Robust Optimization for the Counterpart Open Capacitated Vehicle Routing Problem With Time Windows

Evi Yuliza[1], Fitri Maya Puspita[1*],Siti Suzlin Supadi[2]

[1]*Department of Mathematics, Faculty of Science , Universitas Sriwijaya, Indonesia*
[2]*Institute of Mathematical Sciences, Faculty of Science, University of Malaya, Malaysia*
[*]*Corresponding author. Email: fitrimayapuspita@unsri.ac.id*

ABSTRACT

This paper investigates the robust optimization for the counterpart open capacitated vehicle routing problem (denoted by RCOCVRP) with soft time windows on waste transport problems. The uncertainly of volume of waste and travel time are considered. Garbage collection vehicles in carrying out the process of collecting garbage sometimes there are delays such as congestion and engine damage. We investigates the robust counterpart open capacitated vehicle routing problem with soft time windows models, where the waste transport vehicle does not need to return to the location of origin, the volume of waste carried by the vehicle does not exceed the vehicle capacity and consider the time windows. The robust counterpart open capacitated vehicle routing problemwith soft time windows models solved by cheapest insertion heuristic algorithm. The results of the completion of the RCOCVRP model with soft time windows were completed with the GAMS software. Finally, the results of the calculation of the RCOVRP model with soft time windows using the cheapest insertion heuristics algorithm are also compared with the GAMS software.

Keywords: *Waste Transportation, Soft Time Windows, Robust Optimization, Travel Time.*

1. INTRODUCTION

The problem of route of waste transportation vehicles is a major concern in the city that affects environmental problems. The problem of garbage transportation vehicle routes is a major concern in big cities which has an impact on environmental problems. The existence of residential areas, traditional markets, schools and offices which results in an increase in the volume of waste, such as in Kalidoni sub-district, Palembang city. The robust counterpart model of open capacitated vehicle routing problem (denoted by RCOCVRP)with soft time windows with soft time windows is a robust optimization for data that is uncertain [1], [2]. This model is applied to optimize the route of garbage collection vehicles with uncertain waste volume, time windows and travel time. Garbage collection vehicles depart from a node and do not return at that node. This differs from the classic vehicle routing problem (VRP). Classic VRP departs from an initial node and returns to initial node [3], [4]. Capacitated routing problem (CVRP) is also known as classic VRP related to vehicle capacity [5]–[7]. The problem of route for transporting waste can be considered as a optimization problem[8]–[10].Garbage collection vehicles carry garbage in several Temporary Disposal Sites (TDS) and end up at Final Disposal Sites (FDS). The process of transporting waste at some TDS sometimes occurs delays caused by congestion and machine damage. Garbage transport vehicles carry out the process of transporting waste from 06.00-10.00 and 16.00-20.00. The concept of this model is robust capacitated vehicle routing problem [11], robust open vehicle routing problem [12], robust vehicle routing problem with time windows [13]–[15] and robust counterpart open capacitated vehicle routing problem [16], [17].The problem of route for transporting waste with uncertainly of waste volume can be considered as a robust optimization problem[17], [18], [2].

This study aims to optimize the route of the garbage transporting vehicle in the RCOCVRP model with soft time windows, limited by vehicle capacity, time windows and travel time. Time windows are the time intervals when the vehicle departs and arrives [9]. The RCOCVRP model with soft time windows was solved using the cheapest insertion heuristic algorithm and GAMS software.

The cheapest insertion heuristic algorithm was first introduced to solve the Traveling Salesman Problem (TSP) and is known as the heuristic method [19]. This algorithm has been extended to VRP problems [20]. In this study, the cheapest insertion heuristic algorithm is used to optimize the route of the waste transport vehicle, including minimizing the distance and travel time. The concept of this algorithm is to insert each point in an arc from the starting point to the destination point [21]. The results of the cheapest insertion heuristic algorithm were solved by the GAMS software. GAMS (General Algebraic Modeling Software) is a very powerful optimization tool used to model, solve and analyze optimization problems [22]. Das and Kumar [23] uses GAMS to solve optimization problems and nonlinear programming (NLP) is used for solvers in GAMS such as CONOPT. Yuliza et al [1] has discussed the RCOCVRP model with soft time windows which was solved using LINGO and GAMS software. The resulting solution using GAMS is more optimal than using LINGO, indicated by a smaller calculation of the distance and travel time. In this study, linear programming (LP) was used for solvers in GAMS such as CPLEX.

2. METHOD

Let $G = (V, E)$ be a directed edge weighted graph, where $V = \{1, 2, 3, \dots, n, n+1\}$ denotes a set of vertices with nodes $n+1$ denotes the objective and $E = \{(i,j) | i,j \in V, i \neq j\}$. Non negative c_{ij} is represent the distance of arc $(i,j) \in E$. In this case the distance matrix is symmetric, in others word $c_{ij} = c_{ji}$ for all $i,j \in V$, $i \neq j$. The cheapest insertion heuristic algorithm is as follows:

1. Route starts from the node of origin associated with the destination node.
2. A sub tour is generated between two vertex. A sub tour means a journey from a starting point and ending at a destination point.
3. Change one of the arc between two vertex with a combination of two arcs. Insertion k vertexon arc (i,j) with k vertex is not a member of sub tour and choose the smallest distance. Formula of the smallest distance could be:

   $c_{ik} + c_{kj} - c_{ij}$wherec_{ik} is the distance from vertex *i* to k, c_{kj} is the distance from vertex k to j and c_{ij} is the distance from vertex *i* to *j*.
4. Repeat step 3 until all the vertex are included in the sub tour.

In the case of the route of the garbage transporting vehicle, the vehicle departs from the starting point and does not return to that point so that the cheapest insertion heuristic algorithm is modified with limitations on vehicle capacity and travel time related to time windows and service time. Service time is the time used to load and unload waste from the vehicle. A sub tour obtained from the cheapest insertion heuristic algorithm is solved using the CPLEX solver from GAMS. The RCOCVRP model with soft time windows was solved by the GAMS software. This research framework is as shown in Figure 1.

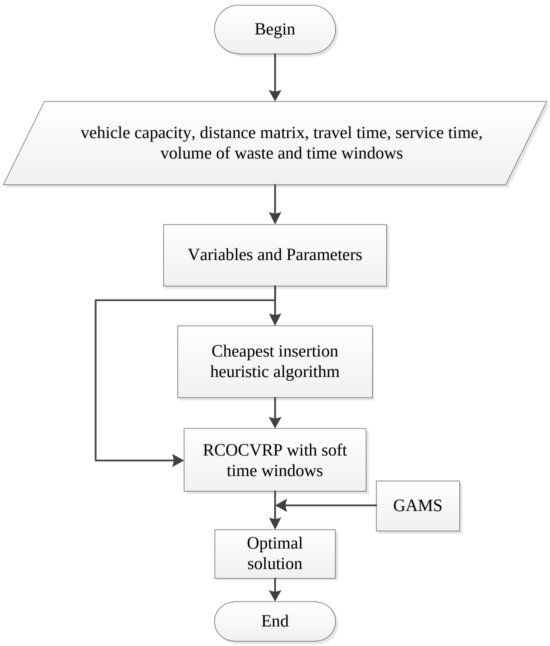

Figure 1. Research Framework

3. RESULT AND DISCUSSION

The research data was obtained from the Palembang City Environment Agency (PCEA) in the form of garbage transportation route data in the city of Palembang in September 2019. The data used in this study is the waste transportation route in Kalidoni sub-district, Palembang City. The following are some of the things that are limited:

1. It is assumed that the dump truck is 8 tons.
2. Assume the service time is approximately 10 minutes.
3. Assuming the time interval is 4 hours.

Table 1. Distance matrix (km)

Nodes	1	2	3	4	5	6	7	8
1	0	0.85	2.3	2.3	2.7	3.5	9.5	7.2
2		0	1.6	1.6	2	2.8	8,7	7
3			0	1.8	2.2	3	9	7.3
4				0	0.45	1.2	7.2	8.5
5					0	0.9	6.9	8.2
6						0	8.6	8.5
7							0	15
8								0

Kalidoni sub-district has 4 working areas, each working area has several TDS. Distance between each TDS and TDS and FDS as shown in Table 1.In this study, two examples of waste volume in working area 3 in each TDS are given such as Table 2 and time windows for each vertex as shown in Table 3.

Table 2. Volume of waste in the working area 3 (ton)

	1	2	3	4	5	6	7
Volume of waste_1	0.95	0.56	1.36	1.4	0.88	0.85	2
Volume of waste_2	2.3	1.2	2.4	2.4	1.5	2.6	1.3

Table 3. Time windows for working area 3

Vertex	Time windows
1	[07:00, 07:30]
2	[07:30, 08:00]
3	[08:00, 08:30]
4	[08:30, 09:00]
5	[09:00, 09:30]
6	[09:30, 10:00]
7	[10.00, 10.30]
8	[10.30, 11.00]

Variables and parameters in the working area 3 for $i = \{1,2,3,\dots,8\}$ dan $j = \{1,2,3,\dots,8\}$ as in Table 4 and 5.The optimal route RCOCVRP with soft time windows model on working area 3 uses the cheapest insertion heuristic algorithm as shown in Table 6 andTable 7 based on the volume of waste in Table 2.

Table 4. Variables anddescriptions

Variable	Description
x_{ij}	Travelling from node *i* to node *j*
t_i	Time arrived of the vehicle when serving node to *i*
y_i	Vehicle load when leaving node to *i*

Table 5. Parameters and descriptions

Parameter	Description
c_{ij}	distance from node *i* to node *j*
Q	vehicle capacity
q_i	quantity of garbage transported at node to *i*
t_{ij}	travel time from node to *i* to node to *j*
a_j	vehicle departure time from node to *j*
b_i	vehicle arrival time at node *i*

Table 6. The results of calculations using the cheapest insertion heuristic algorithm based on the volume of waste_1

Garbage transport routes	Total distance (km)	Total travel time (minute)
TDS1 – TDS2 – TDS5 TDS6 – TDS7 – TDS4 – TDS3 – FDS	28.65	52.995

Table 7. The results of calculations using the cheapest insertion heuristic algorithm based on the volume of waste_2.

Garbage transport routes	Total distance (km)	Total travel time (minute)
TDS1 – TDS2 – TDS5 – TDS3 – FDS	12.35	58.605
TDS4 – TDS6 – TDS7 – FDS	24.8	47.22

The model of RCOCVRP with soft time windows in work area 3 is based on data from Table 1, waste volume_1 from Table 2 and time windows from Table 3 as follows:

Objective function
Minimize

$$0.85x_{12} + 2.3x_{13} + 2.3x_{14} + 2.7x_{15} + 3.5x_{16} + 9.5x_{17} + 7.2x_{18} + 0.85x_{21} + 1.6x_{23} + 1.6x_{24} + 2x_{25} + 2.8x_{26} + 8.7x_{27} + 7x_{28} + 2.3x_{31} + 1.6x_{32} + 1.8x_{34} + 2.2x_{35} + 3x_{36} + 9x_{37} + 7.3x_{38} + 2.3x_{41} + 1.6x_{42} + 1.8x_{43} + 0.45x_{45} + 1.2x_{46} + 7.2x_{47} + 8.5x_{48} + 2.7x_{51} + 2x_{52} + 2.2x_{53} + 0.45x_{54} + 0.9x_{56} + 6.9x_{57} + 8.2x_{58} + 3.5x_{61} + 2.8x_{62} + 3x_{63} + 1.2x_{64} + 0.9x_{65} + 8.6x_{67} + 8.5x_{68} + 9.5x_{71} + 8.7x_{72} + 9x_{73} + 7.2x_{74} + 6.9x_{75} + 8.6x_{76} + 15x_{78} + 7.2x_{81} + x7_{82} + 7.3x_{83} + 8.5x_{84} + 8.2x_{85} + 8.5x_{86} + 15x_{87}$$

subject to

$$x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} = 1$$
$$x_{21} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} = 1$$
$$x_{31} + x_{32} + x_{34} + x_{35} + x_{36} + x_{37} + x_{38} = 1$$

$$x_{41}+x_{42}+x_{43}+x_{45}+x_{46}+x_{47}+x_{48}=1$$
$$x_{51}+x_{52}+x_{53}+x_{54}+x_{56}+x_{57}+x_{58}=1$$
$$x_{61}+x_{62}+x_{63}+x_{64}+x_{65}+x_{67}+x_{68}=1$$
$$x_{71}+x_{72}+x_{73}+x_{74}+x_{75}+x_{76}+x_{78}=1$$
$$x_{81}+x_{82}+x_{83}+x_{84}+x_{85}+x_{86}+x_{87}=1$$
$$\begin{gathered} x_{21}+x_{31}+x_{41}+x_{51}+x_{61}+x_{71}+x_{81} \\ -x_{12}-x_{13}-x_{14}-x_{15} \\ -x_{16}-x_{17}-x_{18}=-1 \end{gathered}$$
$$\begin{gathered} x_{12}+x_{32}+x_{42}+x_{52}+x_{62}+x_{72}+x_{82} \\ -x_{21}-x_{23}-x_{24}-x_{25} \\ -x_{26}-x_{27}-x_{28}=0 \end{gathered}$$
$$\begin{gathered} x_{13}+x_{23}+x_{43}+x_{53}+x_{63}+x_{73}+x_{83} \\ -x_{31}-x_{32}-x_{34}-x_{35} \\ -x_{36}-x_{37}-x_{38}=0 \end{gathered}$$
$$\begin{gathered} x_{14}+x_{24}+x_{34}+x_{54}+x_{64}+x_{74}+x_{84} \\ -x_{41}-x_{42}-x_{43}-x_{45} \\ -x_{46}-x_{47}-x_{48}=0 \end{gathered}$$
$$\begin{gathered} x_{15}+x_{25}+x_{35}+x_{45}+x_{65}+x_{75}+x_{85} \\ -x_{51}-x_{52}-x_{53}-x_{54} \\ -x_{56}-x_{57}-x_{58}=0 \end{gathered}$$
$$\begin{gathered} x_{16}+x_{26}+x_{36}+x_{46}+x_{56}+x_{76}+x_{86} \\ -x_{61}-x_{62}-x_{63}-x_{64} \\ -x_{65}-x_{67}-x_{68}=0 \end{gathered}$$
$$\begin{gathered} x_{17}+x_{27}+x_{37}+x_{47}+x_{57}+x_{67}+x_{68} \\ -x_{71}-x_{72}-x_{73}-x_{74} \\ -x_{75}-x_{76}-x_{78}=0 \end{gathered}$$
$$\begin{gathered} x_{18}+x_{28}+x_{38}+x_{48}+x_{58}+x_{68}+x_{78} \\ -x_{81}-x_{82}-x_{83}-x_{84} \\ -x_{85}-x_{86}-x_{87}=1 \end{gathered}$$

$$950 \le y_1 \le 8000$$
$$560 \le y_2 \le 8000$$
$$1360 \le y_3 \le 8000$$
$$1400 \le y_4 \le 8000$$
$$880 \le y_5 \le 8000$$
$$850 \le y_6 \le 8000$$
$$2000 \le y_7 \le 8000$$
$$0 \le y_8 \le 8000$$
$$y_1-y_2+8000x_{12} \le 7050$$
$$y_2-y_3+8000x_{23} \le 7740$$
$$y_3-y_4+8000x_{34} \le 6640$$
$$y_4-y_5+8000x_{45} \le 6600$$
$$y_5-y_6+8000x_{56} \le 7120$$
$$y_6-y_7+8000x_{67} \le 7150$$
$$y_7-y_8+8000x_{78} \le 8000$$
$$7 \le t_1 \le 7{,}5$$
$$7{,}5 \le t_2 \le 8$$
$$8 \le t_3 \le 8{,}5$$
$$8{,}5 \le t_4 \le 9$$
$$9 \le t_5 \le 9{,}5$$
$$9{,}5 \le t_6 \le 10$$
$$10 \le t_7 \le 10{,}5$$
$$10{,}5 \le t_8 \le 11$$
$$t_1-t_2+0{,}02125x_{12}+0{,}167 \le 0$$
$$t_2-t_3+0{,}04x_{23}+0{,}167 \le 0$$
$$t_3-t_4-0{,}0225x_{34}+0{,}167 \le -0{,}3$$
$$t_4-t_5+0{,}01125x_{45}+0{,}167 \le 0$$
$$t_5-t_6-0{,}2725x_{56}+0{,}167 \le -0{,}3$$
$$t_6-t_7+0{,}215x_{12}+0{,}167 \le 0$$
$$t_7-t_8-0{,}2625x_{78}+0{,}167 \le -0{,}3$$

4. CONCLUSION

The RCOCVRP model with a soft time window with uncertainty of waste volume and travel time is solved using the cheapest insertion heuristic algorithm which yields various solutions. The RCOCVRP model with a soft time window with uncertain waste volume and trip times completed using the CPLEX solver from GAMS yields the optimal solution. Although the total distance and the total travel time calculated using the cheapest insertion heuristic algorithm are smaller than the results from GAMS.

REFERENCES

[1] E. Yuliza, F. M. Puspita, and S. S. Supadi, “The robust counterpart open capacitated vehicle routing problem with time windows on waste transport problems,” *Bull. Electr. Eng. Informatics*, vol. 9, no. 5, pp. 2074–2081, 2020.

[2] E. Yuliza, F. M. Puspita, S. S. Supadi, and S. Octarina, “The robust counterpart open capacitated vehicle routing problem with time windows,” *J. Phys. Conf. Ser.*, vol. 1663, p. 12030, 2020, doi: 10.1088/1742-6596/1663/1/012030.

[3] M. A. H. Akhand, Z. J. Peya, T. Sultana, and Al-Mahmud, “Solving Capacitated Vehicle Routing Problem with route optimization using Swarm Intelligence,” *Proc. Int. Conf. Electr. Inf. Commun. Technol.*, pp. 112–117, 2015, doi: 10.1109/EICT.2015.7391932.

[4] Z. Borcinova, “Two models of the capacitated vehicle routing problem,” *Croat. Oper. Res. Rev.*, vol. 8, pp. 463–469, 2017, doi: 10.17535/crorr.2017.0029.

[5] N. J. Z. Mamat, S. H. Jaaman, and R. Ahmad, “Vehicle routing problem and capacitated vehicle routing problem frameworks in fund allocation problem,” in *AIP Conference Proceedings*, 2016, vol. 1784, pp. 50001-1-50001–7, doi: 10.1063/1.4966820.

[6] N. R. Achuthan, L. Caccetta, and S. P. Hill, “An improved branch-and-cut algorithm for the capacitated vehicle routing problem,” *Transp. Sci.*, vol. 37, no. 2, pp. 153–169, 2003.

[7] G. Laporte, “What You Should Know about the Vehicle Routing Problem,” *Noval Res. Logist.*, vol. 54, no. August, pp. 811–819, 2007.

[8] P. K. Nguyen, T. G. Crainic, and M. Toulouse, “A tabu search for time-dependent multi-zone multi-

trip vehicle routing problem with time windows," *Eur. J. Oper. Res.*, vol. 231, no. 1, pp. 43–56, 2013, doi: 10.1016/j.ejor.2013.05.026.

[9] F. Perea, R. Ruiz, and K. Katragjini, "Integer programming, clustering, and local search approaches for grouping urban waste collection sites," *Bol. Estad. e Investig. Oper.*, vol. 32, no. 3, pp. 203–224, 2016.

[10] M. A. Hannan, M. Akhtar, R. A. Begum, H. Basri, A. Hussain, and E. Scavino, "Capacitated vehicle-routing problem model for scheduled solid waste collection and route optimization using PSO algorithm," *Waste Manag.*, vol. 71, pp. 31–41, 2018.

[11] I. Sungur, F. O. Nez, M. Dessouky, F. Ordónez, and M. Dessouky, "A robust optimization approach for the capacitated vehicle routing problem with demand uncertainty," *IIE Trans.*, vol. 40, pp. 1–33, 2008.

[12] E. Cao, M. Lai, and H. Yang, "Open vehicle routing problem with demand uncertainty and its robust strategies," *Expert Syst. Appl.*, vol. 41, no. 7, pp. 3569–3575, 2014.

[13] A. Agra, M. Christiansen, R. Figueiredo, L. M. Hvattum, M. Poss, and C. Requejo, "The robust vehicle routing problem with time windows," *Comput. Oper. Res.*, vol. 40, no. 3, pp. 856–866, 2013.

[14] S. Braaten, O. Gjønnes, L. M. Hvattum, and G. Tirado, "Heuristics for the robust vehicle routing problem with time windows," *Expert Syst. Appl.*, vol. 77, pp. 136–147, 2017.

[15] C. Hu, J. Lu, X. Liu, and G. Zhang, "Robust vehicle routing problem with hard time windows under demand and travel time uncertainty," *Comput. Oper. Res.*, vol. 94, pp. 139–153, 2018, doi: 10.1016/j.cor.2018.02.006.

[16] F. M. Puspita, Y. Hartono, N. Z. Syaputri, E. Yuliza, and W. D. Pratiwi, "Robust counterpart open capacitated vehicle routing (RC-OCVRP) model in optimization of garbage transportation in District Sako and Sukarami, Palembang City," *Int. J. Electr. Comput. Eng.*, vol. 8, no. 6, pp. 4382–4390, 2018.

[17] Y. Hartono, F. M. Puspita, D. I. Permatasari, and B. Arisha, "LINGO-based on robust counterpart open capacitated vehicle routing problem (RC-OCVRP) model of waste transportation in Palembang," *Int. Conf. Inf. Commun. Technol. ICOIACT*, vol. 18, no. 978-1-5386-954–5, pp. 429–435, 2018.

[18] F. M. Puspita, E. S. Cahyono, S. Rahayu, and B. L. Sintia, "Model of demand robust counterpart open capacitated vehicle routing problem (DRC-OCVRP) simplification by applying preprocessing techniques in rubbish controlling in Sematang Borang District , Palembang," *E3S Web Conf. 68*, vol. 25, pp. 1–6, 2018.

[19] W. N. Tanjung *et al.*, "Cheapest insertion heuristics algorithm to optimize WIP product distributions in FBS Fashion Industry Cheapest insertion heuristics algorithm to optimize WIP product distributions in FBS Fashion Industry," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 277, pp. 1–10, 2017, doi: 10.1088/1757-899X/277/1/012069.

[20] D. Rachmawati and Wilyanto, "Implementation of Modified Cheapest Insertion Heuristic on Generating Medan City Tourism Route Implementation of Modified Cheapest Insertion Heuristic on Generating Medan City Tourism Route," *J. Phys. Conf. Ser.*, vol. 1566, pp. 1–9, 2020, doi: 10.1088/1742-6596/1566/1/012076.

[21] L. V. Hignasari and E. D. Mahira, "Optimization Of Goods Distribution Route Assited By Google Map With Cheapest Insertion Heuristic Algorithm (CIH)," *SINERGI*, vol. 22, no. 2, pp. 132–138, 2018.

[22] R. Singh and A. Kumar, "Estimation of off Shore Wind Power Potential and Cost Optimization of Wind Farm in Indian Coastal Region by Using GAMS," *Proc. 2018 Int. Conf. Curr. Trends Towar. Converging Technol. ICCTCT 2018*, pp. 1–6, 2018, doi: 10.1109/ICCTCT.2018.8550900.

[23] B. Das, "Cost optimization of a hybrid energy storage system using GAMS Department of Electrical Engineering," pp. 89–92, 2017, doi: 10.1109/ICPEDC.2017.8081065.