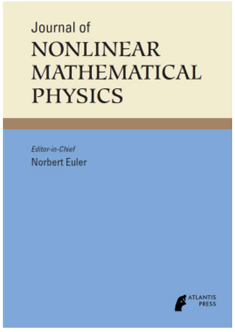

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On a integrable deformations of Heisenberg supermagnetic model

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The Heisenberg supermagnet model which is the supersymmetric generalization of the Heisenberg ferromagnet model is an important integrable system. We consider the deformations of Heisenberg supermagnet model under the two constraint 1. $S^2 = S$ for $S \in USPL(2/1)/S(L(1/1) \times U(1))$ and 2. $S^2 = 3S - 2I$ $S \in USPL(2/1)/S(U(2) \times U(1))$. By means of the gauge transformation, we construct the gauge equivalent counterparts, i.e., the super generalized Hirota equation and Gramman odd nonlinear Schrödinger equation.

Keywords: Heisenberg Supermagnet Model; Gauge Equivalence; Supersymmetry.

2000 Mathematics Subject Classification: 17B80, 37K10, 35Q55

1. Introduction

The Heisenberg ferromagnet (HF) model occurring in the domain of optics and plasma physics is an important integrable systems, and its quantum variant describes a many-particle system with delta-function interaction. HF model has wide applications in the anti-de Sitter/conformal field theories [1, 5], two-dimensional (2D) gravity theory [13], and so on. Takhtajan [20] obtained the Lax representation of HF model and studied its solution through the inverse scattering method. Then, it is presented that HF model is geometrically and gauge equivalent to the nonlinear Schrödinger equation (NLSE) [6, 24]. Much of the work focused on the integrable deformations of HF model including the higher order and inhomogeneous extensions of HF model. Mikhailov and Shabat [15] constructed the first-order integrable deformations of HF model by means of the equivalence between the HF model and the integrable NLSE. In terms of a differential geometric approach, Lakshmanan et al. [8, 16, 18, 25] expressed the higher order deformation of HF model in the form of a higher-order generalized NLSE. Since the inhomogeneous integrable equations have the attractive applications in many fields, Lakshmanan and Ganesan [7] showed that the deformation of HF model is equivalent to the generalized Hirota's equation with linear inhomogeneities. The generalized inhomogeneous HF model and its corresponding gauge equivalent equation is studied in [26]. Recently, Levin et al. [9] investigated the deformed HF model related the quantum 11-vertex R-matrix. By means of the quantum R-matrices, they also established the classical integrable tops which can construct more complicated integrable systems.

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The supersymmetry are of great interest to both mathematicians and physicists. It is encountered in gravity theory, extended supergravity, in the theory of strings and superstrings [2, 3, 17]. As is well known, a number of integrable systems admit a natural extension by odd variables. Thus more motivation comes from superextensions of of many important integrable systems [12, 14, 19] and investigating their structures and integrable properties. Heisenberg supermagnet (HS) models proposed by [10, 11] were the supersymmetric generalizations of the HF model. They have provided a approach to construct the gauge equivalence between the established HS models and the related NLSEs. It has been demonstrated that there is a close relation between HS model and the strong electron correlated Hubbard model. Lately, various deformed integrable HS models were constructed and their integrability properties have been also studied [4, 21–23]. The aim of this paper is to establish another deformations of HS models, which, as far as these authors's knowledge goes, was not previously known. Furthermore, we shall analyze the integrability properties of deformed HS models. By means of the gauge transformation, we derive the gauge equivalent counterparts.

The paper is organized as follows. In section 2, we recall the HS model and its integrable properties. Section 3 is dedicated to introducing the deformed HS models under the two different constraint, and then we investigate their corresponding gauge equivalent counterparts. We end this paper with a summary and discussion in section 4.

2. HS Model

Let us recall the HS model that will be useful in what follows. For a more detailed description we refer to [10, 11].

The HF model is given by

$$\mathbf{S}_t = [\mathbf{S}, \mathbf{S}_{xx}], \tag{2.1}$$

where $\mathbf{S}$ is the spin vector and satisfies the constraint $\mathbf{S}^2 = 1$.

The HS model is the superextensions of the HF model, it can be expressed as follows

$$iS_t = [S, S_{xx}], \tag{2.2}$$

where S is the superspin variable and can be represented

$$\begin{aligned} S &= 2\sum_{a=1}^{4} S_a T_a + 2\sum_{a=5}^{8} C_a T_a, \\ &= \begin{pmatrix} S_3+S_4 & S_1-iS_2 & C_5-iC_6 \\ S_1+iS_2 & -S_3+S_4 & C_7-iC_8 \\ C_5+iC_6 & C_7+iC_8 & 2S_4 \end{pmatrix}, \end{aligned} \tag{2.3}$$

where $S_1, \cdots, S_4$ are the bosonic components and $C_5, \cdots, C_8$ are the fermionic ones, $T_1, \cdots, T_4$ are bosonic generators and $T_5, \cdots, T_8$ are fermionic generators of the superalgebra $uspl(2/1)$.

The concepts of gauge equivalence between integrable systems play an important role in the theory of solitons [24]. The fact is that gauge equivalence exists only for integrable systems possess Lax representations. Understanding the properties of the gauge equivalent counterpart help us know more about the integrable systems. The gauge equivalence can also be used to supersymmetric systems. Under the following two constraints, Makhankov and Pashaev [11] showed the HS modle (2.2) is gauge equivalent to supersymmetric NLSE and Grassman odd NLSE, respectively

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1. $S^2 = S$ for $S \in USPL(2/1)/S(L(1/1)\times U(1))$

$$i\varphi_t + \varphi_{xx} + 2(\varphi\bar{\varphi} + \psi\bar{\psi})\varphi = 0,$$
$$i\psi_t + \psi xx + 2\varphi\bar{\varphi}\psi = 0, \tag{2.4}$$

2. $S^2 = 3S - 2I$ for $S \in USPL(2/1)/S(U(2)\times U(1))$

$$i\psi_{1t} + \psi_{1xx} + 2\bar{\psi}_2\psi_2\psi_1 = 0,$$
$$i\psi_{2t} + \psi_{2xx} + 2\bar{\psi}_1\psi_2\psi_2 = 0, \tag{2.5}$$

where $\varphi(x,t)$ is a Grassman even filed and ψ, ψ_1, ψ_2 are the Grassman odd fileds.

3. Integrable Deformations of the HS Model

Let us consider the deformation of the HS model under the constraint 1. $S^2 = S$, it is easy to get $SS_tS = 0$ and $S[S,S_{xx}]S = 0$. Now we suppose the deformation of the HS model as follows

$$iS_t = f[S,S_{xx}] + g[S,S_x] + uS_x, \tag{3.1}$$

where f is the function of x and t, g and u need to determined later. For the constraint 1, it is not difficult to check $S[S,S_x]S = 0, SS_xS = 0$. We now take

$$U = \lambda S,$$
$$V = i\lambda f[S_x,S] + v(S,S_x), \tag{3.2}$$

where λ is the spectral parameter.

The zero-curvature condition equation

$$U_t - V_x + [U,V] = 0, \tag{3.3}$$

Substituting (3.2) into (3.3), we obtain

$$g = f_x,$$
$$u = -ih,$$
$$v = -(\lambda h + i\lambda^2 f)S,$$
$$\lambda_t = -i\lambda^2 f_x - \lambda h_x. \tag{3.4}$$

where h is the function of x and t.

Then we can rewrite (3.1) as the expression

$$iS_t = f[S,S_{xx}] + f_x[S,S_x] - ihS_x. \tag{3.5}$$

To achieve a better understanding of the the deformation of the HS modle (3.5), let us turn to investigate its gauge equivalent counterpart. It has been known with the constraint 1 and 2, HS model (2.2) is gauge equivalent to super-NLSE and Grassman odd NLSE, respectively. Now one

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shall construct the gauge equivalent counterpart of (3.4). Following the strategy in [11], we take

$$S(x,t) = g^{-1}(x,t)\Sigma g(x,t), \tag{3.6}$$

where $g(x,t) \in USPL(2/1)$. Under the constraint 1 and 2, we take $\Sigma = diag(0,1,1)$ and $\Sigma = diag(1,1,2)$, respectively. Then we introduce the currents

$$J_1 = g_x g^{-1}, J_0 = g_t g^{-1}, \tag{3.7}$$

satisfying the condition

$$\partial_t J_1 - \partial_x J_0 + [J_1, J_0] = 0. \tag{3.8}$$

We now turn to decompose the super algebra $uspl(2/1)$ into two orthogonal parts to obtain J_0 and J_1

$$\mathrm{L} = \mathrm{L}^{(0)} \oplus \mathrm{L}^{(1)}, \tag{3.9}$$

here $[\mathrm{L}^{(\mathrm{i})}, \mathrm{L}^{(\mathrm{j})}\} \subset \mathrm{L}^{(\mathrm{i+j})\mathrm{mod}(2)}$. $L^{(0)}$ is an algebra constructed by means of the generators of the stationary subgroup H. The stationary subgroup H is $S(L(1/1)\times U(1))$ and $S(U(2)\times U(1))$ for constraint 1 and 2, respectively.

We take

$$1. \quad J_1 = i\begin{pmatrix} 0 & \varphi & \psi \\ \bar{\varphi} & 0 & 0 \\ \bar{\psi} & 0 & 0 \end{pmatrix} \in \mathrm{L}^{(1)} \text{ for } S \in USPL(2/1)/S(L(1/1)\times U(1)), \tag{3.10}$$

where $\varphi(x,t)$ and $\psi(x,t)$ are the Grassman even and odd field, respectively.

In terms of (3.6), (3.7) and (3.10), we obtain

$$\begin{aligned} S_t &= g^{-1}(x,t)[\Sigma, J_0]g(x,t), \\ S_x &= g^{-1}(x,t)[\Sigma, J_1]g(x,t), \\ S_{xx} &= g^{-1}(x,t)([[\Sigma, J_1], J_1] + [\Sigma, J_{1x}])g(x,t). \end{aligned} \tag{3.11}$$

Substituting (3.11) into (3.5), we have

$$i[\Sigma, J_0] = f[\Sigma, [[\Sigma, J_1], J_1] + [\Sigma, J_{1x}]] + f_x[\Sigma, [\Sigma, J_1]] - ih[\Sigma, J_1]. \tag{3.12}$$

Under constraint 1, we get $J_0^{(1)}$ by meas of (3.12) and the condition $[\Sigma, J_0^{(0)}] = 0$

$$J_0^{(1)} = i\begin{pmatrix} 0 & i(f\varphi)_x - h\varphi & i(f\psi)_x - h\psi \\ -i(f\bar{\varphi})_x - h\bar{\varphi} & 0 & 0 \\ -i(f\bar{\psi})_x - h\bar{\psi} & 0 & 0 \end{pmatrix}, \tag{3.13}$$

Based on $J_0 = J_0^{(0)} + J_0^{(1)}$, the Eq.(3.8) leads to

$$(J_0^{(0)})_x = [J_1, J_0^{(1)}]. \tag{3.14}$$

Substituting (3.10) and (3.13) into (3.14) and integrating Eq.(3.14), we obtain

$$J_0^{(0)} = \begin{pmatrix} A & 0 & 0 \\ 0 & -if\bar{\varphi}\varphi - i\int_{-\infty}^{x} f_x\bar{\varphi}\varphi dx' & -if\bar{\varphi}\psi - i\int_{-\infty}^{x} f_x\bar{\varphi}\psi dx' \\ 0 & -if\bar{\psi}\varphi - i\int_{-\infty}^{x} f_x\bar{\psi}\varphi dx' & -if\bar{\psi}\psi - i\int_{-\infty}^{x} f_x\bar{\psi}\psi dx' \end{pmatrix}, \tag{3.15}$$

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where $A = if(\varphi\bar{\varphi}+\psi\bar{\psi}) + i\int_{-\infty}^{x} f_x(\varphi\bar{\varphi}+\psi\bar{\psi})dx'$.

By the expression $J_0 = J_0^{(0)} + J_0^{(1)}$, we obtain

$$J_0 = \begin{pmatrix} A & -(f\varphi)_x - ih\varphi & -(f\psi)_x - ih\psi \\ (f\bar{\varphi})_x - ih\bar{\varphi} & -if\bar{\varphi}\varphi - i\int_{-\infty}^{x} f_x\bar{\varphi}\varphi dx' & -if\bar{\varphi}\psi - i\int_{-\infty}^{x} f_x\bar{\varphi}\psi dx' \\ (f\bar{\psi})_x - ih\bar{\psi} & -if\bar{\psi}\varphi - i\int_{-\infty}^{x} f_x\bar{\psi}\varphi dx' & -if\bar{\psi}\psi - i\int_{-\infty}^{x} f_x\bar{\psi}\psi dx' \end{pmatrix}. \tag{3.16}$$

In terms of the gauge transformation, U and V in (3.2) becoms $\hat{U}$ and $\hat{V}$, respectively,

$$\begin{aligned} \hat{U} &= gUg^{-1} + g_x g^{-1} = \lambda\Sigma + J_1, \\ \hat{V} &= gVg^{-1} + g_t g^{-1} = -i\lambda f J_1 - (\lambda h + i\lambda^2 f)\Sigma + J_0. \end{aligned} \tag{3.17}$$

Substituting $\Sigma = diag(0,1,1)$, (3.10) and (3.16) into (3.17), we get

$$\widehat{U} = \begin{pmatrix} 0 & i\varphi & i\psi \\ i\bar{\varphi} & \lambda & 0 \\ i\bar{\psi} & 0 & \lambda \end{pmatrix}, \tag{3.18}$$

$$\widehat{V} = \begin{pmatrix} \widehat{V}_{11} & -(f\varphi)_x - ih\varphi + \lambda f\varphi & -(f\psi)_x - ih\psi + \lambda f\psi \\ (f\bar{\varphi})_x - ih\bar{\varphi} + \lambda f\bar{\varphi} & \widehat{V}_{22} & -if\bar{\varphi}\psi - i\int_{-\infty}^{x} f_x\bar{\varphi}\psi dx' \\ (f\bar{\psi})_x - ih\bar{\psi} + \lambda f\bar{\psi} & -if\bar{\psi}\varphi - i\int_{-\infty}^{x} f_x\bar{\psi}\varphi dx' & \widehat{V}_{33} \end{pmatrix}, \tag{3.19}$$

where

$$\widehat{V}_{11} = if(\varphi\bar{\varphi}+\psi\bar{\psi}) + i\int_{-\infty}^{x} f_x(\varphi\bar{\varphi}+\psi\bar{\psi})dx', \tag{3.20}$$

$$\widehat{V}_{22} = -i\lambda^2 f - \lambda h - if\bar{\varphi}\varphi - i\int_{-\infty}^{x} f_x\bar{\varphi}\varphi dx', \tag{3.21}$$

$$\widehat{V}_{33} = -i\lambda^2 f - \lambda h - if\bar{\psi}\psi - i\int_{-\infty}^{x} f_x\bar{\psi}\psi dx'. \tag{3.22}$$

By means of the zero-curvature condition equation of $\hat{U}$ and $\hat{V}$, we give the super generalized Hirota equation (SGHE)

$$\begin{aligned} & i\varphi_t + i(h\varphi)_x + f[\varphi_{xx} + 2(\varphi\bar{\varphi}+\psi\bar{\psi})\varphi] + 2f_x\varphi_x + 2\varphi\int_{-\infty}^{x} f_x\bar{\varphi}\varphi dx' + \psi\int_{-\infty}^{x} f_x\bar{\psi}\varphi dx' \\ & +\varphi\int_{-\infty}^{x} f_x\psi\bar{\psi}dx' = 0, \\ & i\psi_t + i(h\psi)_x + f(\psi_{xx} + 2\varphi\bar{\varphi}\psi) + 2f_x\psi_x + \varphi\int_{-\infty}^{x} f_x\bar{\varphi}\psi dx' + \psi\int_{-\infty}^{x} f_x\varphi\bar{\varphi}dx' = 0. \end{aligned} \tag{3.23}$$

Equation (3.23) can be regarded as the supersymmetric generalized Hirota equation proposed by Lakshmanan. It should be noted that (3.23) is reduced to generalized Hirota equation [7] with $\gamma = 0$ by taking $h(x,t) = \nu_1 + \mu_1 x, f(x,t) = \nu_2 + \mu_2 x$ in the bosonic limit. The reduction equation can be expressed as follows

$$\begin{aligned} & i\varphi_t + i\mu_1\varphi + i(\nu_1 + \mu_1 x)\varphi_x + (\nu_2 + \mu_2 x)[\varphi_{xx} + 2(\varphi\bar{\varphi})\varphi] \\ & +2\mu_2(\varphi_x + \varphi\int_{-\infty}^{x} \bar{\varphi}\varphi dx') = 0. \end{aligned} \tag{3.24}$$

Taking $f = 1$ and $h = 0$, (3.23) is reduced to super NLSE (2.4).

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We now consider the constraint 2. $S^2 = 3S - 2I$. It is showed that the expression of the corresponding integrable deformation is also (3.5). We should note that $SS_tS = 2S_t, S[S,S_{xx}]S = 2[S,S_{xx}], S[S,S_x]S = 2[S,S_x], SS_xS = 2S_x$. Proceeding the similar procedure as the constraint 1, we consider the deformation equation (3.5) with the constraint 2.

Let us take

$$2. \quad J_1 = i\begin{pmatrix} 0 & 0 & \psi_1 \\ 0 & 0 & \psi_1 \\ \bar{\psi}_1 & \bar{\psi}_2 & 0 \end{pmatrix} \in \mathrm{L}^{(1)} \text{ for } S \in USPL(2/1)/S(U(2)\times U(1)), \tag{3.25}$$

where $\psi_1(x,t), \psi_2(x,t)$ are the Grassman odd fields.

Taking the similar procedure, then we obtain

$$J_0^{(0)} = i\begin{pmatrix} f\psi_1\bar{\psi}_1 + \int_{-\infty}^{x} f_x\psi_1\bar{\psi}_1 dx' & f\psi_1\bar{\psi}_2 + \int_{-\infty}^{x} f_x\psi_1\bar{\psi}_2 dx' & 0 \\ f\psi_2\bar{\psi}_1 + \int_{-\infty}^{x} f_x\psi_2\bar{\psi}_1 dx' & f\psi_2\bar{\psi}_2 + \int_{-\infty}^{x} f_x\psi_2\bar{\psi}_2 dx' & 0 \\ 0 & 0 & B \end{pmatrix}, \tag{3.26}$$

$$J_0^{(1)} = \begin{pmatrix} 0 & 0 & -ih\psi_1 - (f\psi_1)_x \\ 0 & 0 & -ih\psi_2 - (f\psi_2)_x \\ -ih\bar{\psi}_1 + (f\bar{\psi}_1)_x & -ih\bar{\psi}_2 + (f\bar{\psi}_2)_x & 0 \end{pmatrix}, \tag{3.27}$$

where $B = f(\psi_1\bar{\psi}_1 + \psi_2\bar{\psi}_2) + \int_{-\infty}^{x} f_x(\psi_1\bar{\psi}_1 + \psi_2\bar{\psi}_2)dx'$.

Combining (3.27) and (3.26), we obtain

$$J_0 = \begin{pmatrix} if\psi_1\bar{\psi}_1 + i\int_{-\infty}^{x} f_x\psi_1\bar{\psi}_1 dx' & if\psi_1\bar{\psi}_2 + i\int_{-\infty}^{x} f_x\psi_1\bar{\psi}_2 dx' & -ih\psi_1 - (f\psi_1)_x \\ if\psi_2\bar{\psi}_1 + i\int_{-\infty}^{x} f_x\psi_2\bar{\psi}_1 dx' & if\psi_2\bar{\psi}_2 + i\int_{-\infty}^{x} f_x\psi_2\bar{\psi}_2 dx' & -ih\psi_2 - (f\psi_2)_x \\ -ih\bar{\psi}_1 + (f\bar{\psi}_1)_x & -ih\bar{\psi}_2 + (f\bar{\psi}_2)_x & B \end{pmatrix}. \tag{3.28}$$

In terms of the gauge transformation, U and V turn to $\tilde{U}$ and $\tilde{V}$, respectively,

$$\begin{aligned} \tilde{U} &= gUg^{-1} + g_xg^{-1} = \lambda\hat{\Sigma} + J_1, \\ \tilde{V} &= gVg^{-1} + g_tg^{-1} = -i\lambda fJ_1 - (\lambda h + i\lambda^2 f)\hat{\Sigma} + J_0, \end{aligned} \tag{3.29}$$

where $\hat{\Sigma} = diag(1,1,2)$.

By means of (3.25) and (3.28), we rewrite (3.29) as follows

$$\begin{aligned} \tilde{U} &= \begin{pmatrix} \lambda & 0 & i\psi_1 \\ 0 & \lambda & i\psi_2 \\ i\bar{\psi}_1 & i\bar{\psi}_2 & 2\lambda \end{pmatrix}, \\ \tilde{V} &= \begin{pmatrix} \tilde{V}_{11} & if\psi_1\bar{\psi}_2 + i\int_{-\infty}^{x} f_x\psi_1\bar{\psi}_2 dx' & \lambda f\psi_1 - ih\psi_1 - (f\psi_1)_x \\ if\psi_2\bar{\psi}_1 + i\int_{-\infty}^{x} f_x\psi_2\bar{\psi}_1 dx' & \tilde{V}_{22} & \lambda f\psi_2 - ih\psi_2 - (f\psi_2)_x \\ \lambda f\bar{\psi}_1 - ih\bar{\psi}_1 + (f\bar{\psi}_1)_x & \lambda f\bar{\psi}_2 - ih\bar{\psi}_2 + (f\bar{\psi}_2)_x & \tilde{V}_{33} \end{pmatrix}, \end{aligned} \tag{3.30}$$

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where

$$\begin{aligned}
\tilde{V}_{11} &= -i\lambda^2 f - \lambda h + if\psi_1\bar{\psi}_1 + i\int_{-\infty}^{x} f_x\psi_1\bar{\psi}_1 dx',\\
\tilde{V}_{22} &= -i\lambda^2 f - \lambda h + if\psi_2\bar{\psi}_2 + i\int_{-\infty}^{x} f_x\psi_2\bar{\psi}_2 dx'\\
\tilde{V}_{33} &= -2(i\lambda^2 f + \lambda h) + if(\psi_1\bar{\psi}_1 + \psi_2\bar{\psi}_2) + i\int_{-\infty}^{x} f_x(\psi_1\bar{\psi}_1 + \psi_2\bar{\psi}_2) dx'.
\end{aligned} \tag{3.31}$$

From the zero-curvature condition equation of $\tilde{U}$ and $\tilde{V}$, we give the Gramman odd NLSE

$$\begin{aligned}
&i\psi_{1t} + i(h\psi_1)_x + (f\psi_1)_{xx} + 2f\psi_1\bar{\psi}_2\psi_2 - \psi_1\int_{-\infty}^{x} f_x\psi_2\bar{\psi}_2 dx' + \psi_2\int_{-\infty}^{x} f_x\psi_1\bar{\psi}_2 dx' = 0,\\
&i\psi_{2t} + i(h\psi_2)_x + (f\psi_2)_{xx} + 2f\psi_2\bar{\psi}_1\psi_1 - \psi_2\int_{-\infty}^{x} f_x\psi_1\bar{\psi}_1 dx' + \psi_1\int_{-\infty}^{x} f_x\psi_2\bar{\psi}_1 dx' = 0.
\end{aligned} \tag{3.32}$$

Under the reduction $f = 1$ and $h = 0$, (3.32) leads to Gramman odd NLSE (2.5).

4. Summary and discussion

We have investigated the integrable deformations of the HS model. The Lax pairs associated with the deformed models have been deduced. Under the constraint 1. $S^2 = S$ for $S \in USPL(2/1)/S(L(1/1)\times U(1))$ and 2. $S^2 = 3S - 2I$ $S \in USPL(2/1)/S(U(2)\times U(1))$, we construct the integrable deformations of the HS model. The gauge equivalence of the constructed model and the related NLSE is derived. HS model has close relation with the strong electron correlated Hubbard model. Much interest has been stimulated in the study of the strong electron correlated Hubbard model for its important applications in physics. Therefore the applications of the deformations of the HS model in physics worth studying. It should be noted that the gauge equivalent counterpart SGHE (3.23) is reduced to the GHE with $\gamma = 0$ (3.24) in the bosonic limit. The reduction equation is a special case of the GHE. How to construct SGHE with any parameter γ so that it can be reduced to the GHE and how to establish the corresponding deformed HS model still deserves further study.

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