Analysis of Didactic Transposition and HLT as a Rationale in Designing Didactic Situation

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ABSTRACT

This study aimed to analyse the phenomena from didactic transposition process and Hypothetical Learning Trajectory (HLT) in material of method to describe a set. A qualitative method with an interpretive phenomenology analysis approach was used to reveal this didactic phenomenon. A mathematics teacher and 28 students of 7th grade had been participants in the study. Furthermore, the data analysed included the results from review of scholarly mathematics textbook and school mathematics textbooks, observation of the teaching and learning process, diagnostic test, and student interview, and these were analysed using qualitative analysis. The result of this study shows that the transposition of knowledge from mathematical scholarly knowledge to taught knowledge indicate a shift in concept. The flow of material presentation and student learning trajectory in both school mathematics textbooks and the teaching and learning process had weaknesses, which were causing learning obstacles to students. This learning obstacles had an impact on students' difficulties in building conceptual understanding of the method to describe a set material. And then, the findings of didactic transposition analysis become the basis for compiling a new HLT which will serve as a framework in designing didactic situations in the form of didactic designs. So, the didactic transposition analysis and HLT are two things that complement each other in the preparation of didactic situation.

Keywords: Didactic Transposition, HLT, Design, Didactic Situation.

1. INTRODUCTION

In teaching practice, teachers, curriculum designers, and textbook writers are faced with two main problems, namely curriculum management and classroom management [1]. Curriculum management focuses on how to construct scholarly knowledge into teaching objects in the school or in the classroom (knowledge to be taught and taught knowledge), because school mathematics basically evolved scholarly mathematics [1-3]. The process of how scholarly knowledge is transposed into knowledge in a school or classroom context is called didactic transposition [2-4]. This transposition process is important to facilitate students for learning this knowledge and to ensure that there are no gaps between the knowledge learned by students and scholarly knowledge.

The didactic transposition process not only discusses about didactic relationships (students and material), but also pedagogical relationships (teachers and students) [5]. This pedagogical relationship is related to how the teacher tries to teach students [6]. In this connection, the second problem faced by the teacher is classroom management which focuses on how an educator seeks to present didactic situations that can help students build their knowledge [1,2]. Thus, it can be said that the didactic transposition process includes two management at once, namely curriculum management and classroom management.

The efforts to present meaningful mathematics learning require teachers to be able to present an appropriate didactic situation. This didactic situation is certainly determined by how a teacher prepares a didactic design. A didactic design that is developed by the teacher is not formed suddenly but is influenced by a complex social system including the system in government (curriculum) and the school environment [7]. Therefore, as a teacher, it is necessary to view this condition in a comprehensive perspective.

Didactic transpositions exist to provide an overview of the phenomena of educational practice that occur, starting from the process of transposition of knowledge from scholarly knowledge to learnt knowledge to the impact of the transposition process on learning [1,8-9]. By examining this didactic transformation process, education practitioners, especially teachers, will see many phenomena, one of which is a phenomenon that shows the purpose of learning mathematics is not only for the transfer of knowledge to students but also as a means for students to construct their knowledge. Because in fact, mathematical learning is a human activity to obtain mathematical objects by going through the triadic cycle (mental act, ways of thinking, and ways of understanding) [10] and it shows that there is no gap between the concepts constructed by students and concepts in scholarly knowledge. Therefore, the assessment of the didactic transformation process is considered to be able to provide comprehensive information for teachers in designing a didactic situation, more specifically a better didactic situation.

Another aspect of concern in compiling a didactic situation is the Hypothetical Learning Trajectory (HLT), which is a series of predictions of learning situations that contain three components, namely: goals of learning, the thinking and learning process, and learning activities in which they might engage [11-13]. The results of the analysis of the didactic transposition process are also predicted to provide additional information for the preparation of HLT, because in the didactic transposition process, it will be seen how the flow of material presentation and learning trajectory presented by both school mathematics textbooks and the learning process in the classroom.

It is important to know the phenomena that occur in the didactic transposition process and the preparation of HLT, so we conduct research with a focus on examining the didactic transposition process. Specifically examines the flow of material presentation and learning trajectories in the didactic transposition process, and its impact on learning process. This analysis only focuses on the presentation material for class 7 SMP. In addition, at the end of this study, we offer an HLT which will also be a guide in designing a didactic situation.

2. METHOD

Focus of this study was to examine all the facts about the transposition of knowledge from scholarly knowledge to learnt knowledge and the phenomena behind the transposition of knowledge. More specifically, this study examined the flow of material presentation and learning trajectory resulting from the didactic transposition process and its impact on learning process. In addition, the focus in this study was designing the HLT as a basis for designing didactics

design based on the result of recommendations from didactic transposition analysis (Figure 1).
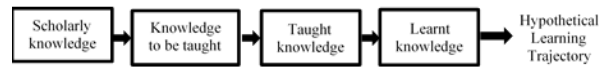

Figure 1. The diagram of didactic transposition analysis and HLT

With the focus of this study, this study was conducted using a qualitative method with a interpretive phenomenological analysis (IPA) approach, which aims to interpret a phenomenon based on human experience [14]. The meanings to be revealed were facts and phenomena about the transposition of knowledge. Therefore, to obtain data about this meaning, so then the data collection process was carried out, consist of review of scholarly mathematics textbook and school mathematics textbooks, observations of teaching and learning process, diagnostic tests, and student interviews.

Participants in this study were 28 students of 7th grade junior high school and a mathematics teacher, while the material that became the object of study was the material of method to describe of set. Furthermore, data analysis uses qualitative data analysis techniques which contain 3 stages, namely managing data, inductive analysis, and interpretation, and data triangulation was a source of data validity [15-16].

3. RESULTS AND DISCUSSION

3.1. Results

The results will be presented in two parts, namely didactic transposition analysis and hypothetical learning trajectory.

3.1.1. Didactic Transposition Analysis

Didactic transposition analysis was carried out sequentially starting from analysis of mathematical scholarly knowledge, analysis of knowledge to be taught, analysis of taught knowledge, and analysis of learnt knowledge.

Mathematical scholarly knowledge was knowledge produced by mathematicians. The concept presented in this scholarly knowledge becomes an ideal concept that becomes a reference in preparing knowledge to be taught or school mathematics. In scholarly knowledge, a set can be described by two methods, namely the tabulation or enumeration method (writing members of set-in curly braces), for example {1,2,3,4} which states the set of the first four integers; and method description (describing the properties of set members), written with $\{x|P(x)\}$ which states the set of all elements x that satisfies property P, for example $\{x \mid x$ is positive

integer and $x < 5$}} [17-19]. The second method was presented to make it easier to describe a set without write down its members.

(describing the properties of all members), for example{set of odd numbers more than 1 and less than 9}, {set of vowels}; and set-forming notation method,

Table 1. Weakness of Material Presentation Flow in School Mathematics Textbooks and Predictions of Its Impact

Weakness of Material Presentation	Impact Prediction
The material about how to describe a set use set-forming notation was considered quite abstract and contains prerequisite concepts that had not been studied by students, namely algebraic concepts. While in textbooks, the algebraic concepts will be studied after the set material.	It was predicted to cause didactic obstacle and ontogenic obstacle. These obstacles were predicted to cause in student having difficulty understanding the material about how to describe a set use set-forming notation method.

Furthermore, by the noosphere (curriculum designer and textbook writer), this material was constructed from scholarly knowledge to knowledge to be taught, so that this material can become the object of teaching in schools. Knowledge to be taught was represented in the school mathematics textbook.

In contrast to mathematical scholarly knowledge, in school mathematics textbooks, the method to describe a set was divided into three methods, namely enumeration method (writing all of the members of the set), for example {3,5,7} which states the set of odd numbers is more than 1 and less than 9, {a, i, u, e, o} which represent a set of vowels and {..., -2, -1,0,1,2, ...} which represent a set of integers; description method

for example $\{x|x < 6$ and x natural number$\}$ [20].

From the analysis results of the material presentation in school mathematics textbooks, it was found a weakness of the material presentation flow, which was predicted to have an impact on learning process (Table 1).

Although the material presentation in school mathematics textbooks contained a weakness, but there were advantages, namely there was a learning trajectory that helps students to recognize various types of number sets and there were various problems with various contexts, thus get used to solving problems about how to describe a set with various methods (Figure 2).

Table 2. Weakness in Teaching and Learning Process and Predictions of Its Impact.

Weaknesses in Teaching and Learning Process	Impact Prediction
1. The teacher assumed that students already know and understand various sets of numbers, so that the teacher did not make the concept construction process on various sets of numbers as part of the student learning trajectory.	It was predicted to cause didactic obstacle. This obstacle was predicted to cause in students having difficulty understanding the various sets of numbers and difficulty writing the members of the numbers sets.
2. The problem presented by the teacher had not been able to accommodate the ability of students to describe sets in various methods. The problem given was only limited to how to describe a set by writing its members.	It was predicted to cause epistemology obstacle. This obstacle was predicted to cause in students' understanding being limited to understanding how to describe a set by writing its members and students did not understand how to describe a set by set-forming notation method or by describing the characteristics of its members because they did not have an understanding of the concept, so that students will have difficulty.
3. It is known that the material about how to describe a set use set-forming notation method was material that is difficult to reach by students' thinking abilities, but the teacher did not prepare anticipations for the prediction of difficulties that will be faced by students.	It was predicted to cause didactic obstacle and ontogenic obstacle. These obstacles were predicted to cause in students not understanding the meaning of the mathematics sentences presented in the set-forming notation, so that students will have difficulty to describe the set by set-forming notation or difficulty determining the members of the set.

1. $A = \{$ whole number less than 5$\}$, if it is described by writing all element then $A = \{0, 1, ..., ..., ...\}$ and if it is described by set-forming notation, then $A = \{x | x < \cdots, \text{and } x \in \cdots\}$.
2. $B = \{x | -2 < x < 3 \text{ and } x \in \text{integer}\}$, if it is described by writing all element then $B = \{..., ..., ..., ...\}$ and if it is described by describing the properties of element then $B = \{$ integer more than ... and less than ... $\}$.

Figure 2. Task in school mathematics textbook

Furthermore, in teaching and learning process, the material of method to describe a set presented in the classroom as taught knowledge basically refer to the material contained in school mathematics textbooks. However, both had different learning trajectory. Based on the results of observation of the teaching and learning process, obtained the following:

1. The teacher explained three method to describe a set, namely writing members of set, describing the characteristics of its members, and set-forming notation.

2. The teacher gave an example of a set which was described by describing the characteristics of its members, for example "$A = \{odd\ number\ less\ than\ 11\}$", then students were required to describe the set by writing set members (enumeration).

From the student's learning trajectory during the teaching and learning process, we got several weaknesses, which was predicted to have impacts on learning process (Table 2).

Furthermore, to obtain information about how students understand the material about method to describe a set as learnt knowledge, a diagnostic test was given to get the information about students' conceptual understanding. The problem was given in two forms, namely determining the members of the set (Figure 3a) and describing a set-in different method (Figure 3b).

1. By using the tabulation method, list all the elements from following sets:
   a. $A = \{$even number between 4 and 13$\}$
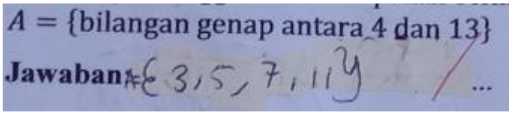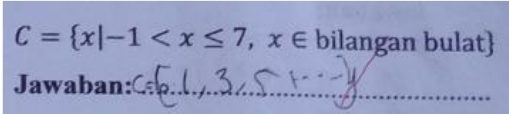   b. $B = \{x | -1 < x \leq 7, x \in \text{integer}\}$

Figure 3a. Task 1

2. Given a set $P = \{2, 4, 6, 8, 10\}$.
   a. Describe a set P by describing by the property to all element of the set!
   b. Describe a set P by set forming notation!

Figure 3b Task 2

The test results indicated that most students were not correct in describing a set-in various method, whether it

Table 3. Students' Answer and Conceptual Understanding

Students' Answers	Conceptual Understanding
1. Students can describe a set either by using tabulation method (writing down its members), describing its properties, and set forming notation.	Students understand the concept of describing a set with various method, using either enumeration, description of member properties, and set-forming notation.
2. Students can describe the set by enumeration method (writing the members of the set), but mistakenly determine the members (problem 1). $A = \{$bilangan genap antara 4 dan 13$\}$ Jawaban: $\{3, 5, 7, 11\}$ ... $C = \{x | -1 < x \leq 7,\ x \in \text{bilangan bulat}\}$ Jawaban: $\{1, 3, 5 \cdots\}$	a. Students did not understand the numbers between two numbers (problem 1a). b. Students did not understand the meaning of the math sentence $-1 < x \leq 7$, so they cannot determine the number x (problem 1b). c. Students did not understand the members of the set of even numbers and the set of integers.
3. Students know how to describe a set using the set-forming notation, but they mistakenly write the mathematical sentence (problem 2).	Students did not understand how to describe a set $P = \{2, 4, 6, 8, 10\}$ in math sentences.

3. The teacher gave an example of a set which is described by the set-forming notation, for example, "Given a set P $= \{x \mid x > 2, x \in primes\ number\}$", then students were required to describe the set by writing set members (enumeration).

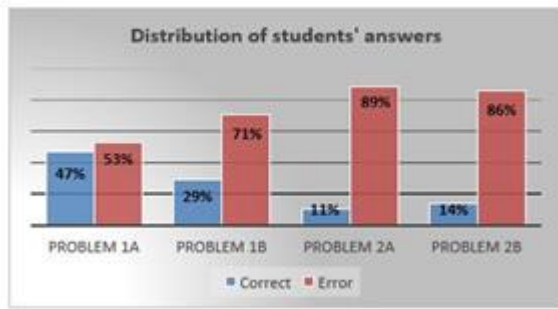was using the enumeration method, the description method, or the set formation notation method (Figure 4).

Figure 4. Distribution of students' answers

Next, six students were selected to be interviewed with the aim of obtaining information about the relationship between the various answers and the various students' conceptual understanding (Table 3).

The variety of students' answers and students' conceptual understanding in Table 3 showed that some students had not understood the material about method to describe a set properly. Student learning trajectory and material presentation were predicted to be one of the causes of learning obstacle that had an impact on students' understanding of concepts (Table 4).

Therefore, several efforts to resolve and anticipate learning obstacles were as follows:

1. The effort to resolve and anticipate the didactical obstacle were (a) reminding (apperception) of the material of numbers as a prerequisite concept, especially the case for determining a number between two numbers using a number line representation; (b) present a didactic situation that can help students recognize the types of set numbers; and (c) present an additional learning trajectory and flow of material presentation that can bridge the concept of describing a set by description method and set-forming notation method.

2. The effort to resolve and anticipate the epistemological obstacle was provide a variety of task with various context related to describe a set. For example, if given a set, students were required to describe a set-in various method, such as task presented in school mathematics textbooks.

3. The effort to resolve and anticipate the onto genic obstacle was material presentation must be conveyed following the level of students' thinking and the basic knowledge that students have. For example, if a set is given which is represented by the form forming notation, then to determine the members of the set, can use the help of a number line.

3.1.2. Hypothetical Learning Trajectory (HLT)

The findings from the results of the didactic transposition analysis offered an overview for compiling

the student's HLT as the basis for designing didactic situation. This HLT contained an additional learning trajectory to anticipate leap of learning trajectory (Figure 5).
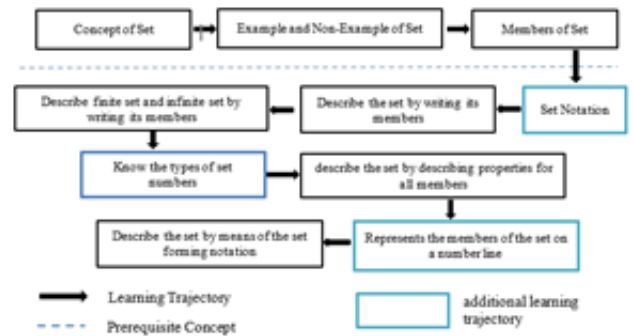

Figure 5. Hypotical learning trajectory

3.2. Discussion

In mathematical scholarly knowledge, a set can be described in two methods, namely the tabulation method (writing members of the set) and description method (describing the characteristics of all members of set). From this mathematical scholarly knowledge, the material was constructed into a teaching object, where mathematical ideas are transferred from a scholarly context to a school or class context. And this was called recontextualization of knowledge [9], [21-22]. This construction process was in accordance with the view of Chevallard [23] which states that school mathematics was basically evolved from scholarly mathematics. The process of how mathematical scholarly knowledge was transposed into school mathematics is called didactic transposition [2],[4]. The process of knowledge transposition from something scholarly to something in the context of this classroom aimed to make the teaching process easier [4]. Thus, this construction process was a necessity that must be faced by curriculum designers, textbook writers, and teachers [1].

Process of knowledge transposition from mathematical scholarly knowledge to knowledge to be taught or external didactic transposition [1] produce material presentation in a different context, which is presented in school mathematics textbook. If in mathematical scholarly knowledge, a set can be described in two methods [17-19], then in knowledge to be taught a set can be described in three methods, consisting of: writing the members of the set (enumeration), describing the properties of members, and set forming notation [20]. Although there is a different context in material, but conceptually there is no fundamental difference between these knowledges.

Associated with the flow of material presentation, there is a weakness. When students are faced with the material presenting sets by set-forming notation method,

for example $B = \{x | -1 < x \leq 7, \ x \in \text{integer}\}$, then students will face school algebra concept indirectly. The notation of $-1 < x \leq 7$ contains a variable x with a certain value, while the set B is a set containing all the values of x that meet the conditions $-1 < x \leq 7$. The weakness was students did not have knowledge of school algebra concepts and even school algebra concepts are learned right after the set material is finished. It showed that there was a mismatch between the flow of the material presentation and the continuity of students' thinking needs.

Brousseau [4] used the term learning obstacle to describe this phenomenon. Referring to the learning obstacle theory, this condition was predicted to cause learning obstacles to students, especially didactic obstacle and onto genic obstacle. Didactic obstacle was caused by gaps between the material presentation and the continuity of children's thinking needs. While, onto genic obstacle was caused by gaps between levels. material difficulties with children's needs [4], [7]. As a result, students will have difficulty reaching the material, where the difficulty arises as part of the way students adapt their knowledge to an environment (presentation of material) [24].

Although learning difficulties or obstacles were something that was difficult to avoid, at least these learning obstacles can be anticipated by the teacher when carrying out the learning process. In other words, the teacher carried out the process of re-construction of knowledge from knowledge to be taught into objects that are actually taught in the classroom (taught knowledge). In this knowledge reconstruction process, teacher was required to depersonalization of knowledge and recontextualization of knowledge [4], [7].

Meanwhile, the reality in the learning process showed that the teacher did not prepare anticipation to resolve these didactic obstacles, so that learning difficulties were actually experienced by students. In fact, in designing the learning process, a teacher was expected to be able to position himself not only as a teacher but also to position himself as a student, so that the teacher can imagine the predictions of student responses to the material presented and prepare anticipation for the predicted responses [4], [7].

Although the material presented by the teacher in the learning process (taught knowledge) was not different from the material presented in the school mathematics textbook, the learning process carried out by the teacher still had weaknesses. In addition to the teacher not preparing anticipations to resolve learning obstacle caused by the presentation of material in school mathematics textbooks, other weaknesses were: (1) the teacher made a leap of learning trajectory. For example, the teacher did not provide a learning trajectory for

students to recognize types of number sets. This condition was predicted can cause didactic obstacle [4], [7]; (2) the teacher did not present a didactic situation for students to carry out the process of institutionalizing their knowledge. In fact, it is important for teachers to present this institutionalization situation to prevent epistemological obstacles, which is caused by gaps in the context of the learning experience that was experienced with the demands of linking the results of the learning process with various contexts outside of those experienced [4], [7].

Furthermore, as the final series of didactic transposition process, an analysis of students' understanding was carried out as a representation of the knowledge that students learn (learnt knowledge). The results of the analysis showed that students had not been able to build a good conceptual understanding of material of method to describe a set. This can be seen from the variety of answers that showed the errors and difficulties of students in solving the problems given. This error was mostly caused by students' lack of understanding of the material. The lack of understanding of a concept was an indication of student difficulties and this difficulty was an indication of learning obstacles [4].

Based on the findings from the results of the didactic transposition analysis from mathematical scholarly knowledge to taught knowledge and findings from the analysis of students' understanding of concepts, as a representation of learnt knowledge, presentation material and didactic situations in school mathematics textbooks and in the teaching and learning process had become learning obstacles for students, which in turn caused students to not understand the concept of the material. Based on this, this finding of study offered a recommendation for improvement next learning design, involve improving the material presentation and student learning trajectory and anticipation to resolve any learning obstacle. These recommendations then become guidelines for compiling didactic designs.

Apart from the findings from the analysis results, the second important thing that must be considered in preparing a didactic design is HLT. In this study, the HLT that is arranged also contains these three components [11-13], where the goal of learning is that students are able to construct their knowledge on the material of describing of a set. This goal was achieved through a series of learning activities that include thinking and learning processes. All of these components were presented in the student learning trajectory starting from reminding the concept of prerequisites, then proceeding to describe a set with a notation, describe a set by enumerating, and so on to describe a set using the set-forming notation.

The difference between this HLT and the student's learning trajectory during the learning process was that

there are additional flow of material presentation and student learning flow to bridge the knowledge possessed by students with material presentation. The flow of material presentation and additional learning trajectory is additional material about set notation introduction, know the types of set number, and represents the members of the set on a number line.

HLT preparation is also inseparable from the recommendations from the findings of the didactic transposition analysis. This shows that didactic transposition analysis and HLT are two complementary things to designing a didactic situation

4. CONCLUSION

Based on the results and discussion, it can be concluded that the transposition of knowledge from mathematical scholarly knowledge to taught knowledge indicate a shift in concept. The flow of material presentation and student learning trajectory in both school mathematics textbooks and the teaching and learning process had weaknesses. These weaknesses were a factor in causing learning obstacles to students. This learning obstacles had an impact on students' difficulties in building conceptual understanding of the method to describe a set material.

The findings of the didactic transposition analysis form the basis for constructing HLT. Thus, it can be said that the results of the didactic transposition analysis offered comprehensive information about the phenomena of the learning situation. The didactic transposition analysis and HLT are two things that complement each other in the preparation of didactic situation.

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