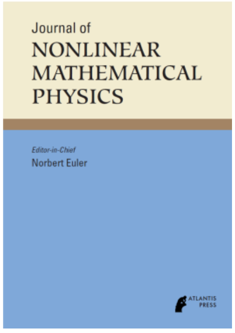

Journal of Nonlinear Mathematical Physics

ISSN (Online): 1402-9251 ISSN (Print): 1776-0852

Journal Home Page: https://www.atlantis-press.com/journals/jnmp

---

Local and Nonlocal Symmetries in Mathematical Physics

Norbert Euler, Enrique G. Reyes

To cite this article: Norbert Euler, Enrique G. Reyes (2017) Local and Nonlocal Symmetries in Mathematical Physics, Journal of Nonlinear Mathematical Physics 24: Supplement 1, 1–2, DOI: https://doi.org/10.1080/14029251.2017.1418049

To link to this article: https://doi.org/10.1080/14029251.2017.1418049

Published online: 04 January 2021

Journal of Nonlinear Mathematical Physics, Vol. 24, Supplement 1 (2017) 1–2

PREFACE

Local and Nonlocal Symmetries in Mathematical Physics

In this special Supplement Issue we present eleven papers on mathematical physics in which symmetries appear in an essential way.

Classical symmetries were introduced by Sophus Lie ([5], [6]); his work has had a revolutionary influence on the development of the theory of differential equations, geometry and mathematical physics in general, and it still has today. We refer to Birkhoff [1], Ovsiannikov [9], Bluman and Cole [2], and Fushchych [3] for earlier reviews and applications of Lie's theory, and to Olver [8], for a contemporary account of the subject rich in applications and historical remarks.

In this current Supplement Issue classical symmetries are used by Bouquet, Conte, Kelsch and Louvet in their analytic study of the buoyancy-drag equation, by Albares, Conde and Estévez in their investigation of a non-isospectral linear problem associated with a recently proposed $(2+1)$ dimensional model describing energy transfer processes in α-helical proteins, and by Nucci in her research on the linearization of the nonlinear pendulum equation. Rogers and Chow show that some symmetry reductions of spatial modulated coupled nonlinear Schrödinger systems give rise to integrable Ermakov and Ermakov-Painlevé subsystems, and Mendoza and Muriel investigate some limits of Lie's methods: they present ordinary differential equations whose algebras of Lie point symmetries are insufficient for integration and they show that the more general λ-symmetries and Sundman transformations can be used to analyze their equations successfully, thereby presenting (yet another) interesting instance in which symmetry methods become very useful complements to numerical tools.

In fact, Lie's original work has over the years developed in many ways. One of its more recent generalizations is the theory of nonlocal symmetries. These symmetries have been used in a somewhat intuitive fashion by several authors, whereas a fully geometric approach to these symmetries has been advanced by Kasil'shchik and Vinogradov [4]. In the current Issue nonlocal symmetries are considered in the contribution by Holba, Krasil'shchik, Morozov and Vojčák. They show that a nonlinear $(2+1)$ dimensional equation that arises in the study of Hamiltonian systems constructed via the Adler-Kostant method admits two-dimensional reductions that possess infinite-dimensional algebras of nonlocal symmetries isomorphic to the Witt algebra. Other ramification of the original class of symmetries studied by Lie is the class of variational symmetries introduced by Noether [7]. In the current Issue, a review and extension of Noether's work is provided in Petrera and Suris' paper on pluri-Lagrangian systems.

Of course the application of symmetry theory is not restricted to differential equations. The current Issue includes a contribution by Chen, Deng and Zhang on the study of symmetries of an important differential-difference equation, namely a version of the Kadomtsev-Petviashvili hierarchy, and another by Gaeta and Lunini on the non-straightforward subject of symmetries of stochastic differential equations.

Co-published by Atlantis Press and Taylor & Francis

Finally, this Issue contains two papers in which symmetries appear in a somewhat different manner than in the contributions mentioned above. Eslami Rad, Magnot and Reyes consider the set of equations of the Kadomtsev-Petviashvili hierarchy and show how to integrate them –thereby effectively finding a hierarchy of infinitely many commuting flows– and, Buring, Kiselev and Rutten review a recent construction by Kontsevich of a differential graded Lie algebra structure on the space of non-oriented graphs and present explicit computations of some cocycles. These computations are relevant for the current Issue as cocycles are known to yield infinitesimal symmetries of Poisson structures.

Norbert Euler (Luleå, Sweden) and Enrique G. Reyes (Santiago, Chile)
November 2017

References

[1] G. Birkhoff, *Hydrodynamics A Study in Logic, Fact and Similitude*, Princeton University Press, Princeton, New Jersey, 1950.

[2] G.W. Bluman and J. Cole, *Similarity Methods for Differential Equations*, Springer-Verlag New York, Heidelberg, Berlin, 1974.

[3] W.I. Fushchych, *Selected Works*, Naukova Dumka, Kiev, 2005, 448 pages: *www.imath.kiev.ua/~fushchych/*

[4] I.S. Krasil'shchik and A.M. Vinogradov, *Nonlocal trends in the geometry of differential equations: symmetries, conservation laws, and Bäcklund transformations*, Acta Appl. Math. 15, (1989) 161–209.

[5] S. Lie, *Theorie der Transformationsgruppen*, B.G. Teubner, Leipzig, vol.I 1888, vol.II 1890 and vol.III 1893.

[6] S. Lie, *Vorlesungen über Differentialgleichungen mit Bekannten Infinitesimalen Transformationen*, B.G. Teubner, Leipzig, 1891.

[7] E. Noether, *Invariante Variationsprobleme*, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Math.-Phys. Kl., (1918) 235–257.

[8] P.J. Olver, *Applications of Lie Groups to Differential Equations* (Second Edition), Springer-Verlag, New York, 1993.

[9] L.V. Ovsiannikov, *Group Analysis of Differential Equations*, Nauka, Moscow, 1978 (in Russian), English transl.: Academic Press, New York, 1982.

Co-published by Atlantis Press and Taylor & Francis