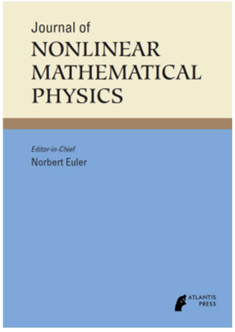

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Arifa Mirza, Mahmood ul Hassan

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Bilinearization and soliton solutions of N=1 supersymmetric coupled dispersionless integrable system

Arifa Mirza[1,2] and Mahmood ul Hassan[1]

[1]*Department of Physics, University of the Punjab*
Quaid-e-Azam Campus, Lahore-54590, Pakistan
arifa.mirza@pucit.edu.pk, mhassan@physics.pu.edu.pk

[2]*Punjab University College of Information Technology, University of the Punjab*
Allama Iqbal Campus, Lahore-54590, Pakistan

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An N=1 supersymmetric generalization of coupled dispersionless (SUSY-CD) integrable system has been proposed by writing its superfield Lax representation. It has been shown that under a suitable variable transformation, the SUSY-CD integrable system is equivalent to N=1 supersymmetric sine-Gordon equation. A superfield bilinear form of SUSY-CD integrable system has been proposed by using super Hirota operator. Explicit expressions of superfield soliton solutions of SUSY-CD integrable system have been obtained by using the Hirota method.

Keywords: integrable systems, supersymmetry, coupled dispersionless system, bilinearization, Hirota method.

1. Introduction

Supersymmetry (SUSY) is a symmetry transformation between bosons and fermions and therefore relates states of different spins [1]- [3]. In classical and quantum field theories, fermions are treated as Grassmann-valued fields. Therefore, we introduce (anticommuting) Grassmann variables along with the commuting ones. That is the bosons are of even grading and fermions are of odd grading. Supersymmetry has been introduced in integrable systems of classical and quantum field theories in order to find the fermionic extensions of known integrable systems and also to understand their underlying algebraic, geometric structures and the physical content in terms of soliton solutions and their interactions [4]- [8]. In these generalizations, the formalism of superspace and superfield has been adopted in order to incorporate fermionic degrees of freedom such that the system remains invariant under supersymmetry transformation.

Dispersionless integrable hierarchies have emerged in diverse areas of theoretical physics with various applications quantum field theories, string theory, conformal field theory and condensed matter theory etc. [9]- [15]. Coupled dispersionless integrable system and its generalizations based on nonabelian Lie groups are also important examples of integrable field theories and in some way are related to dispersionless hierarchies and have found applications in diverse areas of theoretical physics [16]- [29].

In this paper, we study coupled dispersionless integrable system originally presented in [16]. We propose an N=1 supersymmetric generalization of the coupled dispersionless integrable system (SUSY-CD system) in terms of fermionic superfields. We present superfield Lax representations of the system. We also show that the SUSY-CD system is gauge equivalent to the N=1 superfield

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sine-Gordon equation as is the case in the purely bosonic theory. We also present a superfield bilinearization of the SUSY-CD system in terms of super Hirota operators and use it to compute explicit expressions of superfield multisoliton solutions of the system.

2. N=1 SUSY-CD system and superfield Lax representations

To obtain the SUSY field equations, we need to extend the bosonic system with spacetime variables (x,t) to the one with super-spacetime variables (x,t,θ,ζ) where θ and ζ are anticommuting Grassmann variables ($\theta^2 = 0, \zeta^2 = 0$). In superspace, covariant derivatives are defined as $D_x = \partial_\theta + \theta\partial_x$, $D_t = \partial_\zeta + \zeta\partial_t$, where $D_x^2 = \partial_x, D_t^2 = \partial_t$ and D_x anticommutes with D_t. The SUSY transformations are generated by the operators $Q_x = \partial_\theta - \theta\partial_x$, and $Q_t = \partial_\zeta - \zeta\partial_t$. The operators Q_x and Q_t anticommute with covariant derivatives D_x and D_t respectively.

An $N = 1$ SUSY-CD system is defined in terms of fermionic (Grassmann odd) superfields $Q(x,t,\theta,\zeta;\lambda)$ and $R(x,t,\theta,\zeta;\lambda)$ and is given by the following superfield coupled equations

$$D_xD_tQ + RD_xR = 0, \tag{2.1}$$
$$D_xD_tR - RD_xQ = 0. \tag{2.2}$$

The SUSY-CD system (2.1)-(2.2) appears as the compatibility condition of the following superfield Lax pair

$$D_x\Psi = A(x,t,\theta,\zeta;\lambda)\Psi, \qquad D_t\Psi = B(x,t,\theta,\zeta;\lambda)\Psi, \tag{2.3}$$

where $\Psi(x,t,\theta,\zeta)$ is the bosonic (Grassmann even) superfield eigenfunction of the eigenvalue problem and $A(x,t,\theta,\zeta;\lambda)$ and $B(x,t,\theta,\zeta;\lambda)$ are the matrix superfields given by

$$A = \begin{pmatrix} 0 & \frac{g}{4\lambda}(D_xQ + iD_xR) \\ -\frac{g}{4\lambda}(D_xQ - iD_xR) & 0 \end{pmatrix}, \tag{2.4}$$
$$B = \begin{pmatrix} -i\frac{R}{2} & -\lambda g \\ \lambda g & i\frac{R}{2} \end{pmatrix}, \quad \Psi = \begin{pmatrix} X \\ Y \end{pmatrix},$$

together with the following conditions on the fermionic superfield g

$$D_tg = \lambda\left(\frac{X}{Y} - \frac{Y}{X}\right), \tag{2.5}$$
$$D_xg = \frac{1}{4\lambda}\left((D_xQ - iD_xR)\frac{X}{Y} - (D_xQ + iD_xR)\frac{Y}{X}\right),$$

where X and Y are bosonic superfields and λ is a spectral parameter. We have introduced the fermionic superfield g in order to preserve the grading of the superfields. The compatibility condition is equivalent to the superspace zero-curvature condition

$$D_tA + D_xB - \{A,B\} = 0, \tag{2.6}$$

where $\{,\}$ is the anticommutator. The condition (2.6) is equivalent to the SUSY-CD system (2.1-2.2). The SUSY-CD system can also be expressed as the compatibilty condition of another 3×3

superfield Lax representation. In this Lax representation, we introduce 3×3 superfield matrices without introducing superfield g. In order to preserve the grading, we express the superfield $\Psi(x,t,\theta,\zeta)$ as a three component column vector containing Grassmann even and odd functions. The eigenfunction Ψ can now be written as

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix},$$

where ψ_1 and ψ_2 are bosonic superfields and ψ_3 is a fermionic superfield. The matrices $U(x,t,\theta,\zeta;\lambda)$ and $V(x,t,\theta,\zeta;\lambda)$ are 3×3 superfield matrices given by

$$U = \begin{pmatrix} 0 & 0 & \frac{1}{4\lambda}(D_xQ - iD_xR) \\ 0 & 0 & -\frac{1}{4\lambda}(D_xQ + iD_xR) \\ \frac{1}{\lambda}(D_xQ + iD_xR) & -\frac{1}{\lambda}(D_xQ - iD_xR) & 0 \end{pmatrix}, \tag{2.7}$$

$$V = \begin{pmatrix} -iR & 0 & \frac{\lambda}{2} \\ 0 & iR & -\frac{\lambda}{2} \\ -2\lambda & 2\lambda & 0 \end{pmatrix}. \tag{2.8}$$

With these superfield Lax matrices, the zero curvature condition condition $D_tU + D_xV - \{U,V\} = 0$, gives the superfield equations (2.1-2.2).

Since the bosonic CD-system is equivalent to the sine-Gordon equation [17], therefore we show that the SUSY-CD system is equivalent to the superfield sine-Gordon equation. For this purpose, we make the following substitution of superfields Q and R

$$D_xQ = \cos\Phi, \qquad R = D_t\Phi, \tag{2.9}$$

where $\Phi(x,t,\theta,\zeta)$ is a bosonic superfield. By making these substitutions in (2.1) we get

$$-D_t(\cos\Phi) + D_t\Phi D_xD_t\Phi = \sin\Phi D_t\Phi + D_t\Phi D_xD_t\Phi = 0, \tag{2.10}$$

and

$$-D_tD_xD_t\Phi - D_t\Phi\cos\Phi = 0. \tag{2.11}$$

The equations (2.10) - (2.11) are satisfied if superfield Φ obeys the $N = 1$ SUSY sine-Gordon equation

$$D_tD_x\Phi = \sin\Phi. \tag{2.12}$$

This establishes the equivalence of SUSY-CD system with that of SUSY sine-Gordon equation.

3. Component field equations of SUSY-CD system and bosonic limit

In this section, we compute the component fermionic and bosonic field equations by expanding superfields $R(x,t,\theta,\zeta)$ and $Q(x,t,\theta,\zeta)$. We start with the following expansion of the superfields of

SUSY-CD system

$$R(x,t,\theta,\zeta) = \alpha + \theta u + \zeta r + \theta\zeta f, \qquad (3.1)$$
$$Q(x,t,\theta,\zeta) = \beta + \theta v + \zeta q + \theta\zeta h,$$

with the derivative superfields

$$D_x R(x,t,\theta,\zeta) = u + \theta\alpha_x + \zeta f + \theta\zeta r_x, \qquad (3.2)$$
$$D_x Q(x,t,\theta,\zeta) = v + \theta\beta_x + \zeta h + \theta\zeta q_x,$$

where $\alpha(x,t), \beta(x,t), f(x,t)$ and $h(x,t)$ are fermionic fields and $u(x,t)$, $v(x,t)$, $r(x,t)$ and $q(x,t)$, are bosonic fields. The ferminic fields f and h are auxiliary and can be eliminated by corresponding equations. These component fermionic and bosonic fields satisfy the following set of field equations

$$\begin{aligned} q_x - \alpha\alpha_x + u^2 &= 0, \\ r_x - uv + \alpha\beta_x &= 0, \\ v_t - ru &= 0, \\ u_t + rv &= 0, \\ \beta_{xt} + \alpha r_x - 2\alpha uv - \alpha_x r &= 0, \\ \alpha_{xt} - \alpha u^2 + \beta_x r + \alpha v^2 - \alpha q_x &= 0, \end{aligned} \qquad (3.3)$$

where the auxiliary fields f and h are eliminated. At this stage, we expect that these component field equations (3.3) must be equivalent to the component field equations of the $N = 1$ SUSY sine-Gordon equation. If we expand superfield $\Phi(x,t,\theta,\zeta)$ of the SUSY sine-Gordon equation (2.12) as

$$\Phi(x,t,\theta,\zeta) = \omega(x,t) + \theta\phi(x,t) + \zeta\psi(x,t) + \theta\zeta K(x,t), \qquad (3.4)$$

where $\omega(x,t)$ is the usual sine-Gordon field, $\phi(x,t), \psi(x,t)$ are the component fermionic fields and $K(x,t)$ is auxiliary field. When substituted in superfield sine-Gordon equation (2.12), the following set of component field equations are obtained i.e.

$$\begin{aligned} \omega_{xt} &= -\cos\omega\sin\omega - \sin\omega\phi\psi, \\ \phi_t &= \cos\omega\psi, \\ \psi_x &= -\cos\omega\phi. \end{aligned} \qquad (3.5)$$

The component content of the SUSY-CD system (3.3) can also be shown to be equivalent to that of the SUSY sine-Gordon equation (3.5) by making the following set of transformations

$$\begin{aligned} v &= \cos\omega, \qquad q_x = \cos\omega\,\phi\psi - \sin^2\omega, \\ u &= -\sin\omega, \qquad r = \omega_t, \\ \alpha &= \psi, \\ \alpha_x &= -\cos\omega\phi, \\ \beta_x &= -\sin\omega\phi. \end{aligned} \qquad (3.6)$$

By using the set of transformations (3.6), it can be easily checked that the set of equations (3.3) is equivalent to the set of equations (3.5).

In order to get the usual representation of CD systems as given in [16], we take the time derivative of q_x and r_x

$$\begin{aligned} q_{xt} &= 2r_x r + \alpha_t \alpha_x + \alpha \beta_x r, \\ r_{xt} &= -(v^2 + q_x) r - \alpha_t \beta_x. \end{aligned}$$

Let us write $v^2 + q_x = 2p_x$

$$\begin{aligned} 2p_{xt} &= 4r_x r + \alpha_t \alpha_x + 3\alpha \beta_x r, \\ r_{xt} &= -2p_x r - \alpha_t \beta_x. \end{aligned}$$

In the bosonic limit, we get the following expression

$$\begin{aligned} p_{xt} &= 2r_x r, \\ r_{xt} &= -2p_x r, \end{aligned}$$

which is the usual CD system of [16].

4. Bilinearization of SUSY-CD system

In this section, we apply Hirota's bilinearization method to the SUSY-CD system and compute superfield multisoliton solutions of the system. Hirota's method has already been applied to some SUSY integrable equations see e.g. [30]- [36]. In the first step, we transform the superfield equations (2.1-2.2) into a quadratic form in the dependent variables. In order to get a superfield bilinear form of SUSY-CD system, we shall make use of the super Hirota operator. For the functions $a(x,t,\theta,\zeta)$ and $b(x,t,\theta,\zeta)$, the ordinary Hirota operators $\mathbf{D}_x$ and $\mathbf{D}_t$ are defined by

$$\mathbf{D}_x^m \mathbf{D}_t^n (a.b) = (\partial_{x_1} - \partial_{x_2})^m (\partial_{t_1} - \partial_{t_2})^n \{a(x_1,t_1,\theta,\zeta).b(x_2,t_2,\theta,\zeta)\}|_{x_1=x_2=x, t_1=t_2=t}. \tag{4.1}$$

Hirota operators in superspace are defined as

$$S_x(a.b) = (D_{x_1} - D_{x_2})\{a(x_1,t,\theta_1,\zeta).b(x_2,t,\theta_2,\zeta)\}|_{x_1=x_2=x, \theta_1=\theta_2=\theta}, \tag{4.2}$$

and

$$S_t(a.b) = (D_{t_1} - D_{t_2})\{a(x,t_1,\theta,\zeta_1).b(x,t_2,\theta,\zeta_2)\}|_{t_1=t_2=t, \zeta_1=\zeta_2=\zeta}. \tag{4.3}$$

The super Hirota operators S_x and S_t have the following properties

$$\begin{aligned} S_x(a.b) &= (D_x a)b - (-1)^{|a|} a(D_x b), \\ S_x^{2n}(a.b) &= \mathbf{D}_x^n(a.b), \\ S_x^{2n+1}(a.b) &= S_x \mathbf{D}_x^n(a.b), \end{aligned} \tag{4.4}$$

where $|a|$ is the Grassmann parity of function a. The Grassmann parity is unity for fermionic functions (Grassmann odd) and zero for bosonic functions (Grassmann even). Similar properties also hold for the operator S_t. We shall also use an operator S_{xt} which has been defined by its action Grassmann-even function a as [32]

$$S_{xt}(a.a) = aD_x D_t a - (D_x a)(D_t a). \tag{4.5}$$

By introducing the above mentioned super Hirota operators, we now proceed to obtain the superfield bilinear form of the SUSY-CD system. For this pupose, we introduce a suitable transformation of superfields Q and R to the dependent variables F and G given by

$$D_xQ = 1 - 2D_tD_x\log F, \qquad (4.6)$$
$$R = \frac{G}{F},$$

where F is bosonic superfield and G is a fermionic superfield (Grassmann-valued τ-functions). By using above transformation (4.6) in equation (2.1), we obtain the following equation

$$2FF_{t\theta} - 2F_tF_\theta + GG_\theta + \theta\,(2FF_{xt} - 2F_tF_x - GG_x) = 0, \qquad (4.7)$$

where $F_t = \dfrac{\partial F}{\partial t}$, $F_\theta = \dfrac{\partial F}{\partial \theta}$ etc. By using (4.6) in equation (2.2), we obtain the following equation

$$FD_tD_xG - D_xGD_tF + D_tGD_xF + GD_tD_xF + GF = 0. \qquad (4.8)$$

These equations can be expressed in terms of super Hirota operators and provide a superfield Hirota bilinear form of the SUSY-CD system given by

$$2S_x\mathbf{D}_tF.F - S_xG.G = 0, \qquad (4.9)$$
$$S_xS_tG.F - GF = 0. \qquad (4.10)$$

An equivalent bilinearization of SUSY-CD system can be obtained by introducing Grassmann-even τ-functions. We shall see that this bilinearization is same as that of the SUSY sine-Gordon equation.

To relate the bilinearization of SUSY-CD system with that of the SUSY sine-Gordon equation we introduce two bosonic superfields (Grassmann-even τ-functions) F_+, F_- and apply Hirota's bilinearization method to the SUSY-CD system. In the first step, we transform the superfield equations (2.1-2.2) into a quadratic form in the dependent variables. Let us take the transformation of dependent variables Q and R in terms of bosonic superfields F_+ and F_-

$$D_xQ = \frac{1}{2}\left(\frac{F_+}{F_-} + \frac{F_-}{F_+}\right), \qquad (4.11)$$
$$R = iD_t\log\left(\frac{F_-}{F_+}\right).$$

By using the above transformation (4.11) in equation (2.1), we obtain an equation of the form

$$F_-F_+^3S_tF_+.F_- + F_+F_-^3S_tF_-.F_+ + 2S_t(F_-.F_+)(F_+^2S_{xt}F_-.F_- - F_-^2S_{xt}F_+.F_+) = 0. \qquad (4.12)$$

We can split this equation into two parts due to the presence of two τ-functions

$$-2S_t(F_-.F_+)S_{xt}(F_+.F_+) = KF_-^2 - F_-F_+S_t(F_-.F_+), \qquad (4.13)$$
$$2S_t(F_-.F_+)S_{xt}(F_-.F_-) = KF_+^2 + F_-F_+S_t(F_-.F_+),$$

where K is a free separation variable which can be set to zero by a suitable gauge transformation. In such a case, we get

$$2S_{xt}(F_+.F_+) - F_+F_- = 0, \qquad (4.14)$$
$$2S_{xt}(F_-.F_-) - F_-F_+ = 0.$$

So the bilinear form of SUSY-CD system becomes

$$S_{xt}(F_+.F_+ - F_-.F_-) = 0, \qquad (4.15)$$
$$S_{xt}(F_+.F_+) - \frac{1}{2}F_-F_+ = 0.$$

Similarly, same bilinear representation is obtained from equation (2.2). We note that the bilinear form of SUSY-CD system (4.15) is same as that of the bilinear form of SUSY sine-Gordon equation given in ref. [32]. By a suitable choice of superfield τ-functions one can relate the bilinear form SUSY-CD system with that of the SUSY sine-Gordon equation.

5. Soliton solutions of SUSY-CD system

Starting with the bilinear form of SUSY-CD system (4.14), we can compute soliton solution of the SUSY-CD system. Following [32], we start with the τ-Functions

$$F_\pm = e^{\zeta\theta/2} \pm e^{(kx+wt)-\{(\xi+\zeta)(\theta+\lambda)\}/2}. \qquad (5.1)$$

Using expressions of (5.1) in (4.11), we obtain the solutions of SUSY-CD system

$$D_xQ = \coth 2\beta, \qquad (5.2)$$
$$R = -i\vartheta\,\mathrm{csch}\,2\beta,$$

where $2\beta = \zeta\theta/2 + (\xi+\zeta)(\theta+\lambda)/2 - (kx+wt)$ and $\vartheta = \zeta w - \theta - \lambda/2$. The two soliton solution is given by

$$F_\pm = e^{\zeta\theta/2} \pm e^{\eta_1}e^{-(\xi+\zeta_1)(\theta+\lambda_1)/2} \pm e^{\eta_2}e^{-(\xi+\zeta_2)(\theta+\lambda_2)/2} \qquad (5.3)$$
$$+A_{12}\left(1 - \frac{k_1^2+3k_1k_2+k_2^2}{2(k_1-k_2)}\zeta_1\zeta_2\right)e^{\eta_1+\eta_2}e^{(\xi+\alpha_{12}\zeta_1+\alpha_{21}\zeta_2)(\theta+\alpha_{12}\lambda_1+\alpha_{21}\lambda_2)/2},$$

where $\eta = \dfrac{kx-t}{k+\eta_0}$, $\alpha_{ij} = \dfrac{k_i+k_j}{k_i-k_j}$ and $A_{ij} = \left(\dfrac{k_i-k_j}{k_i+k_j}\right)^2$. As is the case in SUSY sine-Gordon equation, the fermionic contribution in the interaction term appears due to supersymmetry.

6. Concluding remarks

In this paper, we have presented an $N = 1$ supersymmetric generalization of coupled dispersionless (SUSY-CD) integrable system by writing down two linear systems of superfield equations in superspace. By using a suitable transformation, we have also shown that the SUSY-CD system is equivalent to an $N = 1$ SUSY sine-Gordon equation. We have presented two bilinearizations of SUSY-CD system, one with fermionic and bosonic superfield τ-functions, and the other with two bosonic superfield τ-functions. We have shown that the later bilinearization is the same as that of the SUSY sine-Gordon equation. We have also computed the soliton solutions of SUSY-CD system. The work can be further extended to study the superfield Darboux transformation and the construction of Wronskian determinant multisoliton solutions of the SUSY-CD system.

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