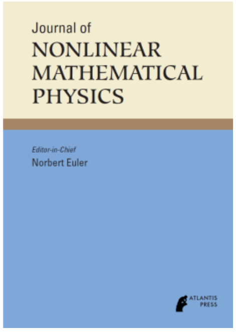

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THE FIRST COHOMOLOGY OF THE SUPERCONFORMAL ALGEBRA $K(1|4)$

ELENA POLETAEVA

Department of Mathematics
University of Texas-Pan American
Edinburg, TX 78539, USA
elenap@utpa.edu

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The infinitesimal deformations of the embedding of the Lie superalgebra of contact vector fields on the supercircle $S^{1|4}$ into the Poisson superalgebra of symbols of pseudodifferential operators on $S^{1|2}$ are explicitly calculated.

Keywords: Superconformal algebra; Poisson superalgebra; cohomology.

Mathematics Subject Classification 2000: 17B56, 17B65

1. Introduction

1.1. *General setting of the problem*

Several years ago Ovsienko and Roger suggested the following problem. Let $\rho : \mathfrak{h} \to \mathfrak{g}$ be an embedding of Lie algebras over a field k. A map $\rho + t\rho_1 : \mathfrak{h} \to \mathfrak{g}$, where $\rho_1 \in Z^1(\mathfrak{h}, \mathfrak{g})$ and $t \in k$, is a Lie algebra homomorphism up to quadratic terms in t. Such infinitesimal deformations of the embedding are classified by linearly independent elements of $H^1(\mathfrak{h}, \mathfrak{g})$, see [13]. Let $c_1, \ldots, c_n$ be cocycles representing a basis of $H^1(\mathfrak{h}, \mathfrak{g})$. The generic infinitesimal deformation of the embedding $\mathfrak{h} \to \mathfrak{g}$ is of the form $t_1c_1 + \cdots + t_nc_n$. The obstructions to integrability of infinitesimal deformations are described by elements of $H^2(\mathfrak{h}, \mathfrak{g})$. The idea of Ovsienko and Roger: one can canonically associate to the embedding $\rho : \mathfrak{h} \to \mathfrak{g}$ a commutative associative algebra, whose generators are the nontrivial cohomology classes in $H^1(\mathfrak{h}, \mathfrak{g})$, and the relations between the generators correspond to the obstructions to integrability of infinitesimal deformations, and they are given by elements of $H^2(\mathfrak{h}, \mathfrak{g})$. We thus get an algebra $k[t]/I$, where $t = (t_1, \ldots, t_n)$ and I is the ideal generated by the coefficients of the obstructions. The algebra $k[t]/I$ only depends on the triple $\rho : \mathfrak{h} \to \mathfrak{g}$.

For interesting examples (the deformations of the natural embeddings of the Lie algebra $\mathfrak{vect}(S^1)$ into the Poisson algebra of the Laurent series on S^1 and into the Lie algebra of symbols of pseudodifferential operators on S^1), see [14, 15]. One more example is considered in [1].

Certain superizations of the constructions considered in [14, 15] were obtained in [2–6]. In [2, 3, 5], the authors described the infinitesimal deformations of the natural embedding of the Lie superalgebra $K(1|N)$ of contact vector fields on the supercircle $S^{1|N}$ into the Lie superalgebra of symbols of pseudodifferential operators on $S^{1|N}$ for $N = 1, 2$ and 3, respectively. In [5] the authors also discussed the difficulties in generalizing their result from $S^{1|3}$ to the case $S^{1|N}$ with $N \geq 4$. In [6] and [4] the authors computed the obstructions to integrability and classified nontrivial deformations of these embeddings for $N = 1$ and 2, respectively.

Here we consider the infinitesimal deformations of the embedding of $K(1|4)$ into the Poisson superalgebra of symbols of pseudodifferential operators on $S^{1|2}$.

1.2. *Basic notions*

The Lie superalgebra $K(1|N)$ — the complexified space of contact vector fields on the supercircle $S^{1|N}$ (with even variable t and odd variables $\xi = (\xi_1, \dots, \xi_r)$ and $\eta = (\eta_1, \dots, \eta_r)$ if $N = 2r$ plus one more variable θ if $N = 2r + 1$) with Laurent polynomials as coefficients is characterized by its action on a contact 1-form. Let $W(1|N)$ be the Lie superalgebra of all superderivations of $\mathbb{C}[t, t^{-1}] \otimes \Lambda(N)$, where $\Lambda(N)$ is the Grassmann algebra in N indeterminates ξ, η (and, perhaps, θ). By definition,

$$K(1|N) = \{D \in W(1|N) \mid D\Omega_N = f\Omega_N \text{ for some } f \in \mathbb{C}[t, t^{-1}] \otimes \Lambda(N)\},$$

where

$$\Omega_N = dt + \sum_{1 \leq i \leq r} (\xi_i d\eta_i + \eta_i d\xi_i) + \begin{cases} 0 & \text{if } N = 2r, \\ \theta d\theta & \text{if } N = 2r + 1 \end{cases}$$

is a differential 1-form (see [8, Sec. 1; 10, Sec. 2]). Note that there are two types of supercircles: one is associated with the trivial vector bundle over the circle, the other one with the Whitney sum of the trivial bundle and the Möbius bundle; the supermanifold $S^{1|N}$ is associated with the rank N trivial bundle over the circle. We consider not all Fourier images of the smooth functions on $S^{1|N}$, but only polynomial ones.

Superalgebras $K(1|N)$ are simple, except for $N = 4$ when the derived superalgebra $K'(1|4) = [K(1|4), K(1|4)]$ is simple. It is known to physicists as the (centerless) "big $N = 4$ superconformal algebra". Note that $K(1|N)$ has no (nontrivial) central extensions if $N > 4$, it has one central extension if $N \leq 3$, while $K(1|4)$ has two central extensions. The Lie superalgebra $K'(1|4)$ is the only superconformal algebra which has three independent central extensions, see [8, 11].

The superalgebra $K(1|2N)$ can be realized as a subsuperalgebra of the Poisson superalgebra $P(2|2N)$ of symbols of pseudodifferential operators on $S^{1|N}$, see [16, 18] and references therein. $K(1|2N + 1)$ has a similar realization, and it will be discussed in detail in another paper. Note that the realization, which we consider here is different from the realization of $K(1|N)$ in [2–6], where the authors consider the natural embedding of $K(1|N)$ into the Lie superalgebra of symbols of pseudodifferential operators on $S^{1|N}$, which contracts to the Poisson superalgebra $P(2|2N)$.

It is a remarkable fact that if $N = 2$, one of the nontrivial central extensions $\hat{K}'(1|4)$ also admits an embedding into the deformed Poisson superalgebra $P_h(2|4)$ which contracts to $P(2|4)$. There exists a one-parameter family of irreducible representations of $\hat{K}'(1|4)$ associated with this embedding in a superspace spanned by four fields (coefficients of the monomials in $\xi = (\xi_1, \xi_2)$), see [16].

In this work, to compute $H^1(K'(1|4), P(2|4))$, we restrict an arbitrary 1-cocycle to a certain subsuperalgebra $\Gamma(\sigma)$ of $K'(1|4)$.

1.3. *What is* $\Gamma(\sigma) := \Gamma(\sigma_1, \sigma_2, \sigma_3)$

Let $\sigma_1, \sigma_2, \sigma_3 \in \mathbb{C}$ be such that $\sigma_1 + \sigma_2 + \sigma_3 = 0$. Let V_i be a 2-dimensional vector space and ψ_i be a nondegenerate skew-symmetric form on V_i for each i. Let $\Gamma(\sigma)_{\bar{0}} = \mathrm{sp}(\psi_1) \oplus \mathrm{sp}(\psi_2) \oplus \mathrm{sp}(\psi_3)$ and $\Gamma(\sigma)_{\bar{1}} = V_1 \otimes V_2 \otimes V_3$ with the natural $\Gamma(\sigma)_{\bar{0}}$-action on $\Gamma(\sigma)_{\bar{1}}$. Let $\mathfrak{P}_i : V_i \times V_i \to \mathrm{sp}(\psi_i)$ be $\mathrm{sp}(\psi_i)$-invariant bilinear mappings given by

$$\mathfrak{P}_i(x_i, y_i)z_i = \psi_i(y_i, z_i)x_i - \psi_i(z_i, x_i)y_i, \quad \text{where } x_i, y_i, z_i \in V_i.$$

The commutator of two elements of $\Gamma(\sigma)_{\bar{1}}$ is given by the formula

$$\begin{aligned}[x_1 \otimes x_2 \otimes x_3, y_1 \otimes y_2 \otimes y_3] &= \sigma_1 \psi_2(x_2, y_2)\psi_3(x_3, y_3)\mathfrak{P}_1(x_1, y_1) \\ &\quad + \sigma_2 \psi_1(x_1, y_1)\psi_3(x_3, y_3)\mathfrak{P}_2(x_2, y_2) \\ &\quad + \sigma_3 \psi_1(x_1, y_1)\psi_2(x_2, y_2)\mathfrak{P}_3(x_3, y_3).\end{aligned}$$

The superalgebra $\Gamma(\sigma_1, \sigma_2, \sigma_3)$ is simple if and only if $\prod \sigma_i \neq 0$, and $\Gamma(\sigma_1, \sigma_2, \sigma_3) \cong \Gamma(\sigma_1', \sigma_2', \sigma_3')$ if and only if the sets $\{\sigma_i'\}$ and $\{\sigma_i\}$ are obtained from each other by a permutation and multiplication of all elements of one set by a nonzero complex number, so we can fix $\sigma_1 = 2$ and set $\lambda = \sigma_2/\sigma_3$, so $\Gamma(\sigma_1, \sigma_2, \sigma_3)$ is a one-parameter family of deformations of the Lie superalgebra $\mathrm{osp}(4|2)$.

Note that this family of Lie superalgebras has different notation in the literature. It was introduced in [12] and denoted by $\Gamma(A, B, C)$. In [9, p. 33] it is denoted by $D(2, 1; \lambda)$.

For each $\alpha \in \mathbb{C}$, the Lie superalgebra $\Gamma(2, -1 - \alpha, \alpha - 1)$ can be realized as a subsuperalgebra of the Poisson superalgebra $P(2|4)$ (see [17, 18]):

$$\Gamma(2, -1 - \alpha, \alpha - 1) \subset K'(1|4) \subset P(2|4).$$

In [18], the infinitesimal deformations of the embedding of $\Gamma(2, -1 - \alpha, \alpha - 1)$ into $P(2|4)$ are described: $\dim H^1(\Gamma(2, -1 - \alpha, \alpha - 1), P(2|4)) = 2$. To compute the obstructions to the integrability of these infinitesimal deformations is an open problem.

Let $\Gamma = \Gamma(2, -1, -1) \cong \mathrm{osp}(4|2)$. Let c be a 1-cochain for the $K'(1|4)$-module $P(2|4)$. Let $c|_\Gamma$ be its restriction to Γ. We will show that $\dim H^1(K'(1|4), P(2|4)) = 3$ and that the map $\varphi : c \to c|_\Gamma$ defines a surjective homomorphism $\varphi : H^1(K'(1|4), P(2|4)) \to H^1(\Gamma, P(2|4))$. We will also show that $\dim H^1(K(1|4), P(2|4)) = 3$.

2. Poisson Superalgebra $P(2|4)$

The *Poisson algebra P of pseudodifferential symbols on the circle* is formed by the formal series

$$A(t,\tau) = \sum_{-\infty}^{n} a_i(t)\tau^i,$$

where $a_i(t) \in \mathbb{C}[t, t^{-1}]$, and the even variable τ corresponds to ∂_t, see [14, 15]. The Poisson bracket is defined as follows:

$$\{A(t,\tau), B(t,\tau)\} = \partial_\tau A(t,\tau) \cdot \partial_t B(t,\tau) - \partial_t A(t,\tau) \cdot \partial_\tau B(t,\tau).$$

Let $\Lambda(2N) = \mathbb{C}[\xi_1, \ldots, \xi_N, \eta_1, \ldots, \eta_N]$. The *Poisson superalgebra of pseudodifferential symbols on* $S^{1|N}$ is $P(2|2N) = P \otimes \Lambda(2N)$. The Poisson bracket is defined as follows:

$$\{A, B\} = \partial_\tau A \cdot \partial_t B - \partial_t A \cdot \partial_\tau B + (-1)^{p(A)+1} \sum_{1\leq i \leq N} (\partial_{\xi_i} A \cdot \partial_{\eta_i} B + \partial_{\eta_i} A \cdot \partial_{\xi_i} B).$$

2.1. *The superalgebra* $K(1|4)$

Lie superalgebras $K(1|N)$ have different names and notation in the literature. Physicists call these superalgebras "superconformal" by analogy with the Witt algebra $\mathfrak{witt} = \mathrm{der}\mathbb{C}[t, t^{-1}]$ of conformal transformations. Physicists who studied superstrings were mainly interested in nontrivial central extensions of "superconformal" superalgebras, and they used the term "superconformal" for centrally extended superalgebras although it is not known if these algebras are conformal in any sense.

In [8], it is explained to which extent the simple "superconformal" superalgebra is conformal, a not self-contradicting definition superizing the notion of "deep" algebras (due to O. Mathieu), and the term "stringy" is suggested instead to them, and their "relatives", i.e. central extensions and algebras of derivations.

In [10], the author uses notation $W_{(N)}$, $K_{(N)}$ and $K'_{(4)}$.

Consider a $\mathbb{Z}$-grading $P(2|2N) = \bigoplus_i P_{(i)}(2|2N)$, of an associative superalgebra, defined by

$$\deg t = \deg \eta_i = \deg \tau = \deg \xi_i = 1 \quad \text{for } i = 1, \ldots, N.$$

With respect to the Poisson super bracket we see that

$$\{P_{(i)}(2|2N), P_{(j)}(2|2N)\} \subset P_{(i+j-2)}(2|2N).$$

Thus $P_{(2)}(2|2N)$ is a subsuperalgebra of $P(2|2N)$, and $P_{(2)}(2|2N) \cong K(1|2N)$; see [16, 18].

Note that $K(1|4)$ is spanned by the following elements:

$$\begin{array}{llll}
E_i^1 = t^{i+2}\tau^{-i}, & F_i^1 = t^{i-2}\tau^{-i}\xi_1\xi_2\eta_1\eta_2, & T_i^1 = t^{i+1}\tau^{-i}\eta_1, & T_i^2 = t^{i+1}\tau^{-i}\eta_2,\\
E_i^2 = t^{i}\tau^{-i}\xi_1\xi_2, & F_i^2 = t^{i}\tau^{-i}\eta_1\eta_2, & T_i^3 = t^{i+1}\tau^{-i}\xi_1, & T_i^4 = t^{i+1}\tau^{-i}\xi_2,\\
E_i^3 = t^{i}\tau^{-i}\xi_1\eta_2, & F_i^3 = t^{i}\tau^{-i}\xi_2\eta_1, & D_i^1 = t^{i-1}\tau^{-i}\xi_1\xi_2\eta_2 & D_i^2 = t^{i-1}\tau^{-i}\xi_1\xi_2\eta_1\\
H_i^1 = t^{i}\tau^{-i}\xi_1\eta_1, & H_i^2 = t^{i}\tau^{-i}\xi_2\eta_2, & D_i^3 = t^{i-1}\tau^{-i}\xi_2\eta_1\eta_2, & D_i^4 = t^{i-1}\tau^{-i}\xi_1\eta_1\eta_2,
\end{array}$$

where $i \in \mathbb{Z}$. The Lie superalgebra $K'(1|4)$ is defined from the exact sequence

$$0 \to K'(1|4) \to K(1|4) \to \mathbb{C}F_1^1 \to 0.$$

2.2. *Realization of $\Gamma(\sigma)$ as a subsuperalgebra of $P(2|4)$*

We proved in [18, Proposition 3.2] that for each $\alpha \in \mathbb{C}$ there exists an embedding

$$\Gamma(2, -1-\alpha, \alpha-1) \xrightarrow{\rho_\alpha} K'(1|4) \subset P(2|4).$$

The image $\Gamma_\alpha = \rho_\alpha(\Gamma(2, -1-\alpha, \alpha-1))$ is spanned by the following elements:

$$\begin{array}{lll}
E_\alpha^1 = t^2, & F_\alpha^1 = \tau^2 - 2\alpha t^{-2}\xi_1\xi_2\eta_1\eta_2, & H_\alpha^1 = t\tau, \\
E_\alpha^2 = \xi_1\xi_2, & F_\alpha^2 = \eta_1\eta_2, & H_\alpha^2 = \xi_1\eta_1 + \xi_2\eta_2, \\
E_\alpha^3 = \xi_1\eta_2, & F_\alpha^3 = \xi_2\eta_1, & H_\alpha^3 = \xi_1\eta_1 - \xi_2\eta_2, \\
T_\alpha^1 = t\eta_1, & T_\alpha^2 = t\eta_2, & T_\alpha^3 = t\xi_1, \\
T_\alpha^4 = t\xi_2, & D_\alpha^1 = \tau\xi_1 + \alpha t^{-1}\xi_1\xi_2\eta_2, & D_\alpha^2 = \tau\xi_2 - \alpha t^{-1}\xi_1\xi_2\eta_1, \\
D_\alpha^3 = \tau\eta_1 + \alpha t^{-1}\xi_2\eta_1\eta_2, & D_\alpha^4 = \tau\eta_2 - \alpha t^{-1}\xi_1\eta_1\eta_2. &
\end{array}$$

We have also proved in [18, Theorem 4.1] that $H^1(\Gamma_\alpha, P(2|4))$ is spanned by the classes of the 1-cocycles θ_1 and θ_2 given as follows:

$$\begin{array}{lll}
\theta_1(F_\alpha^1) = 2t^{-1}\tau, & \theta_1(H_\alpha^1) = 1, & \theta_1(D_\alpha^1) = t^{-1}\xi_1, \\
\theta_1(D_\alpha^2) = t^{-1}\xi_2, & \theta_1(D_\alpha^3) = t^{-1}\eta_1, & \theta_1(D_\alpha^4) = t^{-1}\eta_2,
\end{array}$$

$$\begin{array}{ll}
\theta_2(E_\alpha^1) = t\tau^{-1} - \tau^{-2}\xi_1\eta_1 & \\
\qquad - \tau^{-2}\xi_2\eta_2 - 2t^{-1}\tau^{-3}\xi_1\xi_2\eta_1\eta_2, & \theta_2(E_\alpha^2) = t^{-1}\tau^{-1}\xi_1\xi_2, \\
\theta_2(F_\alpha^1) = t^{-1}\tau + t^{-2}\xi_1\eta_1 & \\
\qquad + t^{-2}\xi_2\eta_2 + 2(1+\alpha)t^{-3}\tau^{-1}\xi_1\xi_2\eta_1\eta_2, & \theta_2(H_\alpha^1) = 1, \\
\theta_2(F_\alpha^2) = -t^{-1}\tau^{-1}\eta_1\eta_2, & \theta_2(T_\alpha^3) = \tau^{-1}\xi_1 - t^{-1}\tau^{-2}\xi_1\xi_2\eta_2, \\
\theta_2(T_\alpha^4) = \tau^{-1}\xi_2 + t^{-1}\tau^{-2}\xi_1\xi_2\eta_1, & \theta_2(D_\alpha^1) = t^{-1}\xi_1, \\
\theta_2(D_\alpha^2) = t^{-1}\xi_2, & \theta_2(D_\alpha^3) = -(1+\alpha)t^{-2}\tau^{-1}\xi_2\eta_1\eta_2, \\
\theta_2(D_\alpha^4) = (1+\alpha)t^{-2}\tau^{-1}\xi_1\eta_1\eta_2. &
\end{array}$$

3. The First Cohomology of $K(1|4)$

Theorem 3.1. *The space $H^1(K'(1|4), P(2|4))$ is spanned by the classes of the 1-cocycles c_1, c_2 and c_3 given as follows ($i \in \mathbb{Z}$):*

$$\begin{array}{ll}
c_1(E_i^1) = -it^{i+1}\tau^{-i-1}, & c_1(F_i^1) = -it^{i-3}\tau^{-i-1}\xi_1\xi_2\eta_1\eta_2 \quad (i \neq 1), \\
c_1(E_i^2) = -it^{i-1}\tau^{-i-1}\xi_1\xi_2, & c_1(F_i^2) = -it^{i-1}\tau^{-i-1}\eta_1\eta_2, \\
c_1(E_i^3) = -it^{i-1}\tau^{-i-1}\xi_1\eta_2, & c_1(F_i^3) = -it^{i-1}\tau^{-i-1}\xi_2\eta_1, \\
c_1(H_i^1) = -it^{i-1}\tau^{-i-1}\xi_1\eta_1, & c_1(H_i^2) = -it^{i-1}\tau^{-i-1}\xi_2\eta_2,
\end{array}$$

$$c_1(T_i^1) = -it^i\tau^{-i-1}\eta_1, \qquad c_1(T_i^2) = -it^i\tau^{-i-1}\eta_2,$$
$$c_1(T_i^3) = -it^i\tau^{-i-1}\xi_1, \qquad c_1(T_i^4) = -it^i\tau^{-i-1}\xi_2,$$
$$c_1(D_i^1) = -it^{i-2}\tau^{-i-1}\xi_1\xi_2\eta_2, \quad c_1(D_i^2) = -it^{i-2}\tau^{-i-1}\xi_1\xi_2\eta_1,$$
$$c_1(D_i^3) = -it^{i-2}\tau^{-i-1}\xi_2\eta_1\eta_2, \quad c_1(D_i^4) = -it^{i-2}\tau^{-i-1}\xi_1\eta_1\eta_2,$$

$$c_2(E_i^1) = t^{i+1}\tau^{-i-1} - (i+1)t^i\tau^{-i-2}(\xi_1\eta_1+\xi_2\eta_2), \quad c_2(F_i^1) = -t^{i-3}\tau^{-i-1}\xi_1\xi_2\eta_1\eta_2 \quad (i \neq 1),$$
$$c_2(E_i^2) = t^{i-1}\tau^{-i-1}\xi_1\xi_2, \qquad c_2(F_i^2) = -t^{i-1}\tau^{-i-1}\eta_1\eta_2,$$
$$c_2(H_i^1) = it^{i-2}\tau^{-i-2}\xi_1\xi_2\eta_1\eta_2, \qquad c_2(H_i^2) = it^{i-2}\tau^{-i-2}\xi_1\xi_2\eta_1\eta_2,$$
$$c_2(T_i^1) = \left(i+\frac{1}{2}\right)t^{i-1}\tau^{-i-2}\xi_2\eta_1\eta_2, \qquad c_2(T_i^2) = -\left(i+\frac{1}{2}\right)t^{i-1}\tau^{-i-2}\xi_1\eta_1\eta_2,$$
$$c_2(T_i^3) = t^i\tau^{-i-1}\xi_1 - \left(i+\frac{1}{2}\right)t^{i-1}\tau^{-i-2}\xi_1\xi_2\eta_2, \qquad c_2(T_i^4) = t^i\tau^{-i-1}\xi_2 + \left(i+\frac{1}{2}\right)t^{i-1}\tau^{-i-2}\xi_1\xi_2\eta_1,$$
$$c_2(D_i^3) = -t^{i-2}\tau^{-i-1}\xi_2\eta_1\eta_2, \qquad c_2(D_i^4) = -t^{i-2}\tau^{-i-1}\xi_1\eta_1\eta_2,$$

$$c_3(E_i^1) = i(i+1)(i+2)t^{i-1}\tau^{-i-3}\xi_1\xi_2\eta_1\eta_2, \quad c_3(F_i^1) = \frac{1}{i-1}t^{i-1}\tau^{-i+1} \quad (i \neq 1),$$
$$c_3(E_i^2) = -it^{i-1}\tau^{-i-1}\xi_1\xi_2, \qquad c_3(F_i^2) = -it^{i-1}\tau^{-i-1}\eta_1\eta_2,$$
$$c_3(E_i^3) = it^{i-1}\tau^{-i-1}\xi_1\eta_2, \qquad c_3(F_i^3) = it^{i-1}\tau^{-i-1}\xi_2\eta_1,$$
$$c_3(H_i^1) = it^{i-1}\tau^{-i-1}\xi_2\eta_2, \qquad c_3(H_i^2) = -it^{i-1}\tau^{-i-1}\xi_1\eta_1.$$
$$c_3(T_i^1) = i(i+1)t^{i-1}\tau^{-i-2}\xi_2\eta_1\eta_2, \qquad c_3(T_i^2) = -i(i+1)t^{i-1}\tau^{-i-2}\xi_1\eta_1\eta_2,$$
$$c_3(T_i^3) = i(i+1)t^{i-1}\tau^{-i-2}\xi_1\xi_2\eta_2, \qquad c_3(T_i^4) = -i(i+1)t^{i-1}\tau^{-i-2}\xi_1\xi_2\eta_1,$$
$$c_3(D_i^1) = t^{i-1}\tau^{-i}\xi_1, \qquad c_3(D_i^2) = -t^{i-1}\tau^{-i}\xi_2,$$
$$c_3(D_i^3) = t^{i-1}\tau^{-i}\eta_1, \qquad c_3(D_i^4) = -t^{i-1}\tau^{-i}\eta_2.$$

Proof. Consider $\mathrm{gl}(2) \cong \mathrm{Span}(\xi_i\eta_j \mid i,j = 1,2) \subset K'(1|4)$. The diagonal subalgebra of gl(2) consists of $h = h_1\xi_1\eta_1 + h_2\xi_2\eta_2$, where $h_1, h_2 \in \mathbb{C}$. Let $\epsilon_i(h) = h_i$, where $i = 1,2$. Obviously, $\mathrm{Span}(\xi_1,\xi_2)$ is the standard gl(2)-module, $\mathrm{Span}(\eta_1,\eta_2)$ is its dual, ξ_i and η_i have weights ϵ_i and $-\epsilon_i$, respectively. Note that $H^1(K'(1|4), P(2|4))$ is a trivial gl(2)-module, since every Lie (super)algebra acts trivially on its own cohomology (see [7, p. 28]). Hence we have to compute only 1-cocycles of weight 0. Note that

$$H^1(K'(1|4), P(2|4)) = \bigoplus_{k\in\mathbb{Z}} H^1(K'(1|4), P_{(k)}(2|4))$$

and

$$H^1(K'(1|4), P(2|4)_{(k)}) = \bigoplus_{n\in\mathbb{Z}} H^{1,n}(K'(1|4), P_{(k)}(2|4)),$$

where the $\mathbb{Z}$-grading of $H^1(K'(1|4), P(2|4)_{(k)})$ is given by the conditions

$$\deg t = 1, \quad \deg \tau = -1, \quad \deg \xi_i = \deg \eta_i = 0 \quad \text{for } i = 1, 2.$$

Let $c \in C^{1,n}(K'(1|4), P_{(k)}(2|4))$ be a 1-cochain of weight zero. Let $\theta = c|_\Gamma$ be the restriction of c to Γ. If c is a cocycle (a coboundary), then θ is a cocycle (a coboundary). It follows from the description of $H^1(\Gamma, P(2|4))$ that if $k \neq 0$ or $n \neq 0$ and θ is a 1-cocycle, then it is a coboundary. Let $k = 0$ and $n = 0$.

(1) Suppose that $c|_\Gamma = \theta_1$. Then

$$\begin{aligned} &c(E^1_{-2}) = 2t^{-1}\tau, \quad c(E^1_{-1}) = 1, \qquad\quad c(E^1_0) = 0, \\ &c(T^1_{-1}) = t^{-1}\eta_1, \quad c(T^2_{-1}) = t^{-1}\eta_2, \quad c(T^3_{-1}) = t^{-1}\xi_1, \quad c(T^4_{-1}) = t^{-1}\xi_2. \end{aligned} \tag{3.1}$$

Note that

$$\begin{aligned} c(E^1_i) = {} & a_{i,1}t^{i+1}\tau^{-i-1} + a_{i,2}t^i\tau^{-i-2}\xi_1\eta_1 + a_{i,3}t^i\tau^{-i-2}\xi_2\eta_2 \\ & + a_{i,4}t^{i-1}\tau^{-i-3}\xi_1\xi_2\eta_1\eta_2, \end{aligned} \tag{3.2}$$

for some $a_{i,k} \in \mathbb{C}$. Note also that

$$\{E^1_i, E^1_j\} = 2(j-i)E^1_{i+j+1}.$$

The cocycle equation reads: $dc = 0$, where for (homogeneous) $X, Y \in K'(1|4)$ we have

$$dc(X,Y) = \begin{cases} \{X, c(Y)\} + \{Y, c(X)\} - c(\{X,Y\}) & \text{if } p(X) = p(Y) = \bar{1}, \\ \{X, c(Y)\} - \{Y, c(X)\} - c(\{X,Y\}) & \text{if } p(X) = \bar{0}, \quad p(Y) = \bar{1}, \\ \{X, c(Y)\} - \{Y, c(X)\} - c(\{X,Y\}) & \text{if } p(X) = p(Y) = \bar{0}. \end{cases}$$

Then from

$$(dc)(E^1_i, E^1_j) = \{E^1_i, c(E^1_j)\} - \{E^1_j, c(E^1_i)\} - c(\{E^1_i, E^1_j\}) = 0$$

we have

$$\begin{aligned} &(j-i)a_{i+j+1,1} = (j+1)a_{j,1} - (i+1)a_{i,1}, \quad (j-i)a_{i+j+1,2} = (i+j+2)(a_{j,2} - a_{i,2}), \\ &(j-i)a_{i+j+1,3} = (i+j+2)(a_{j,3} - a_{i,3}), \qquad (j-i)a_{i+j+1,4} = (2i+j+3)a_{j,4} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad - (i+2j+3)a_{i,4}. \end{aligned} \tag{3.3}$$

Then from Eq. (3.1) it follows that $a_{i,1} = -i$ for all i, and $a_{i,k} = 0$ for all i if $k = 2, 3$, and also $a_{i,4} = ai(i+1)(i+2)$ for all i and an arbitrary $a \in \mathbb{C}$. Assume that $a = 0$. We will show that then $c = c_1$. Note that if c is an arbitrary 1-cocycle for the $K'(1|4)$-module $P(2|4)$ such that $c|_\Gamma = \theta_1$, then $(c - c_1)|_\Gamma = 0$, and it follows from part (3) of the proof that $c - c_1$ is a multiple of c_3.

To compute c, we will use the Hochschild–Serre spectral sequence with respect to the subalgebra Γ. Recall the definition of this spectral sequence (see [7, pp. 40–44]).

We consider the following filtration on the space of cochains $C^k = C^k(K'(1|4), P(2|4))$:

$$F^0C^k = C^k \supset F^1C^k \supset \cdots \supset F^pC^k \supset \cdots \supset F^{k+1}C^k = 0,$$

where

$$F^pC^k = \{c \in C^k \mid c(X_1, \ldots, X_i, \ldots, X_k) = 0 \text{ if } X_1, \ldots, X_{k+1-p} \in \Gamma\},$$

where $0 \le p \le k+1$. Set

$$Z_r^{p,q} = \{c \in F^pC^{p+q} \mid dc \in F^{p+r}C^{p+q+1}\}, \quad E_r^{p,q} = Z_r^{p,q}/(Z_{r-1}^{p+1,q-1} + dZ_{r-1}^{p-r+1,q+r-2}).$$

Notice that the differential d induces the differentials

$$d_r^{p,q} : E_r^{p,q} \to E_r^{p+r,q-r+1}$$

and $E_{r+1}^{p,q} = H^{p,q}(E_r)$, so that

$$H^k(K'(1|4), P(2|4)) = \bigoplus_{p+q=k} E_\infty^{p,q}.$$

Note that

$$E_1^{p,q}(K'(1|4), P(2|4)) = H^q(\Gamma, \mathrm{Hom}(\Lambda^p(K'(1|4)/\Gamma), P(2|4))),$$

see [7, p. 40]. If $k = 1$, then

$$H^1(K'(1|4), P(2|4)) = E_\infty^{0,1} \oplus E_\infty^{1,0}.$$

Note that $c \in E_1^{0,1}$, and

$$0 \xrightarrow{d_1^{-1,1}} E_1^{0,1} \xrightarrow{d_1^{0,1}} E_1^{1,1}, \quad 0 \xrightarrow{d_2^{-2,2}} E_2^{0,1} \xrightarrow{d_2^{0,1}} E_2^{2,0}, \quad 0 \xrightarrow{d_3^{-3,3}} E_3^{0,1} \xrightarrow{d_3^{0,1}} 0.$$

The condition $c \in E_2^{0,1}$ requires that $(d_1^{0,1}c)(X, Y) = 0$, if $Y \in \Gamma$. Note that

$$(d_1^{0,1}c)(E_i^1, T_{-1}^k) = \{E_i^1, c(T_{-1}^k)\} - \{T_{-1}^k, c(E_i^1)\} + (i+2)c(T_i^k) = 0 \quad \text{for } k = 1, 2, 3, 4 \tag{3.4}$$

implies that

$$c(T_i^k) = -it^i\tau^{-i-1}\eta_k \text{ if } k = 1, 2, \quad c(T_i^{k+2}) = -it^i\tau^{-i-1}\xi_k \text{ if } k = 1, 2.$$

Since

$$(d_1^{0,1}c)(T_i^3, T_{-1}^4) = \{T_i^3, c(T_{-1}^4)\} + \{T_{-1}^4, c(T_i^3)\} + (i+1)c(E_i^2) = 0, \tag{3.5}$$

we have

$$c(E_i^2) = -it^{i-1}\tau^{-i-1}\xi_1\xi_2.$$

Since

$$(d_1^{0,1}c)(T_i^1, T_{-1}^2) = \{T_i^1, c(T_{-1}^2)\} + \{T_{-1}^2, c(T_i^1)\} + (i+1)c(F_i^2) = 0,$$

we derive $c(F_i^2) = -it^{i-1}\tau^{-i-1}\eta_1\eta_2$.

Since

$$(d_1^{0,1}c)(E_i^2, T_{-1}^k) = \{E_i^2, c(T_{-1}^k)\} - \{T_{-1}^k, c(E_i^2)\} + ic(D_i^{3-k}) \pm c(T_{i-1}^{5-k}) = 0 \quad \text{if } k = 1, 2,$$

we have

$$c(D_i^{3-k}) = -it^{i-2}\tau^{-i-1}\xi_1\xi_2\eta_k, \quad \text{if } k = 1, 2.$$

Similarly, from $(d_1^{0,1}c)(F_i^2, T_{-1}^{k+2}) = 0$ we derive

$$c(D_i^{5-k}) = -it^{i-2}\tau^{-i-1}\xi_k\eta_1\eta_2, \quad \text{if } k = 1, 2.$$

Next, since

$$(d_1^{0,1}c)(T_i^3, T_{-1}^2) = \{T_i^3, c(T_{-1}^2)\} + \{T_{-1}^2, c(T_i^3)\} + (i+1)c(E_i^3) = 0,$$

it follows that $c(E_i^3) = -it^{i-1}\tau^{-i-1}\xi_1\eta_2$, and the condition $(d_1^{0,1}c)(T_i^4, T_{-1}^1) = 0$ implies $c(F_i^3) = -it^{i-1}\tau^{-i-1}\xi_2\eta_1$. From the condition

$$(d_1^{0,1}c)(T_i^1, T_{-1}^3) = \{T_i^1, c(T_{-1}^3)\} + \{T_{-1}^3, c(T_i^1)\} + (i+1)c(H_i^1) - c(E_{i-1}^1) = 0,$$

it follows that $c(H_i^1) = -it^{i-1}\tau^{-i-1}\xi_1\eta_1$, and from the condition $(d_1^{0,1}c)(T_i^4, T_{-1}^2) = 0$, it follows that $c(H_i^2) = -it^{i-1}\tau^{-i-1}\xi_2\eta_2$. Finally, the condition

$$(d_1^{0,1}c)(D_i^3, T_{-1}^3) = \{D_i^3, c(T_{-1}^3)\} + \{T_{-1}^3, c(D_i^3)\} + (1-i)c(F_i^1) + c(H_{i-1}^2) = 0$$

implies that if $i \neq 1$, then $c(F_i^1) = -it^{i-3}\tau^{-i-1}\xi_1\xi_2\eta_1\eta_2$. Thus $c = c_1$, and since $d_1^{0,1}c = 0$, then $c_1 \in E_2^{0,1}$. Examination of the higher differential $d_2^{0,1}$ shows that $c_1 \in E_3^{0,1} = E_\infty^{0,1}$.

(2) Suppose that c' is a 1-cocycle for the $K'(1|4)$-module $P(2|4)$ whose restriction to Γ coincides with the 1-cocycle θ_2. Let $c = c' + \frac{1}{2}dc_0$, where $c_0 = t^{-2}\tau^{-2}\xi_1\xi_2\eta_1\eta_2$. Then

$$c(E_0^1) = t\tau^{-1} - \tau^{-2}(\xi_1\eta_1 + \xi_2\eta_2), \quad c(E_{-2}^1) = t^{-1}\tau + t^{-2}(\xi_1\eta_1 + \xi_2\eta_2), \quad c(E_{-1}^1) = 1, \tag{3.6}$$

$$\begin{aligned} c(T_{-1}^1) &= -\frac{1}{2}t^{-2}\tau^{-1}\xi_2\eta_1\eta_2, & c(T_{-1}^2) &= \frac{1}{2}t^{-2}\tau^{-1}\xi_1\eta_1\eta_2, \\ c(T_{-1}^3) &= t^{-1}\xi_1 + \frac{1}{2}t^{-2}\tau^{-1}\xi_1\xi_2\eta_2, & c(T_{-1}^4) &= t^{-1}\xi_2 - \frac{1}{2}t^{-2}\tau^{-1}\xi_1\xi_2\eta_1. \end{aligned} \tag{3.7}$$

Let $c(E_i^1)$ be given by (3.2). Then Eqs. (3.3) and (3.6) imply $a_{i,1} = 1$, $a_{i,2} = a_{i,3} = -i-1$ for all i, and $a_{i,4} = ai(i+1)(i+2)$ for all i and an arbitrary $a \in \mathbb{C}$.

Assume that $a = 0$. We will show that then $c = c_2$. Hence $c' = c_2 - \frac{1}{2}dc_0$. Note that if c' is an arbitrary 1-cocycle for the $K'(1|4)$-module $P(2|4)$ such that $c'|_\Gamma = \theta_2$, then $(c' - c_2 + \frac{1}{2}dc_0)|_\Gamma = 0$, and it follows from part (3) of the proof that $c' - c_2 + \frac{1}{2}dc_0$ is a multiple of c_3. Note that $c \in E_1^{0,1}$. The condition $c \in E_2^{0,1}$ requires that

$(d_1^{0,1}c)(X,Y)=0$, if $Y\in\Gamma$. Equations (3.4) and (3.7) imply

$$c(T_i^k)=\pm\left(i+\frac{1}{2}\right)t^{i-1}\tau^{-i-2}\xi_{3-k}\eta_1\eta_2 \qquad \text{if } k=1,2,$$

$$c(T_i^{k+2})=t^i\tau^{-i-1}\xi_k\mp\left(i+\frac{1}{2}\right)t^{i-1}\tau^{-i-2}\xi_1\xi_2\eta_{3-k} \quad \text{if } k=1,2.$$

From Eq. (3.5) we derive

$$c(E_i^2)=t^{i-1}\tau^{-i-1}\xi_1\xi_2.$$

Similarly, the condition $(d_1^{0,1}c)(T_i^1,T_{-1}^2)=0$ implies that $c(F_i^2)=-t^{i-1}\tau^{-i-1}\eta_1\eta_2$. From the condition $(d_1^{0,1}c)(E_i^2,T_{-1}^k)=0$ we have

$$c(D_i^{3-k})=0 \quad \text{if } k=1,2.$$

From the condition $(d_1^{0,1}c)(F_i^2,T_{-1}^{k+2})=0$ we derive

$$c(D_i^{5-k})=-t^{i-2}\tau^{-i-1}\xi_k\eta_1\eta_2 \quad \text{if } k=1,2.$$

Next, $(d_1^{0,1}c)(T_i^3,T_{-1}^2)=0$ implies that $c(E_i^3)=0$, and $(d_1^{0,1}c)(T_i^4,T_{-1}^1)=0$ implies that $c(F_i^3)=0$. Since $(d_1^{0,1}c)(T_i^1,T_{-1}^3)=0$, it follows that $c(H_i^1)=it^{i-2}\tau^{-i-2}\xi_1\xi_2\eta_1\eta_2$, and since $(d_1^{0,1}c)(T_i^4,T_{-1}^2)=0$, it follows that $c(H_i^2)=it^{i-2}\tau^{-i-2}\xi_1\xi_2\eta_1\eta_2$. Finally, $(d_1^{0,1}c)(D_i^3,T_{-1}^3)=0$ implies that if $i\neq 1$, then $c(F_i^1)=-t^{i-3}\tau^{-i-1}\xi_1\xi_2\eta_1\eta_2$. Thus $c=c_2$, and since $d_1^{0,1}c=0$, then $c_1\in E_2^{0,1}$. Examination of the higher differential $d_2^{0,1}$ shows that $c_2\in E_3^{0,1}=E_\infty^{0,1}$.

(3) Suppose that c is a 1-cocycle for the $K'(1|4)$-module $P(2|4)$ such that $c|_\Gamma=0$. Then

$$c(E_{-2}^1)=c(E_{-1}^1)=c(E_0^1)=0,\quad c(T_{-1}^k)=0 \quad \text{for } k=1,\ldots,4. \tag{3.8}$$

Let $c(E_i^1)$ be given by Eq. (3.2). It follows from Eqs. (3.3) and (3.8) that $a_{i,k}=0$ for all i and $k=1,2,3$ and $a_{i,4}=ai(i+1)(i+2)$ for all i and an arbitrary $a\in\mathbb{C}$. Note that $c\in Z_1^{1,0}$, and

$$E_1^{0,0}\xrightarrow{d_1^{0,0}}E_1^{1,0}\xrightarrow{d_1^{1,0}}E_1^{2,0},\quad 0\xrightarrow{d_2^{-1,1}}E_2^{1,0}\xrightarrow{d_2^{1,0}}0.$$

Note also that

$$E_1^{1,0}(K'(1|4),P(2|4))=H^0(\Gamma,\mathrm{Hom}((K'(1|4)/\Gamma),P(2|4))).$$

The condition $c\in E_1^{1,0}$ requires that $dc(X,Y)=0$, if $Y\in\Gamma$. Since $dc(E_i^1,T_{-1}^k)=0$, it follows that

$$c(T_i^k)=\pm ai(i+1)t^{i-1}\tau^{-i-2}\xi_{3-k}\eta_1\eta_2 \qquad \text{if } k=1,2,$$

$$c(T_i^{k+2})=\pm ai(i+1)t^{i-1}\tau^{-i-2}\xi_1\xi_2\eta_{3-k} \quad \text{if } k=1,2.$$

The condition $dc(T_i^3,T_{-1}^4)=0$ implies that $c(E_i^2)=-ait^{i-1}\tau^{-i-1}\xi_1\xi_2$, and the condition $dc(T_i^1,T_{-1}^2)=0$ implies that $c(F_i^2)=-ait^{i-1}\tau^{-i-1}\eta_1\eta_2$. From $dc(E_i^2,T_{-1}^k)=0$

we have

$$c(D_i^{3-k}) = \mp a t^{i-1}\tau^{-i}\xi_{3-k} \quad \text{if } k = 1, 2,$$

and from $dc(F_i^2, T_{-1}^{k+2}) = 0$ we derive

$$c(D_i^{5-k}) = \mp a t^{i-1}\tau^{-i}\eta_{3-k} \quad \text{if } k = 1, 2.$$

Next, $dc(T_i^3, T_{-1}^2) = 0$ implies that $c(E_i^3) = ait^{i-1}\tau^{-i-1}\xi_1\eta_2$, and $dc(T_i^4, T_{-1}^1) = 0$ implies that $c(F_i^3) = ait^{i-1}\tau^{-i-1}\xi_2\eta_1$. From $dc(T_i^1, T_{-1}^3) = 0$ we see that $c(H_i^1) = ait^{i-1}\tau^{-i-1}\xi_2\eta_2$ and from $dc(T_i^4, T_{-1}^2) = 0$ we see that $c(H_i^2) = -ait^{i-1}\tau^{-i-1}\xi_1\eta_1$. Finally, $dc(D_i^3, T_{-1}^3) = 0$ implies that if $i \neq 1$, then $c(F_i^1) = \frac{a}{i-1}t^{i-1}\tau^{-i+1}$. Thus $c = ac_3$, and $c \in E_1^{1,0}$. Examination of the higher differential $d_1^{1,0}$ shows that $ac_3 \in E_2^{1,0} = E_\infty^{1,0}$. □

Corollary 3.2. *Let $c \in C^1(K'(1|4), P(2|4))$. The map $\varphi : c \to c|_\Gamma$ defines a surjective homomorphism $\varphi : H^1(K'(1|4), P(2|4)) \to H^1(\Gamma, P(2|4))$.*

Corollary 3.3. *The space $H^1(K(1|4), P(2|4))$ is spanned by the classes of 1-cocycles $\bar{c}_1, \bar{c}_2$ and $\bar{c}_3$, where*

$$\bar{c}_k|_{K'(1|4)} = c_k, \quad \bar{c}_k(F_1^1) = -t^{-2}\tau^{-2}\xi_1\xi_2\eta_1\eta_2 \ \textit{for}\ k = 1, 2, \quad \textit{and}$$
$$\bar{c}_3|_{K'(1|4)} = 0, \quad \bar{c}_3(F_1^1) = 1.$$

Proof. Note that $\bar{c}_3$ is a 1-cocycle. In fact, since $\{F_1^1, X\} \in K'(1|4)$, it follows that

$$d\bar{c}_3(F_1^1, X) = \{F_1^1, \bar{c}_3(X)\} - \{X, \bar{c}_3(F_1^1)\} - \bar{c}_3(\{F_1^1, X\}) = 0 \quad \text{for any element } X \in K(1|4).$$

Let c be a 1-cocycle for the $K(1|4)$-module $P(2|4)$. Note that

$$c(F_1^1) = b_{1,1} + b_{1,2}t^{-1}\tau^{-1}\xi_1\eta_1 + b_{1,3}t^{-1}\tau^{-1}\xi_2\eta_2 + b_{1,4}t^{-2}\tau^{-2}\xi_1\xi_2\eta_1\eta_2 \tag{3.9}$$

for some $b_{1,i} \in \mathbb{C}$. Assume that $c|_\Gamma = \theta_1$ so that $c|_{K'(1|4)} = c_1$ or $(c - \frac{1}{2}dc_0)|_\Gamma = \theta_2$, where $c_0 = t^{-2}\tau^{-2}\xi_1\xi_2\eta_1\eta_2$, so that $c|_{K'(1|4)} = c_2$. Then from

$$dc(F_1^1, T_i^n) = \{F_1^1, c(T_i^n)\} - \{T_i^n, c(F_1^1)\} - c(\{F_1^1, T_i^n\}) = 0 \quad \text{for } n = 1, \ldots, 4 \tag{3.10}$$

we have $b_{1,2} = b_{1,3} = 0$ and $b_{1,4} = -1$. Hence $c = \bar{c}_k + b_{1,1}\bar{c}_3$ for $k = 1, 2$. Finally, if $c|_\Gamma = 0$, and hence $c|_{K'(1|4)} = ac_3$, where $a \in \mathbb{C}$, then Eqs. (3.9) and (3.10) imply that if $n = 1$, then $b_{1,2} = a$, $b_{1,3} = b_{1,4} = 0$, and if $n = 2$, then $b_{1,3} = a$, $b_{1,2} = b_{1,4} = 0$. Thus, $a = 0$. Hence, $c|_{K'(1|4)} = 0$ and $c = b_{1,1}\bar{c}_3$. □

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