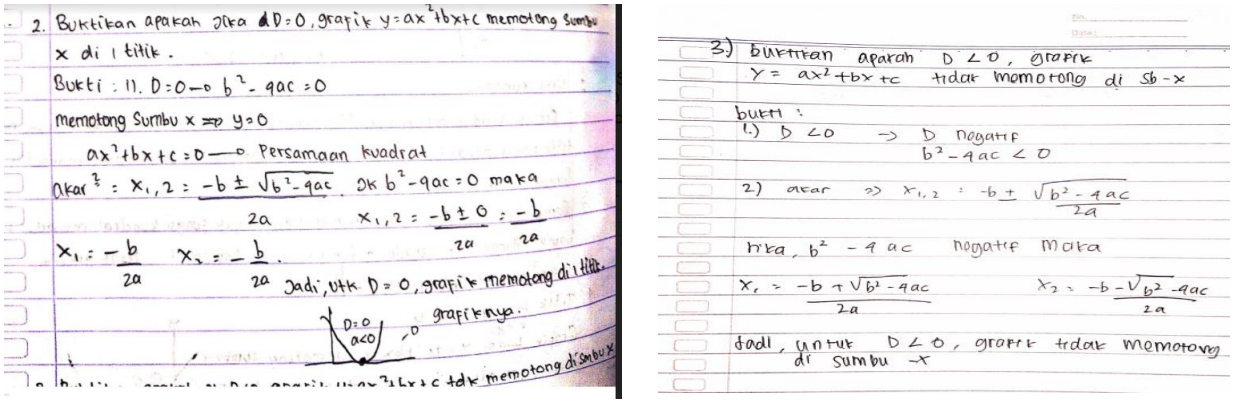
Figure 1. Students' works for proving peak point of quadratic function

Figure 1 shows the works of students named IT in proving peak point of quadratic function. They rewrote the proof that given in worksheet then they wrote an explaining for the operation and the properties that used for proving steps. In this first meeting, students should get a brief explanation about proof and prooving [23-25]. Likewise they are learning the symbol of proved in the learning process.

The next learning materials are 2 questions about proof of quadratic function by given peak point and an intersect point. The first question had already explained and the 2nd question to train students to prove. According to researchers, because of this first meeting were the first time for students in learning proof and proving, student must be have difficulties as to explain the steps of proving and some students also forgot the proved symbol to show that the final result were proved [23,24]. In the learning process, students had already learned from proof about representation symbolic and representation verbal.

At the second meeting, there was enchanment in students' performance in proof-based learning. The instruments was still made in form of student worksheets, google meet, and learning videos. In the worksheets, the materials were characteristic of quadratic function, proof of maximum and minimum peak point, proof of discriminant quadratic function, and the relation of discriminant with maximum and minimum peak point through visual representation. The videos on youtube were the material that explain about worksheets. And later on still same, namely learning was using google meet for discussion with students.

The characteristics of quadratic function, namely open graphic, peak point, and axis of symmetry which were discussed. The proof of minimum peak point had already discussed. Students tasked to proof the maximum peak point quadratic function. Some students can also prove, and most of them made the proof in the right ways. They also add conclusion after they done with proving. After introducing proofs continuously, students become very familiar and confident with proof [25,26].



(a) (b)

Figure 2. Students' works in proving discriminant quadratic function

Figure 2(a) shows the works of student named FL in proving "if $D = 0$, then the graph of the quadratic function will be intersect x-axis at one point". The steps of proof that FL showed were right and also gave a picture that represent the graphic intersect at x-axis. Another question was proving " if $D < 0$, then the graph of the quadratic function will not intersect x-axis". Figure 2(b) shows the works of student named ADP in proving, but there was no argument that state "the roots of negatif are imaginer", and only made conclution. This way of proving that shows on figure 2(b) were false. It is okay to make mistakes in proving, because steps of proving is not easy to understand with two meeting [1,26-28].

The next meeting was the implementation of the test. It was given to assess students' mathematical representation ability. The test was carried out for 1 hour and 15 minutes for answering 3 questions that monitored via google meet. All the students activated camera, and will be monitored. If there is a question, students can ask through voice on Google meet.

In the data analysis stage, the result of the test will be analysis using the indicator of mathematical representation ability describe in table 1.

Table 1. Indicator of mathematical representation ability

Code	Representation	Indicator
RG	Visual/Graph	1. Students are able to present a graph of given a function
		2. Students are able to connect a problem with a graphic
RS	Symbolic	1. Students are able to understand, make, and process equations
		2. Students are able to explain mathematical symbols
		3. Students are able to make interpretations of mathematical symbols
RV	Verbal	1. Students are able to make arguments in words that explain the meaning of a representation
		2. Students are able to write problem solving steps

Based on the analysis of students answer, students had grades of the visual/graph ability higher than symbolic and verbal represenation. The assessment will be carried out with scoring guidelines. The average score of students was 43.82 with low category (Table 2).

Table 2. The assessment result of the test

Mathematical Representation Ability	Visual/ Graph	Symbolic	Verbal	Total
Maximum Score	28.57	28.57	42.86	83.33
Minimum Score	0	0	0	8
Average Score	17.24	8.74	17.98	43.82
Grades	60.34	30.60	41.95	43.82
Category	Intermediate	Very Low	Low	Low

The analysis of all indicator that mention in Table 1 will be made detail. The average grades will be categorize in each part of indicator that shown in Table 3.

Proof-based learning was relatively new to students. Moreover, this instruction was conducted online, hence the process was challenging for them. Students' mathematical representation ability also enchanced on the process proof-based learning. Moreover, it was done online, because the teacher cannot guide students directly in proving.

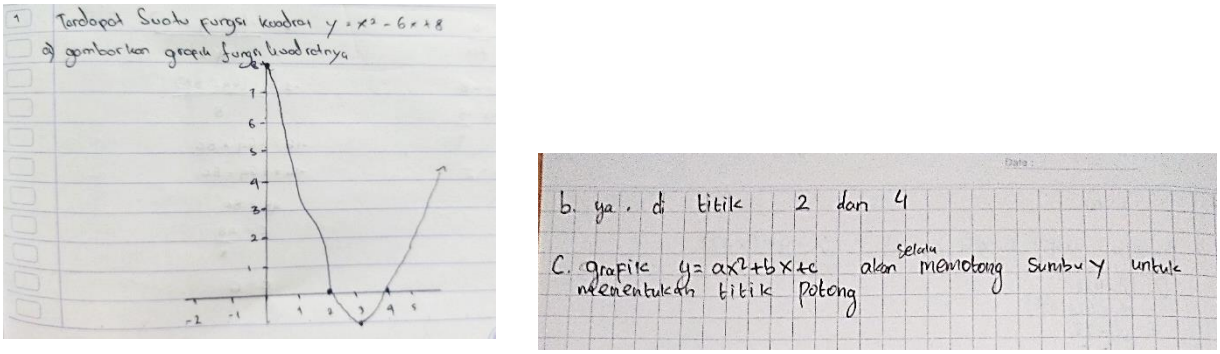
3.1. Visual/Graph Representation

Students' visual representation quite high compared to the symbolic and verbal representation as same result as [14,18]. The visual representation ability assessed in this study was the ability to graph and connect it to solve the problem. Based on Table 2, visual representation have average score of 17.24 (maximum score 28.57) with grades of 60.34. This shows the ability of visual/graph representation in intermediate category. Table 3 show that average of the ability to present a graph of given a function has the highest grades at 75.86.

Table 3. The analysis each part of indicator

Code	RG1	RG2	RS1	RS2	RS3	RV1	RV2
Average Grades	75.86	44.83	30.60	37.93	23.28	44.83	40.52

Figure 3 below show how students's answer in visual representation.



(a) (b)

Figure 3. Students' answer

Figure 3(a) shows student made the graphic representation are quite good. Student didn't make the peak point clearly and the graph is somewhat croocked. Figure 3(b) shows student's misconception drawing a graphic. Student answered the question 1b correctly, but the argument are false. The cartesian coordinate must consist of (x, y) in order to make a function. Student's answer for question 1c had no supporting argument for making the answer clear. Students were expected to be able to argue from general form of the quadratic function. 3.2. Symbolic Representation

The symbolic representation ability assessed in this study was the ability to (1) understand, make, and process equation, (2) able to explain mathematical symbols, and (3) able to make interpretations of

mathematical symbols. Based on Table 2 above, symbolic representation have average score of 8.74 out of 28.57 with grades of 30.60. This shows the ability of symbolic representation in very low category. This result obtained were similar to [13,14,18].

We can see from Figure 4, the student named MFA answered the question test number 2 correctly. Students understand how to process calculation given the peak point, but most of students were false in concept of general form of function given. Students thought that the given had no meaning. Students must connect the repesentation verbal ability to solve this question. The symbolic ability must teach abstractly for students and need scaffolding [29].

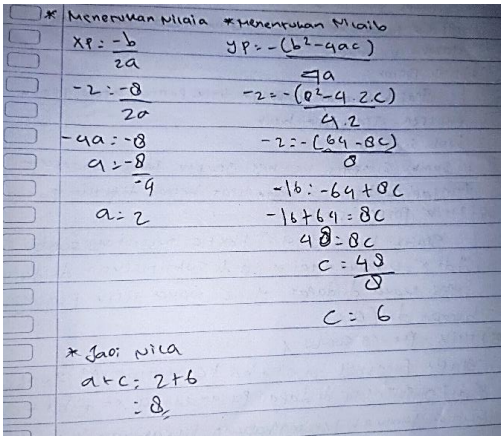


Figure 4. Students' answer 2

3.3. Verbal Representation

The symbolic representation ability assessed in this study was the ability to make arguments in word that can explain a representation and able to write problem solving steps. Based on Table 2 above, verbal representation have average score of 17.88 (maximum 42.86) with grades of 41.95. This shows the ability of symbolic representation in low category. The result given were similar to [13, 14].

Based on figure 4, the students' answer for questions in the test number 3. They required symbolic and verbal representation for solving this type question. Figure 4 shows how student's named AMN was wrong in processing the symbol "a". The answered should be " $a = 3$ ". Their mistakes when transforming the mathematical process and mistakes in algebraic. It can affect the result of the test. Most of students also made the same mistakes in the symbol. This kind of mistakes are common mistakes [30]. The verbal ability that show in figure 7 were correct. Student were able to write the problem solving steps correctly. The scoring for verbal representation was not affected by the mistakes in symbolic ability

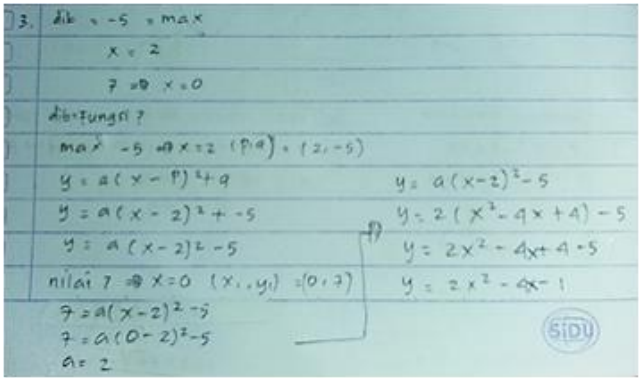


Figure 4. Students' answer 2

4. CONCLUSION

Based on the test of mathematical representation ability through proof-based learning for visual representation in intermediate category (60.34), symbolic representation in very low category (30.60), and verbal representation in low category (41.95). The visual representation relatively higher than symbolic and verbal representations because the visual learning materials were still conveyed in its entirety through online learning. Overall students' mathematical representation ability belongs to low cateogry (43.83). Most of students made mistakes in completing proof, especially reasons that were not sufficient to prove a statement. This caused online learning lack of scaffolding, and only the results were known. Through this learning, students undertand more deeply the relationship between the quadratic function through proving. Constant employment of proof-based learning gave positive result to symbolic and verbal representations. Moreover visual representation also improved during learning the quadratic function material.

For future research this online learning of proof-based teaching can be a refernce to compare optimizations for online and offline learning. Also for research that that links the relationship between visual, symbolic, and verbal representations. It is better for teachers to apply proof-based learning, because students need to get used to prove. In the future, researchers will also continue to develop proof-based learning in online and offline learning. The mathematical representation ability will also continue to be improved along with the development of learning materials.

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