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Using Representational Form:

Cognitive Processes in Mathematical Modeling With Pre-Service Mathematics Teacher

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ABSTRACT
Cognitive modelling has psychological goals to analyse of cognitive processes during modeling processes and understanding of these cognitive processes and to promote of mathematical thinking processes by using models as mental images or even physical pictures or by emphasizing modeling as mental process such as abstraction or generalization. Using representational form is one of the important component of cognitive modeling. This cognitive process refers to the ability of understanding and expressing mathematical content in different ways. This research use sample item of using representational form. The total of research subject is 123 pre-service mathematics teachers. The results shows that less than 50% of research subject can answer correctly and give the reason based on their competence in using representational form. According to interview data, we found that the obstacle of extracting information from given mathematical representations such as graphs in this sample item and others is translating mathematical content from one mathematical representational form to another.

Keywords: *Cognitive Process, Modeling, Representational Form, Mathematical Representation.*

1. INTRODUCTION

One goal of learning mathematics is to form new schemas in cognitive structure by considering the existing schema so that assimilation occurs. In everyday life, education and culture, it is something that cannot be avoided because culture is a unity comprehensive and comprehensive, applies to a society and education is a basic need for every individual in society. Cognitive theory is based on computer metaphors. So that every The incoming information (via sensory registers) will be processed similarly processing in the computer [1]. Finally information if possible, will be stored in the form of a knowledge representation structure. Meanwhile, information in learning mathematics refers more to mathematical objects [2]. Cognitive processing is a term which is used by a psychologist in describing the activity of mental which deals with perception, thought, memory, and information processing which allows a person to acquire knowledge, solve problems, as well as planning for the future, or all processes related to how individuals learn, pay attention, observe, imagine, estimate, assess, and think about the environment. There are six components in cognitive processes are included: mathematical communication, using representational forms, mathematical problem solving, mathematical argumentation, modeling, and technical abilities and skills [3]. In this study, we will focus on one component of these six cognitive processes which is using representational forms. This cognitive process refers to the ability of expressing and understanding mathematical content area in different ways. Mathematical content might be given in a mathematical language verbally or in symbolic, written form, or by means of figural representation [3].

In addition, National Council of Teachers of Mathematics (NCTM) states five standards of mathematical abilities that pre service teachers must master, namely communication skills, problem solving skills, connection skills, reasoning skills, and representation skills [4]. This means that

pre service teachers' mathematical representation skills are important for pre service teachers to have and must be developed further. The ability of pre service teachers to represent mathematical representations is needed by pre service teachers to be able to find and make a tool or way of thinking in expressing mathematical ideas from initially abstract to concrete so that it will be easier to understand [5-7]. From the reference [8] states that there are several reasons why mathematical representation is one of the standard stages, namely, fluency in translating between many different types of representations is a basic ability that pre service teachers need to have to build a concept in mathematical thinking. Thus, there needs to be an action that can train pre service teachers' mathematical representation skills so that they are able to understand a context and relate it to mathematical problems [8].

Mathematical modeling is one of the steps to solve mathematical problems, the process is to change problems that exist in everyday life into the form of mathematical problems [9-11]. In mathematical modeling problems, pre service teachers are asked to be able to formulate a problem, and then formulate what mathematical model is right for the problem [12]. At the stage of formulating a mathematical model, the ability of mathematical representation is needed to translate what is known from the problem into the form of a mathematical model. When pre service teachers think about turning a problem into a mathematical form, pre service teachers' reasoning will automatically interpret mathematical solutions so that mathematical representation skills will also emerge. Based on the description above, this study aims to explore and describe the cognitive processes especially using the representational form in mathematical modeling with pre-service mathematics teacher

2. METHOD

2.1 Research design

This study is intended to explore and provide an overview of the cognitive processes especially using the representational form in mathematical modeling with pre-service mathematics teacher. The type of research used is descriptive exploratory with a qualitative approach[13].

2.2 Subject

The research subjects were two class of undergraduate student that are 123 pre service teachers in mathematics education program study.

2.3 Instrument

The main instrument in this study were a written test and an interview guide sheet. The question of written test addresses the relationship between time/season and number of zoo visitors. In this sample item, pre service teachers need to identify a certain graph representing from given description of the particular relationship. Therefore, they have to understand particular properties of graphs, such as slope, minima, and maxima with respect to different in representational forms. So, this question is used to explore the cognitive processes especially using the representational form in mathematical modeling with pre-service mathematics teacher.

2.4 Data Collection and Analysis

The data collection procedure included a mathematics modeling test and interview. Moreover, we use methods and time triangulations were used to get credible data[13], consist of activities (1) data reduction, (2) data presentation, and (3) drawing conclusions[14]. As the basis for the analysis used indicators of the cognitive processes in mathematical modeling process in this research are presented in Table 1[15].

Table 1.The representational form in mathematical modeling

No	Cognitive Process	Behavior
1.	Knowing	- Recall definitions, geometric properties and notation. - Recognize mathematical objects, shapes, and expressions. - Retrieve information from the mathematical problem. - Use measuring instruments; use units of measurement appropriately; estimate measures; convert units
2.	Applying	- Select an efficient/appropriate method or strategy for solving problems. - Display mathematical information and data in diagrams, tables, charts, or graphs. - Select equivalent representations for a given mathematical entity or relationship. - Follow and execute a set of mathematical instructions. Given specifications, draw figures and shapes.
3.	Reasoning	- Decompose geometric figures to simplify in solving a problem from given information. Display mathematical information and data in graphs. - Create connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas. - Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.

3. RESULTS AND DISCUSSION

3.1 Pre-service teachers' in using representational form of cognitive processes

This is the instrument test to assess the pre service mathematics teachers' in using the representational form as a cognitive process.

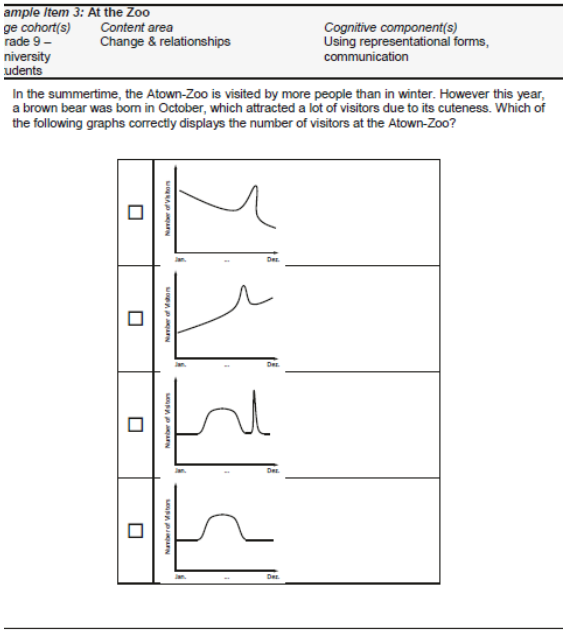
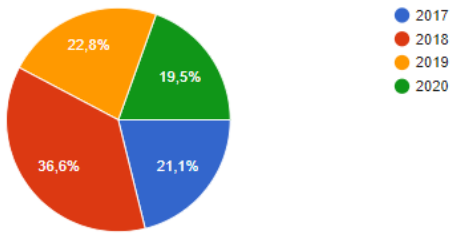


Figure 1. The item test

The problem was given for pre service teachers (4 batch of undergraduate students in mathematics education program). Totally were 123 students. The pie chart in Figure 2 below shows the percentages every batch that answered this modeling problem.

Figure 2. The percentage of pre service teacher every batch



Based on the figure above, we can see that the highest percentage in batch 2018 that is 36.6%, while for other batch has similar percentages around 20%. It means that every batch has quite similar of number participants that taken this mathematical modeling problem. The answer of pre service teachers shows below (Figure 3)

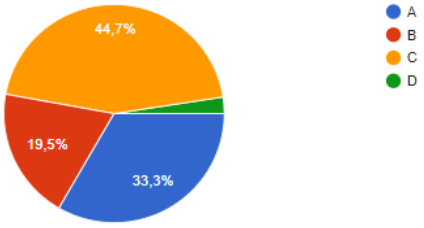


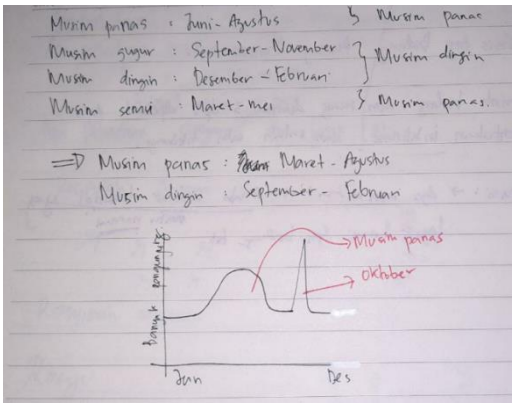
Figure 3. The percentage of pre service teacher's answer

3.2 The Cognitive process in using representation form regarding mathematical modeling

3.2.1 Knowing

What do you want to do or find? This step is a difficult step because we often do having difficulty deciding what to do. In real world situations no one gives us anything math problem to solve. Usually we have to vote on between large amounts of data and identify a particular aspect that is we want to learn. Next, we must precisely define the problem so that it can translate the verbal statement that describes problems in math symbols.

Figure 4. Knowing in cognitive process



Based on the figure 4, we can see that simplify problem by reducing the number of factors considered. Then, the relationship among the remaining variables must be determined. So, the

complexity of a problem can be reduced by assumes relatively simple relationships. There are two the main activities in this step are as follows: a) classifying variables. What factors influence the behavior identified in step 1? List the factors these as variables. The variable that the models want to explain is called dependent variable. The remaining variables are called independent variables. There are two reasons for ignoring independent variables. First, influence of the variable is relatively small when compared to other factors involved in behavior. Furthermore, we can ignore a factor whose influence is the same as any other factor, b) Determine the relationship between the variables that have been selected. Before we can hypothesize the relationship between variables, we generally have to make some additional simplifications. A problem can be complex enough that we cannot at first see the relationship among all the variables. In that case, we are can create a sub model, that is, we study one or more variables free separately. Finally we will connect all submodel together. In the next learning activity, we will learn a technique, for example comparison, that will helps in hypothesizing the relationship between variables. [17].

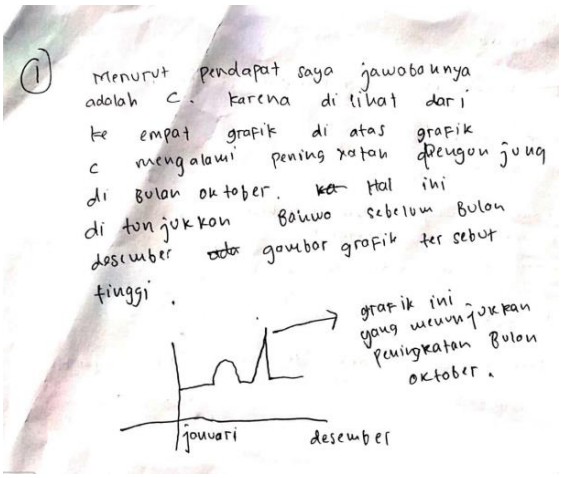


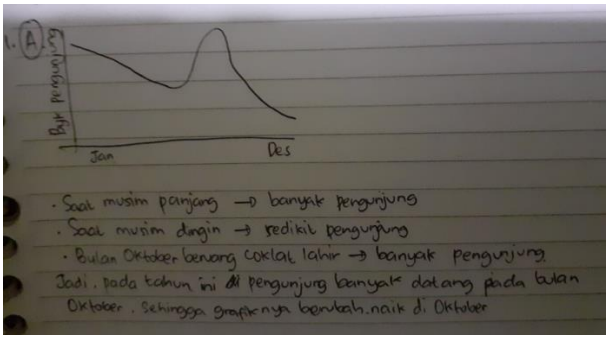
Figure 5. Knowing in cognitive process

3.2.2 Applying

The second process in cognitive process is that applying. Now, combine all sub models to become a model. In some case, the model can consist of mathematical equations or inequalities that must be resolved to find the information we are looking for. Often times a problem statement requires a model solution optimal or best. At this step, we can find no solution to the model or unable to interpret the model. In that case, we must return to step 2 to make additional assumptions to simplify. Or we even have to go back to step 1 to redefine it problem. From the figure 5, we can see these behaviors emerged. However, there are some pre service mathematics teachers (33% participants) that unable to select the appropriate the representational form for the problem then the display graph is not suitable with the given information and situation as the figure 6 below.

Figure 6. The mistaken applying in cognitive process

To get the information about their thinking in this wrong answer. We did an interview (R means researcher and P is pre service mathematics teacher) as follows.



R : Can you tell about how you display the problem situation becomes this graph (answer the graph B)?

P : *Yes, it is based on the information that the October is that the maximum month of zoo visitor and in my assumption that the people is less in winter which is in Desember.*

R : What about January?

P : *Hmmm, I think I'm wrong in this month (laugh). I think, I only the consider the October as the maximum point in this graph.*

The interview statement above is very clear that P realize that she did a mistake at the starting point in the graph because, she only focus on the maximum point of the graph on October. But, she did not the others seasons. Some pre services teacher has the similar thinking with this argumentation that is only consider the maximum point of the graph. It seems that the obstacle of extracting information from given mathematical representations such as graphs in this sample item and others is translating mathematical content from one mathematical representational form to another [18].

3.2.3 Reasoning

Reasoning mathematically is the ability to think in logic and systematic. When we use model, we have to test that model. There are several questions that are can be submitted before we carry out data collection (a step which can be costly and time consuming). First, what the model answers the problem identified in step 1 or the model it becomes separate from the main issue when we build the model. Second, whether the model can be used easily, for example, do we can collect the data needed to implement the model? Third, does the model make sense? Don't we make up math error in step 3 or made an assumption error on step 2? If the questions above have been answered satisfactorily then we can test the model using the data obtained from observation. We need to be careful when using data the. The assumptions made in step 2 make the data possible only applies to certain intervals and does not apply to interval other. For example, Newton's second law only applies if the object's velocity is higher smaller than the speed of light. We must also be careful in drawing conclusions about the results model testing. We cannot make guesses in general, only because the model applies to the special data we get. A model is not can become a law (provision) only because it is proven to be valid over and over for certain data cases. Through data gathered, we are convinced that our model makes sense.

> The answer is graph c because in January and December are winter so that the graph is in the same position, then it increases slowly after January to summer, the maximum point is in October when a brown bear is born

Figure 7. Reasoning cognitive process

From the figure above, we can see the third process in cognitive process that is reasoning. The behavior emerge is that 1) Create connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas, and 2) Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties [15].

From the result, the using representational form shown by pre service mathematics teacher is an expression of mathematical ideas as an attempt to find solutions to the graph problem in this item research which will be resolved. Not much different from reference [6] there are four ideas used in understand the concept of representational form regarding cognitive process, namely: (1) representation can be viewed as internal abstraction of mathematical ideas or cognitive schemata that are built through experience, (2) as a mental reproduction of a mental state previously, (3) as a presentation of structure through pictures, symbols and symbols, and (4) as knowledge about something that represents something else.

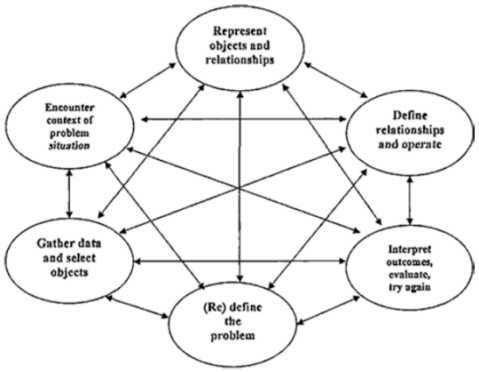


Figure 8. The nodes of modeling process

As illustrated by Lesh and Doerr [18], the actual modeling activity does not move in a linear path through the boxes and sub processes of the modeling cycle. The pre service mathematics thinking, “bounce around”. Mathematical modeling as a method of problem solving requires students to first understand the problem then represent or represent problem into the form of an image. Hence, mathematical modeling will make it easy for students to solve problems in the form of story problems, because this mathematical modeling is in accordance with the student's way of thinking which is concrete, enactive-iconic or concrete-pictorial. Learning by using this mathematical modeling provides concrete, engaging learnin activity designing models based on the problems at hand. Relating to the cognitive processes, knowing, applying and reasoning embedded in this modeling cycle. Particularly, the knowing process emerge in the gather the data, define the relationship object, whereas the applying process happened in the represent object and relationship, and the the reasoning comes up in the interpret outcome, encounter context and redefined the problem.

4. CONCLUSION

The process of thinking students think about turning a problem into a mathematical form, students' reasoning will automatically interpret mathematical solutions so that mathematical representation skills will also take place. To evaluate items pre service teachers not only have to be well-known with the content of mathematics that will being assessed, but they also need to draw on a range of cognitive skills. The cognitive process is that knowing, applying and reasoning. In this study, we found that the obstacle of extracting information from given mathematical representations such as graphs in this

sample item is that translating mathematical content from one mathematical representational form to another.

This study was limited to one component of cognitive processes in mathematical modeling that is using representational form. So that it is still very open to do the extent of the next research for explore other components, such as mathematical communication, mathematical argumentation, modeling, using representational forms, mathematical problem solving, and technical abilities and skills.

REFERENCES

[1] W. Blum, et al, O. (Eds.). Bildungs standards Mathematik: konkret. Sekundarstufe I: Aufgabenbeispiele, Unter richts an regungen, Fortbildungsideen. Berlin, Germany: Cornelsen Scriptor. 2006.

[2] I. Neumann, et al, Modeling and assessing mathematical competence over the lifespan. Journal for educational research online 5. 2013. 2, S. 80-109.

[3] R. B. Ferri, Theoretical and empirical diff erentiations of phases in the modeling process. Zentralblatt der Mathematikdidaktik, 38(2), 2006, 86–95.

[4] NCTM, Principles and Standards for School Mathematics. Reston, VA: NCTM, 2000

[5] S.D. Fatmaryanti, and Sarwanto. Profil Kemampuan Representasi Mahasiswa Pendidikan Fisika Universitas Muhammadiyah Purworejo. Jurnal Pendidikan Fisika dan Keilmuan. 1(1), 2015, 20 – 22.

[6] A. Goldin, Representation in Mathematical Learning and Problem Solving. New Jersey: Lawrence Erlbaun Associated, Inc, 2002.

[7] B.C. Luitel, Multiple Representation of Mathematical Learning, 2001, (online),(http://www.matedu.cinvestav.mx/adalira.pdf).

[8] J. A. Eli, M. J. Mohr-schroeder, and C. W. Lee, Mathematical Connections and Their Relationship to Mathematics Knowledge for Teaching Geometry, *Sch. Sci. Math.*, vol. 113, no. 3, pp. 120–134, 2013.

[9] W. Blum, and D. Leiß, How do students and teachers deal with modeling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), Mathematical modeling : Education, engineerin and economics (pp. 222–231), 2007, Chichester: Horwood.

[10] R.B. Ferri, Personal experiences and extra-mathematical knowledge as an influence factor on modeling routes of pupils. In D. Pitta-Pantazi & G. Philippou (Eds.), Proceedings of CERME 5 (pp. 2020–2079), 2007, Larnaca.

[11] Sugiyono, *Metode penelitian kuantitatif, kualitatif, dan R&D. Bandung: Alfabeta*. 2016.

[12] W. Wiersma and S. Jurs, *Research methods in educational an introduction, ninth edition. USA: Pearson.* 2009.

[15] ----------. 2014. Mathematics cognitive processes. Idonesian Ministry of Education

[16] H.M. Doerr, Experiment, simulation and analysis: An integrated instructional approach to the concept of force. International Journal of Science Education, 19(3), 1997, 265–282.

[17] H.M. Doerr, and D. Pratt, The learning of mathematics and mathematical modeling. In M. K. Heid & G. W. Blume (Eds.), Research on technology in the teaching and learning of mathematics: Syntheses and perspectives: Vol. 1. Mathematics learning, teaching and policy (pp. 259–285), 2008, Charlotte: Information Age.

[18] R.A. Lesh, and H.M. Doerr, Alternatives to trajectories and pathways to describe development in modeling and problem solving. In W. Blum, R. Borromeo Ferri, & K. Maaß (Eds.), Mathematikunterricht im Kontext von Realität, Kultur un Lehrerprofessionalität (pp. 138–147), 2012, Fachmedien Wiesbaden: Springer Spektrum.J