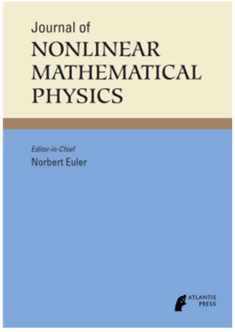

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Conservation Laws and optimal system of extended quantum Zakharov-Kuznetsov equation

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In this paper, the (2+1)-dimensional extended quantum Zakharov-Kuznetsov equation is further explored. The equation is shown to be self-adjoint and conserved vector is constructed according to the related theorem. Then the corresponding optimal system of one-dimensional subgroups is determined. Similarity reductions of the equation under optimal system of subgroups are performed. As a result, the (2+1)-dimensional extended quantum Zakharov-Kuznetsov equation is reduced into a linear PDE with two independent variables.

Keywords: Extended quantum Zakharov-Kuznetsov equation; conserved vector; optimal system; similarity transformation; reduction.

2000 Mathematics Subject Classification: 35G20, 35L65, 58J70

1. Introduction

Several years ago, Zakharov and Kuznetsov [38] established an equation for nonlinear ion-acoustic waves (IAWs) in a magnetized plasma composed of cold ions and hot isothermal electrons. The quantum plasmas and their new features have attracted much attention from both the experimental and theoretical point of view, due to its important role in the charged carrier behaviour when the de Broglie wavelength exceeds the Debye wavelength and approaches the Fermi wavelength [14, 20, 25, 27, 28, 33–36, 38]. The behaviour of the weakly nonlinear ion-acoustic waves in the presence of an uniform magnetic field is governed by the quantum Zakharov-Kuznetsov (QZK) equation. So many authors have considered the effect of the magnetic field in different quantum plasma models [1–5, 7, 9, 12, 13, 15, 17–19, 22, 24, 30, 31, 39].

The (2+1)-dimensional Zakharov-Kuznetsov (ZK) equation was examined by using the sine-cosine method, the extended tanh method, the homotopy analysis method [1], the simplified form

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of Hirota's method [33, 34] and the mapping method [15]. The (2+1)-dimensional generalized Zakharov-Kuznetsov (gZK) equation with nonlinear dispersion and time-dependent coefficients was studied by the solitary wave ansatz method [4]. The (3+1)-dimensional QZK equation was examined by using the auxiliary equation method [20] and the extended F-expansion method (EFE) [3]. The authors [20] employed the reductive perturbation method to formally derive an extended quantum Zakharov-Kuznetsov (extended QZK) equation, which was studied by generalized expansion method [27] and Jacobi elliptic sine and cosine functions [36]. The Lie symmetry approach and the simplest equation method were used to the Zakharov-Kuznetsov modified equal width equation with power law nonlinearity [7] and a class of Generalized (2+1)-dimensional Zakharov-Kuznetsov equation [13].

Wazwaz [35] investigated a new extended (2+1)-dimensional QZK equation, a new (3+1)-dimensional QZK equation and the (3+1)-dimensional extended QZK equation.

The new extended (2+1)-dimensional QZK equation is as follows:

$$u_t + auu_x + b(u_{xxx} + u_{yyy}) + c(u_{xyy} + u_{xxy}) = 0. \tag{1.1}$$

where a, b and c are real-valued constants while $u(x,y,t)$ represents the electrostatic wave potential in plasmas that is a function of the spatial variables x, y and the temporal variable t. The first term in (1.1) is the temporal evolution term, while the coefficient of a is the nonlinear term and the coefficients of b and c represents the spatial dispersions in multi-dimensions.

The authors applied the simplified form of Hirota's method to determine multiple soliton solutions and explosive solutions for the new extended equations above [33, 34]. Then Eq. (1.1) was also studied by Lie symmetry method [32].

In this paper, we will do further research for the extended quantum Zakharov-Kuznetsov equation (QZK) on the basis of the literature [32].

Conservation laws play an important role in the study of differential equations, because conservation laws describe physical conserved quantities, such as mass, energy, momentum and angular momentum, as well as charge and other constants of motion [6, 19, 23]. They have been used in investigating the existence, uniqueness and stability of solutions of nonlinear partial differential equations [29]; and been applied to numerical methods [8, 16] etc. Thus, it is essential to study the conservation laws of partial differential equations.

The plan of the paper is as follows. In the section 2, conservation laws for extended QZK equation are constructed for the first time by using the new conservation theorem of Ibragimov. Then in section 3, an optimal system of one-dimensional subalgebras is found. In section 4, Similarity reductions of the equation under optimal system of subgroups are performed. As a result, the (2+1) dimensional extended quantum Zakharov-Kuznetsov equation is reduced into the linear PDE with two independent variables. Finally, a conclusion is given.

2. Conservation laws of the extended QZK equation

In this section, we obtain conservation laws for Eq. (1.1) using the new conservation theorem due to Ibragimov [10, 11].

2.1. *Preliminaries*

The notation and pertinent results are consistent with the literature. For details, the reader is referred to [10, 11, 21, 37].

We denote a rth order $(r \geq 1)$ system of m PDEs of n independent variables $x = (x^1, x^2, \cdots, x^n)$ with components x^i and m dependent variables $u = (u^1, u^2, \cdots, u^n)$ with components u^β by

$$F_\alpha(x, u, u_{(1)}, \cdots, u_{(r)}) = 0,\ \alpha = 1, 2, \cdots, m. \tag{2.1}$$

The system (2.1) admits a Lie point symmetry with generator

$$X = \xi^i(x,u)\frac{\partial}{\partial x^i} + \eta^\beta(x,u)\frac{\partial}{\partial u^\beta}, \tag{2.2}$$

if $XF_\alpha = 0$ on the solution space of (2.1).

The vector $C = (C^1, C^2, \cdots, C^n)$ is a conserved vector of (2.1) if

$$divC \equiv D_i(C^i) = 0, \tag{2.3}$$

on the solution space of (2.1). The expression (2.3) is a conservation law of (2.1). Here D_i is the total derivative with respect to x^i.

Theorem 2.1. *[10, 11] Lie point symmetry operator* (2.2) *of a system of Eq.* (2.1) *leads to a conserved vector $C = (C^1, C^2, \cdots, C^n)$, constructed by the formula*

$$\begin{aligned} C^i = L\xi^i + W^\alpha &\left[\frac{\partial L}{\partial u_i^\alpha} - D_j(\frac{\partial L}{\partial u_{ij}^\alpha}) + D_jD_k(\frac{\partial L}{\partial u_{ijk}^\alpha}) - \cdots\right] \\ &+ D_j(W^\alpha)\left[\frac{\partial L}{\partial u_{ij}^\alpha} - D_k(\frac{\partial L}{\partial u_{ijk}^\alpha}) + \cdots\right] + D_jD_k(W^\alpha)\left[\frac{\partial L}{\partial u_{ijk}^\alpha}\right] + \cdots, \end{aligned}$$

where $W^\alpha = \eta^\alpha - \xi^j u_j^\alpha$, $L = v^\alpha F_\alpha(x, u, v, u_{(1)}, v_{(1)}, \cdots, u_{(r)}, v_{(r)})$, $(v = (v^1, v^2, \cdots, v^m)$ is the adjoint variable, $\alpha = 1, 2, \cdots, m)$ are Lie characteristic function and formal Lagrangian, respectively.

2.2. *Conservation laws of the extended QZK equation*

Theorem 2.2. *Eq.* (1.1) *is self-adjoint.*

Proof. We write the Lagrangian equation for Eq. (1.1) in the following form:

$$L = v[u_t + auu_x + b(u_{xxx} + u_{yyy}) + c(u_{xyy} + u_{xxy})], \tag{2.4}$$

where v is the adjoint variable. According to Eq. (2.4), we obtain

$$\frac{\partial L}{\partial u} = au_xv,\ \frac{\partial L}{\partial u_t} = v,\ \frac{\partial L}{\partial u_x} = auv,\ \frac{\partial L}{\partial u_{xxx}} = \frac{\partial L}{\partial u_{yyy}} = bv,\ \frac{\partial L}{\partial u_{xyy}} = \frac{\partial L}{\partial u_{xxy}} = cv.$$

Adjoint equation of Eq. (1.1) is written as

$$\begin{aligned} F^* &= \frac{\delta}{\delta u}[vF] = 0; \\ F^* &= \frac{\partial L}{\partial u} - D_x\frac{\partial L}{\partial u_x} - D_t\frac{\partial L}{\partial u_t} - (D_x)^3\frac{\partial L}{\partial u_{xxx}} \\ &\quad - (D_y)^3\frac{\partial L}{\partial u_{yyy}} - D_xD_yD_y\frac{\partial L}{\partial u_{xyy}} - D_xD_xD_y\frac{\partial L}{\partial u_{xxy}} = 0, \end{aligned}$$

namely

$$F^* = -[v_t + auv_x + b(v_{xxx} + v_{yyy}) + c(v_{xyy} + v_{xxy})] = 0. \tag{2.5}$$

Setting $v = u$ in Eq. (2.5), then we yield extended QZK equation

$$u_t + auu_x + b(u_{xxx} + u_{yyy}) + c(u_{xyy} + u_{xxy}) = 0.$$

Therefore, the extended quantum Zakharov-Kuznetsov equation is self-adjoint.

□

Then, we construct new conservation laws for Eq. (1.1) in the light of the new conservation theorem by Ibragimov [10,11]. According to Lie symmetry group method, the Lie point symmetries of Eq. (1.1) are given in [32] as following

$$X_1 = \frac{\partial}{\partial x},\ X_2 = \frac{\partial}{\partial y},\ X_3 = \frac{\partial}{\partial t},\ X_4 = x\frac{\partial}{\partial x} + 3t\frac{\partial}{\partial t} + y\frac{\partial}{\partial y} - 2u\frac{\partial}{\partial u},\ X_5 = t\frac{\partial}{\partial x} + \frac{1}{a}\frac{\partial}{\partial u}.$$

(1) We first consider the Lie point symmetry $X_1 = \frac{\partial}{\partial x}$ of Eq. (1.1). The components of the conserved vector are given by:

$$\begin{aligned} c^x &= uu_t - cu_x u_{yy} + cu_y uxx + cu_y u_{xy}; \\ c^y &= -bu_x uyy + cuxxu_y + bu_y u_{xy} - cu_y u_{xxx} - cu_y u_{xxy} - buu_{xyy}; \\ c^t &= -uu_x. \end{aligned}$$

(2) Likewise, the components of the conserved vector associated with the Lie point symmetry $X_2 = \frac{\partial}{\partial y}$ are given by:

$$\begin{aligned} c^x &= -a^2 u_y - bu_y u_{xx} + bu_x u_{xy} + cu_x u_{yy} - buu_{xxy} - cuu_{xyy} - cuu_{yyy}; \\ c^y &= uu_t + au^2 u_x + bu_{xxx} - cu_y u_{xx} + cu_x u_{xy} + cu_x u_{yy}; \\ c^t &= -uu_y. \end{aligned}$$

(3) Corresponding to the Lie point symmetry $X_3 = \frac{\partial}{\partial t}$, we get the following conserved vectors

$$\begin{aligned} c^x &= -au^2 u_t - bu_{xx} u_t - cu_{xy} u_t - cu_{yy} u_t + bu_{xt} u_x + cu_{tx} u_y + cu_{ty} u_x + cu_{ty} u_y \\ &\quad - buu_{xxt} - cuu_{txy} - cuu_{tyy}; \\ c^y &= -cu_t u_{xx} - cu_{xt} u_t - bu_{yy} u_t + cu_x u_{tx} + cu_y u_{tx} + cu_x u_{ty} + bu_y u_{ty} - cuu_{txx} \\ &\quad - cuu_{txy} - buu_{tyy}; \\ c^t &= au^2 u_x + buu_{xxx} + buu_{yyy} + cuu_{xyy} + cuu_{xxy}. \end{aligned}$$

(4) For the Lie point symmetry $X_4 = x\frac{\partial}{\partial x} + 3t\frac{\partial}{\partial t} + y\frac{\partial}{\partial y} - 2u\frac{\partial}{\partial u}$, the components of the conserved vector are given by:

$$\begin{aligned}
c^x &= xu(u_t + auu_x + bu_{yyy}) - (2u + xu_x + yu_y + 3tu_t)(au^2 + bu_{xx} + cu_{xy} + cu_{yy}) \\
&\quad + (3u_x + xu_{xx} + yu_{xy} + 3tu_{tx})(bu_x + cu_y) + (3u_y + xu_{xy} + yu_{yy} + 3tu_{ty})(cu_x + cu_y) \\
&\quad - (4u_{xx} + yuxxy + 3tu_{txx})bu - (4u_{xy} + yuxyy + 3tu_{txy})cu \\
&\quad - (4u_{yy} + yuyyy + 3tu_{tyy})cu; \\
c^y &= yu(u_t + auu_x + bu_{xxx}) - (2u + xu_x + yu_y + 3tu_t)(cu_{xx} + cu_{xy} + bu_{yy}) \\
&\quad + (3u_x + xu_{xx} + yu_{xy} + 3tu_{tx})(cu_x + cu_y) + (3u_y + xu_{xy} + yu_{yy} + 3tu_{ty})(cu_x + bu_y) \\
&\quad - (4u_{xx} + xu_{xxx} + 3tu_{txx})cu - (4u_{xy} + xu_{xxy} + 3tu_{txy})cu \\
&\quad - (4u_{yy} + xu_{xyy} + 3tu_{tyy})bu; \\
c^t &= 3btuu_t u_{xxx} + 3(b+c)tuu_{yyy} + 3ctuu_{xyy} - 3atu^2uu_x - 2xuu_x - yu_y - 2u^2.
\end{aligned}$$

(5) Finally, we consider the Lie point symmetry $X_5 = t\frac{\partial}{\partial x} + \frac{1}{a}\frac{\partial}{\partial u}$, and obtain the conserved vector whose components are

$$\begin{aligned}
c^x &= btuu_{yyy} + ctuu_{xyy} - ctu_x u_{xx} - ctu_x u_{yy} + ctu_y u_{xx} + \frac{b}{a}u_{xx} + \frac{c}{a}u_{xy} \\
&\quad + \frac{c}{a}u_{yy} + tuu_t + u^2; \\
c^y &= \frac{c}{a}u_{xx} + \frac{c}{a}u_{xy} + \frac{b}{a}u_{yy} - ctu_x u_{xy} - btu_x u_{yy} + ctu_y u_{xx} - ctuu_{xxx} - ctuu_{xxy}; \\
c^t &= \frac{1}{a}u - tu_x u.
\end{aligned}$$

3. Optimal system of one-dimensional subalgebras for extended QZK equation

In this section we present the optimal system of one-dimensional subalgebras for Eq. (1.1). The method which we use for obtaining optimal system of one-dimensional subalgebras is given in [26]. The adjoint transformations are given by

$$Ad(\exp(\varepsilon X_i))X_j = X_j - \varepsilon[X_i, X_j] + \frac{\varepsilon^2}{2}[X_i, [X_i, X_j]] - \cdots,$$

where $[X_i, X_j] = X_i X_j - X_j X_i$ is the commutator for the Lie algebra and ε is a parameter.

Then we construct the optimal system of one-dimensional subalgebras of Eq. (1.1). The adjoint representation table of Lie algebra is constructed in the following Table 1.

Table 1. Adjoint representation of infinitesimal generators.

$Ad(\exp(\varepsilon *))(*)$	X_1	X_2	X_3	X_4	X_5
X_1	X_1	X_2	X_3	$X_4 - \varepsilon X_1$	X_5
X_2	X_1	X_2	X_3	$X_4 - \varepsilon X_2$	X_5
X_3	X_1	X_2	X_3	$X_4 - 3\varepsilon X_3$	$X_5 - a\varepsilon X_1$
X_4	$e^{\varepsilon}X_1$	$e^{\varepsilon}X_2$	$e^{3\varepsilon}X_3$	X_4	$e^{-2\varepsilon}X_5$
X_5	X_1	X_2	$X_3 + a\varepsilon X_1$	$X_4 + 2\varepsilon X_5$	X_5

Theorem 3.1. *An optimal system of one-dimensional Lie subalgebras for Eq.* (1.1) *is provided by*

$$mX_3+nX_4+X_5,\ X_4,X_3,\ X_3-X_2,\ X_3+X_2,\ lX_1+X_2,\ X_1,$$

where $m,n,l\in\mathbf{R}$ *are arbitrary nonzero constants.*

Proof. Given a nonzero vector

$$X=\beta_1X_1+\beta_2X_2+\beta_3X_3+\beta_4X_4+\beta_5X_5.$$

And then simplify as many of the coefficients β_i as possible by utilizing suitable adjoint maps.

Case 1:

First suppose that $\beta_5\neq 0$. Scaling X if necessary, we assume that $\beta_5=1$. Applying $Ad(\exp(\frac{\beta_2}{\beta_4}X_2))$ and $Ad(\exp(\frac{\beta_1}{\beta_4}X_1))$ to it yields

$$\widetilde{X}=Ad(\exp(\frac{\beta_2}{\beta_4}X_2))\circ Ad(\exp(\frac{\beta_1}{\beta_4}X_1))X=\beta_3X_3+\beta_4X_4+X_5.$$

No further simplifications are possible. Then every one-dimensional subalgebra generated by X with $\beta_5\neq 0$ is equivalent to the subalgebra spanned by

$$\beta_3X_3+\beta_4X_4+X_5,$$

where $\beta_3,\beta_4\in\mathbf{R}$ are arbitrary nonzero constants.

Case 2:

The remaining one-dimensional subalgebras are spanned by vectors of the above form with $\beta_5=0,\ \beta_4\neq 0$. We can take $\beta_4=1$. So, the nonzero vector $X=\beta_1X_1+\beta_2X_2+\beta_3X_3+X_4$ is equivalent to $\widetilde{X}$ under adjoint map:

$$\widetilde{X}=Ad(\exp(\frac{\beta_3}{3}X_3))\circ Ad(\exp(\beta_2X_2))\circ Ad(\exp(\beta_1X_1))X=X_4.$$

So every one-dimensional subalgebra generated by X with $\beta_5=0,\ \beta_4\neq 0$ is equivalent to the subalgebra spanned by X_4.

Case 3:

If $\beta_5=0,\ \beta_4=0$, and $\beta_3\neq 0$, we scale to make $\beta_3=1$. Thus, X is equivalent to $\widetilde{X}$ under adjoint representation.

$$\begin{aligned}\widetilde{X}&=Ad(\exp(\varepsilon X_4))\circ Ad(\exp(\frac{-\beta_1}{a}X_1))X\\&=e^{\varepsilon}\beta_2X_2+e^{3\varepsilon}X_3.\end{aligned}$$

This is a scalar multiple of $\widetilde{X}=e^{-2\varepsilon}\beta_2X_2+X_3$. So, depending on the sign of β_2, we can make the coefficient of X_2 either $+1,\ -1$ or 0. Thus every one-dimensional subalgebra generated by X with $\beta_5=0,\ \beta_4=0$, is equivalent to the subalgebra spanned by

$$X_3,\ X_3-X_2,\ X_3+X_2.$$

Case 4:

The remaining one-dimensional subalgebras are spanned by vectors of the above form with $\beta_5=\beta_4=\beta_3=0$. We can take $\beta_2=1$, then $\widetilde{X}=\beta_1X_1+X_2$. If we act on $\widetilde{X}$ by $Ad(\exp(\varepsilon X_4))$,

$$\widetilde{X} = Ad(\exp(\varepsilon X_1))X = \beta_1 X_1 + X_2.$$

Then every one-dimensional subalgebra generated by X with $\beta_5 = \beta_4 = \beta_3 = 0$ is equivalent to the subalgebra spanned by

$$\beta_1 X_1 + X_2,$$

where $\beta_1 \in \mathbf{R}$ is arbitrary nonzero constant.

Case 5:

The remaining case, $\beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$, is similarly seen to be equivalent to X_1.

So the optimal system of one-dimensional Lie subalgebras for Eq. (1.1) is provided by

$$mX_3 + nX_4 + X_5,\ X_4,\ X_3,\ X_3 - X_2,\ X_3 + X_2,\ lX_1 + X_2,\ X_1,$$

where $m, n, l \in \mathbf{R}$ are arbitrary nonzero constants.

□

4. Reduction of the extended QZK equation

In this section we use the obtained optimal symmetries to reduce Eq. (1.1).

(1) $X_3 - X_2 = \frac{\partial}{\partial t} - \frac{\partial}{\partial y}$.

Integration of the invariant surface condition

$$\frac{dx}{0} = \frac{dy}{-1} = \frac{dt}{1} = \frac{du}{0},$$

gives similarity transformation $u = \phi(f,g)$, where the similarity variables are $f = x,\ g = t + y$. Substitute similarity transformation $u = \phi(f,g)$ into Eq. (1.1), and reduced equation is obtained as follows

$$\phi_g + a\phi\phi_f + b(\phi_{fff} + \phi_{ggg}) + c(\phi_{fgg} + \phi_{ffg}) = 0.$$

(2) $X_3 + X_2 = \frac{\partial}{\partial t} + \frac{\partial}{\partial y}$.

Solving the invariant surface condition

$$\frac{dx}{0} = \frac{dy}{1} = \frac{dt}{1} = \frac{du}{0},$$

yields the similarity transformation $u = \phi(f,g)$, with the similarity variables $f = x,\ g = t - y$. Substituting similarity transformation $u = \phi(f,g)$ into Eq. (1.1), leads to the reduced equation

$$\phi_g + a\phi\phi_f + b(\phi_{fff} - \phi_{ggg}) + c(\phi_{fgg} - \phi_{ffg}) = 0.$$

(3) $\beta_1 X_1 + X_2 = \beta_1 \frac{\partial}{\partial x} + \frac{\partial}{\partial y}$.

Considering the invariant surface condition

$$\frac{dx}{\beta_1} = \frac{dy}{1} = \frac{dt}{0} = \frac{du}{0},$$

we obtain similarity transformation $u = \phi(f,g)$ with the similarity variables $f = y - \beta_1 x,\ g = t$. Substituting similarity transformation $u = \phi(f,g)$ into Eq. (1.1), yields the reduced equation

$$\phi_g - a\beta_1\phi\phi_f + (b - b\beta_1^3 - c\beta_1 + c\beta_1^2)\phi_{fff} = 0.$$

5. Conclusion

In this paper, the composite variational principle has been applied to the extended quantum Zakharov-Kuznetsov equation of (2+1)-dimension. Using these symmetries, we prove that the extended QZK equation of (2+1)-dimension is self-adjoint and the conservation laws for the extended QZK equation of (2+1)-dimension are constructed. Then the optimal system of one-dimensional subalgebras is determined. Under some corresponding similarity transformation with similarity invariants, the extended quantum Zakharov-Kuznetsov equation of (2+1)-dimension is reduced into linear PDE with two independent variables. The conservation laws of some more complex equations should be studied.

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