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## EXPERIMENTAL STUDY OF THE VELOCITY FIELD IN SOLITARY WATER WAVES

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We describe experiments that have been conducted to investigate the velocity field in a solitary water wave. The horizontal and vertical velocity components were measured. The experimental results show that the horizontal velocity component is monotonically increasing with the distance to the wave crest line but the vertical velocity is not monotonic.

*Keywords:* Velocity; solitary water wave; experiment.

Mathematics Subject Classification 2000: 22E46, 53C35, 57S20

### 1. Introduction

The steady finite-amplitude solitary waves were first noticed by Scott Russell in 1834. Forty years later Boussinesq [5] and Rayleigh [19] provide an approximation solution of the wave. In 1895, Korteweg and de Vries [17] derived a theory for irrotational solitary shallow water waves of small amplitude. Recently, Alvarez-Samaniego and Lannes [1] discuss the various regimes suitable for the modeling of water waves. In contrast to the case of periodic traveling waves, for solitary waves it is known that the method of linearization is not appropriate even for waves of small amplitude (see [16, 18, 20]). The existence theory for solitary waves (of small and large amplitude) propagating at the surface of water in irrotational flow and with a flat bed was developed in [2, 3], which extended from the earlier works [4, 14].

Concerning theoretical investigations of the flow beneath an irrotational solitary wave, these were undertaken recently in [9–12]. The horizontal velocity under solitary wave is already rigorously demonstrated; however, the structure of the vertical one is lacking. Clamond [7] developed a numerical method to show the structure of the velocity and related field.

In this paper, the solitary waves were generated by a piston-type wavemaker and the wave-board motions can be prescribed by the computer. The particle orbits beneath a solitary wave will be observed using a high-speed camera which can get a good quality of experimental data. The horizontal and vertical spatial distribution calculated from the experiment of particle trajectory and experimental results show that the horizontal velocity distribution is monotonic increase from the still water to the head of solitary wave. For the vertical velocity distribution, it is not the non-monotonicity. The experimental horizontal velocity distribution presented is in close agreement with the theoretical findings [7, 9, 12, 13]. The experimental evidences of vertical velocity distribution can be used to validate the numerical and theoretical theories.

## 2. Experimental Facility

The experiments were carried out in a wave tank at Tainan Hydraulics Laboratory of National Cheng Kung University, Taiwan. This tank was 21 m long and 0.7 m deep, with a width of 0.5 m, and its glass sidewalls facilitate the recording with a camera and permit the visual observation of the evolution of a wave. The target solitary waves were generated at one end of the flume by a programmable high-resolution wavemaker. A plane beach with slope 1/20 covered with a smooth layer of concrete starts 11 m from the wave paddle. A schematic diagram of the experimental definition is shown in Fig. 1.

The arrangement of the measurement apparatus was deployed with wave gauges and high-speed camera. Figure 2 shows a photograph of the laboratory flume and of the measurement facilities employed in the reported experiments. The elevation of the local water surface was recorded by employing three capacitance-type wave gauges located between 3.56 and 7.63 m downstream of the wavemaker. It is noted that the wave probe located at  $x = 3.56$  m is referred to the reference gauge in all experiments. All wave gauges were calibrated through a standard method which concerns the change of water level to adjust the response voltage of each gauge before and after the experiments to ensure its linearity and stability. The linearity of gauge response was given by a correlation coefficient of 0.9997–0.99991.

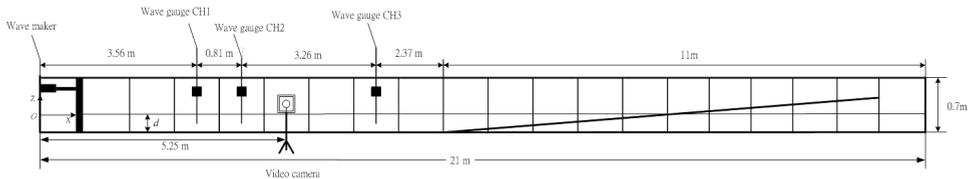


Fig. 1. Sketch of wave flume layouts and the definitions of the physical variables, with the exact positions of the wave gauges.

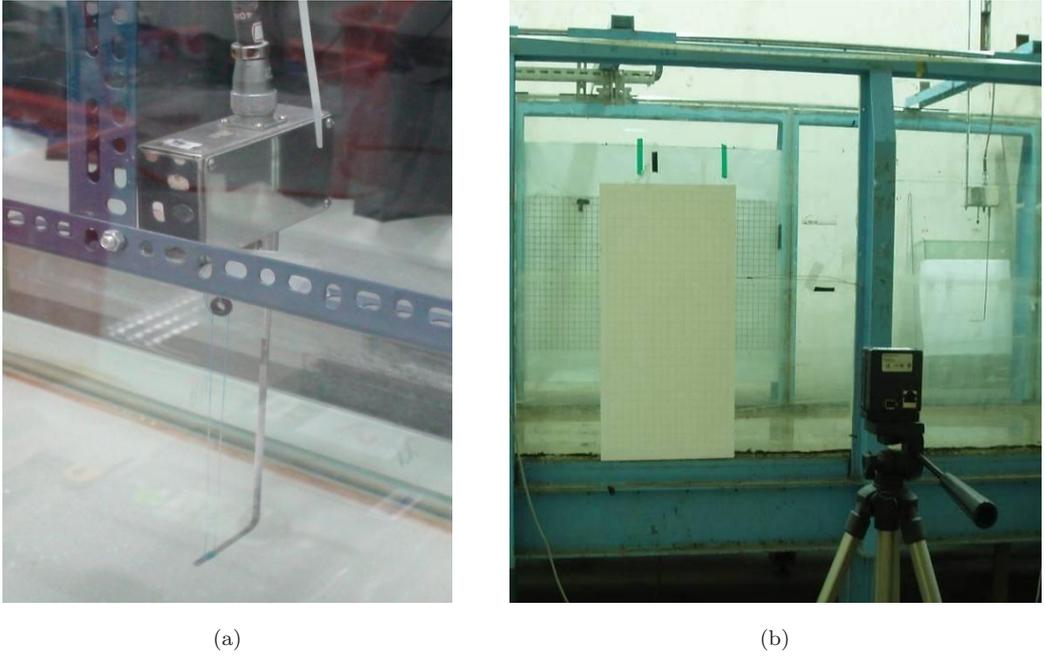


Fig. 2. Prospective view of the wave tank and the measurement apparatus in the present experiment: (a) wave gauge; (b) camera.

### 3. Solitary Wave Generation

To push or withdraw a certain volume of water within certain duration is the basic idea to generate a solitary wave. In this study, the waves were generated by a hydraulically driven, dry back, piston-type wavemaker with a maximum stroke of 1.0 m. A programmable controller that can be accessed easily by a PC controlled the motion of the wave-board at 25 Hz through a 16 bits AD/DA card. Different wave-board motions can be prescribed by the computer. Target solitary waves were generated at one end of the flume with a programmable, hydraulically driven, dry back, piston-type wavemaker. Among the two existing nonlinear algorithms developed respectively by Goring [15] and Synolakis [21], the former was employed here to generate solitary waves. The measured solitary wave profiles also agreed with the first-order theory of Boussinesq [5] are shown in Fig. 3, suggesting that the experimental results are highly repeatable and reliable. Note that Boussinesq theory is not applicable as we do not deal with waves of small amplitude. However, the wave profile agrees to a great extent with the theoretical predictions in [2]. The abscissa is a non-dimensional time variable  $t \cdot \sqrt{g/d}$  ( $d$ : water depth), as adopted by Synolakis [21]. The ordinate is the wave profile, normalized with the offshore constant water depth,  $\eta/H$  ( $H$ : wave height). It is noted that the origin of normalized time is shifted to match the time when wave crest passes the reference gauge. Clearly, small variations at the tail for higher wave nonlinearity are found and slight asymmetry of wave shape with respect to wave crest is observed. This phenomenon is due to the algorithm restriction of Goring [15]. Comparisons show that this facility validates the control of solitary wave generation.

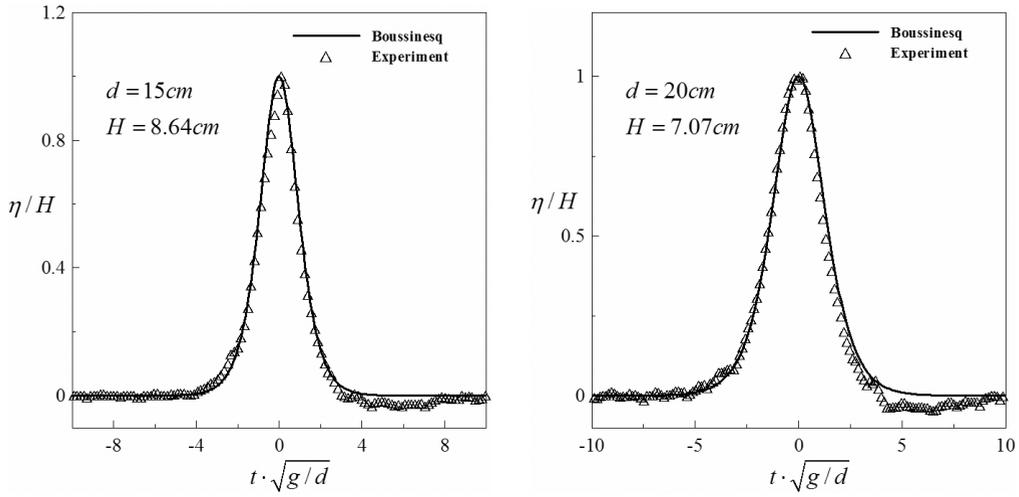


Fig. 3. Comparison of the time history of the water elevation displacement for repeated experiments measured at the wave gauge. The dashed line denotes the theoretical profile, while the solid line is the experimental data. The free surface displacements of several trials at the wave gauge and the theoretical predictions are in good agreement.

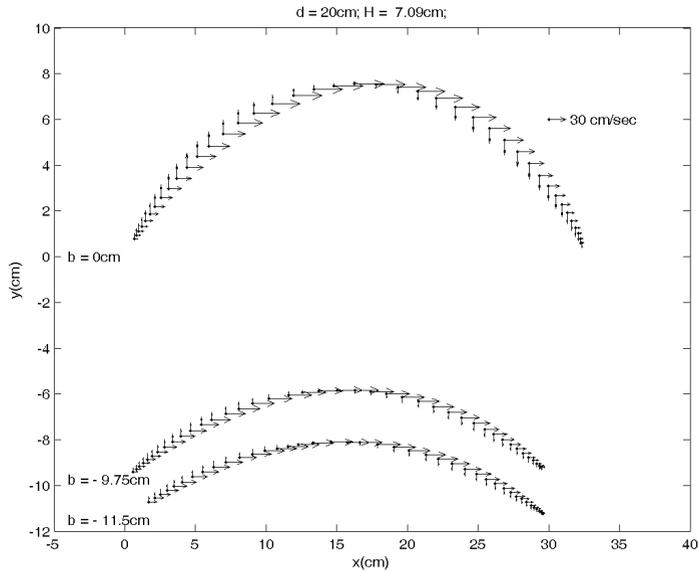
#### 4. Experimental Procedure

Solitary waves of which relative incident wave heights,  $H/d$ , four tests were generated, where  $H$  is the wave height and  $d$  is the constant water depth. The test conditions are listed in Table 1. Time series of local water surface elevation were recorded by capacitance-type wave gauges distributed at three stations along the tank. Each sensor of the gauges was mounted at 0.25 m away from the sidewall. The resolution of the gauges is 12 bits on a dynamic range of 0.6 m. As these gauges were entirely out of water before the wave passing by, their outputs were carefully examined to avoid wrong signals. In this laboratory, signal synchronization from numerous parallel inputs and signal decay due to the long-distance transmission were two important data-acquisition problems. To cope with these challenges, measuring data and wave paddle motion were all recorded simultaneously with 50 Hz sampling for 60 s using a Multi-Nodes-Data-Acquisition-System (MNDAS), developed by THL. The still water depth was also measured after each test to maintain the same initial condition. The repeatability of all experiments was satisfactory with a maximum derivation in reference wave height of approximately 3% between repetitions. The experimental data were synchronized before analyses.

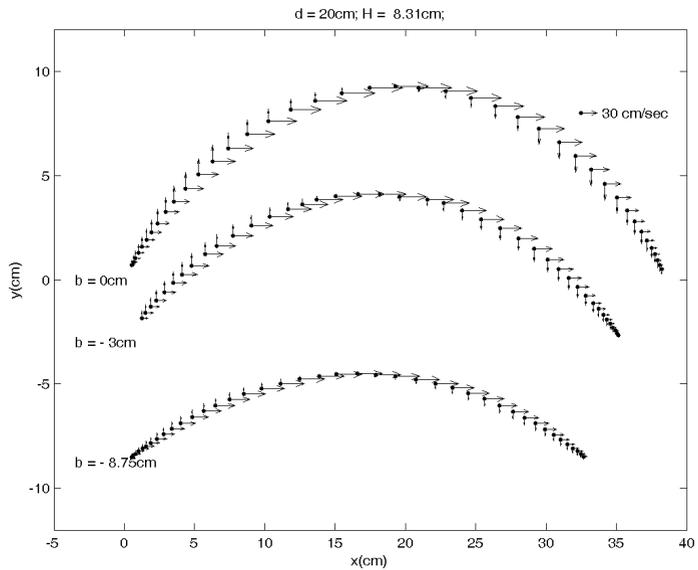
Table 1. The test conditions.

Case	Water depth $d$ (cm)	Water height $H$ (cm)
1	20	7.09
2	20	8.31
3	30	5.16
4	30	7.85

The water particles were simulated with fluorescent spherical polystyrene beads (PS) with a diameter of about 0.1 cm. The density of primitive PS is about  $1.05 \text{ g/cm}^3$  heavier than the water. After boiling with water, the PS density is approximately equal to water density  $1.000 \text{ g/cm}^3$  and will remain neutrally stable in a fixed position in water. Images were captured by a high-speed camera (A301fc-type, Basler Company), which can take 80 frames per second. The camera was controlled by BCamGraber program and linked to



(a)



(b)

Fig. 4. The orbits of water particles obtained from the experimental measurements of the PS motions at different water levels  $b$  in the four experimental wave cases.

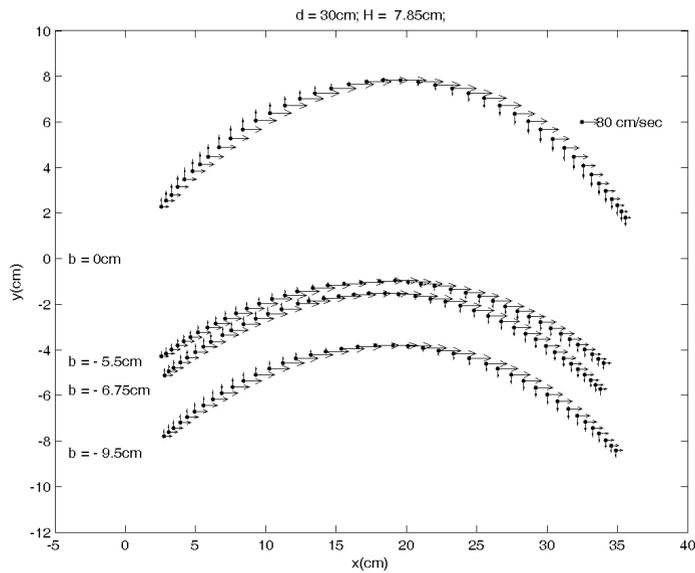
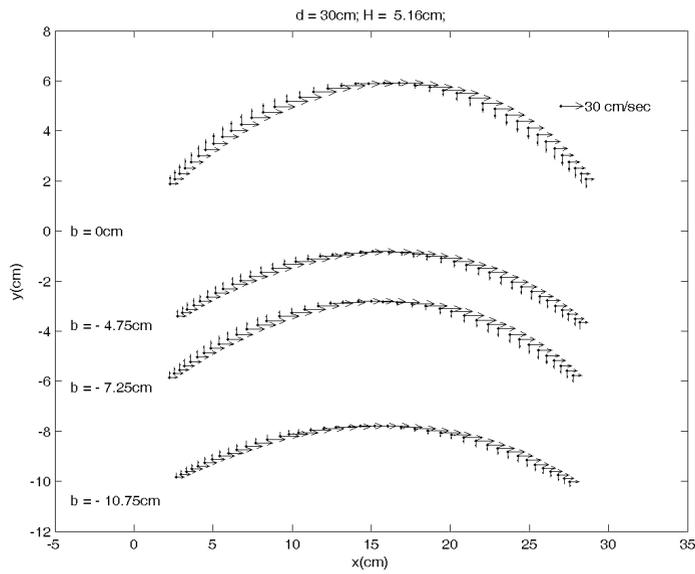
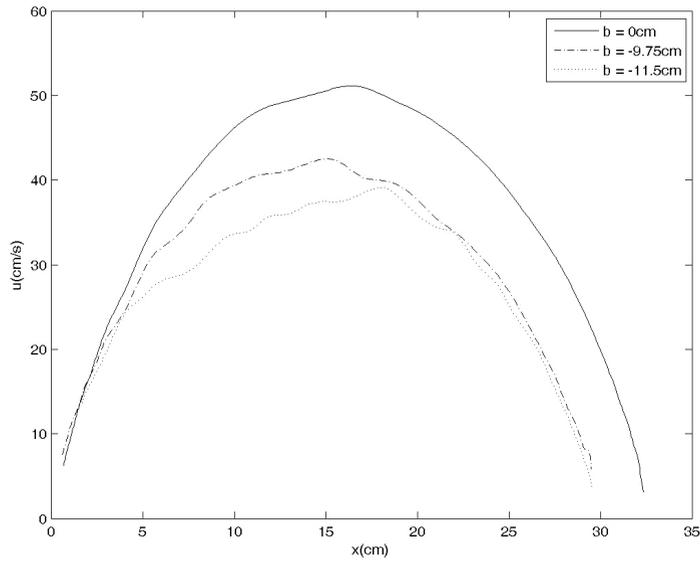


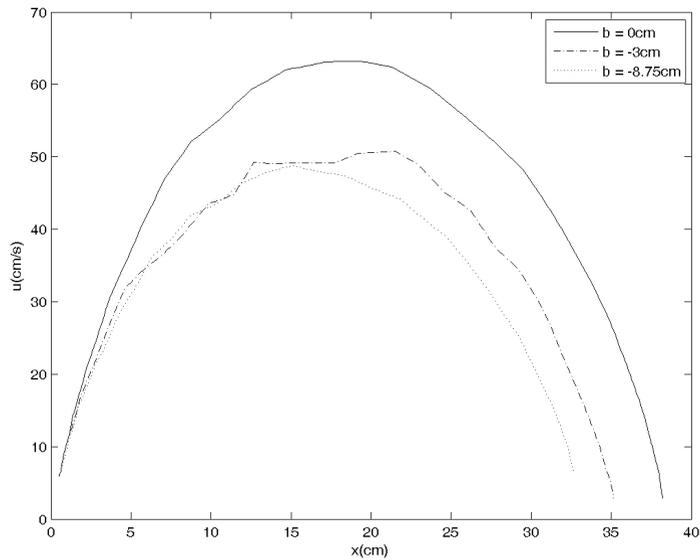
Fig. 4. (Continued)

a 1394 CARD to collect and analyze the data. Four powerful lamps (110 V and 500 W) were set up to reinforce the brightness of the images of PS motion in the water waves for easier identification. A transparent acrylic-plastic sheet (1 m  $\times$  45 cm  $\times$  2 mm) plotted with 2 cm  $\times$  2 cm square grids was placed in the still water centered along the width of the tank. It was first photographed before being removed from the tank. The network grids in the photograph were programmed into the computer and used to analyze the continuous images

of particle trajectories captured by the high-speed camera. A copper-pole (150 cm long with a diameter of 0.5 cm), calibrated at 0.1 cm intervals and perforated below 70 cm with 20 holes having a diameter of 0.3 cm, was erected vertically in front of the viewing glass in the still water tank. The PS was pushed out horizontally from the holes of the copper-pole at different water levels into the still water. Then, the copper-pole was slowly removed from the tank before the waves were generated to avoid interfering with the incident waves and PS motion.

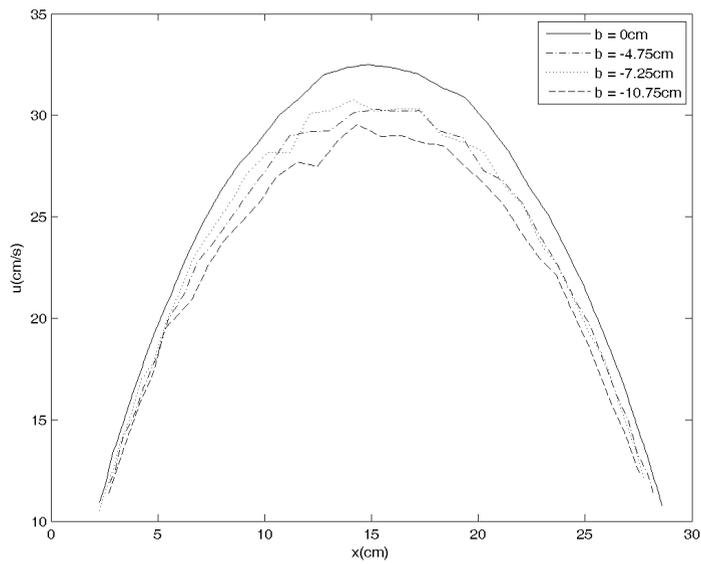


(a)

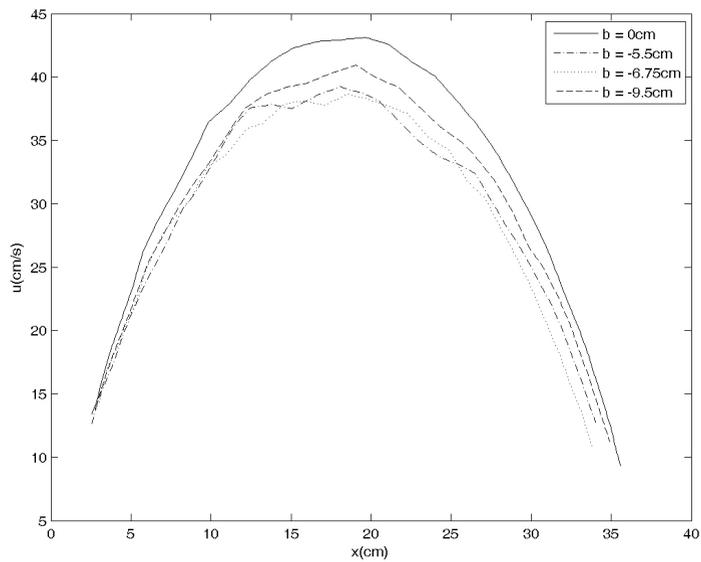


(b)

Fig. 5. The horizontal distribution beneath solitary waves.



(c)



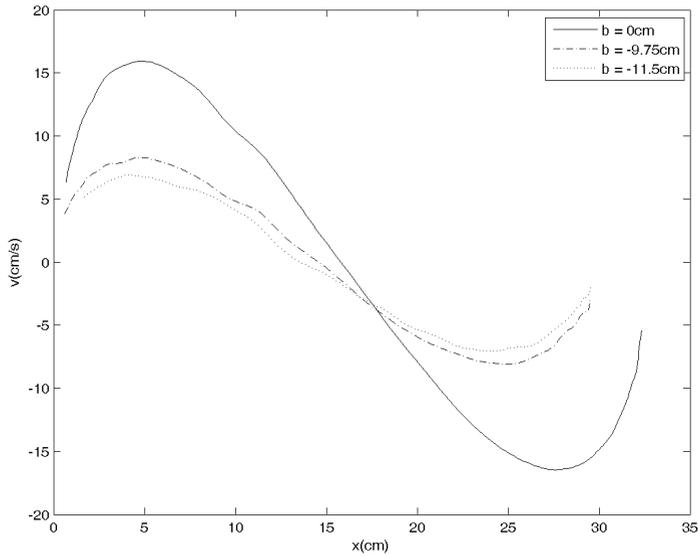
(d)

Fig. 5. (Continued)

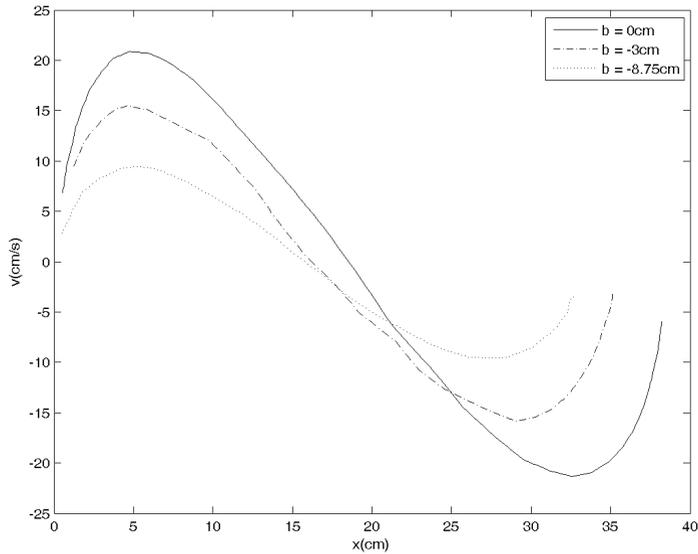
## 5. Discussion and Conclusions

The particle motions and velocity vector beneath the solitary waves are shown in Figs. 4–6, respectively. In the case of periodic irrotational traveling waves it is known that the particles perform a forward–backward motion, with no closed particle paths in the absence of an underlying current; cf. [8, 13] for theoretical considerations and [6] for experimental data. It is obvious that for solitary waves the particle orbit does not comprise any backward motion because of the existence of a displacement that persists with it along the wave direction;

this phenomenon is shown in Figs. 4(a) and 4(d). The velocity of any water particle can be obtained from the experimental particle trajectories using  $\vec{V} = \vec{i}u + \vec{j}v = \vec{i}x_t + \vec{j}y_t$ . The velocity vector that would occur in a succession of widely separated solitary waves for surface and subsurface particles is shown in Fig. 4. Figures 5 and 6 illustrate the horizontal and vertical velocity components of particles at free surface ( $b = 0$ ) and subsurface. It can be seen that the horizontal velocity  $u$  increases monotonically from still water to the head of solitary wave and vice versa along any horizontal line. The horizontal velocity component

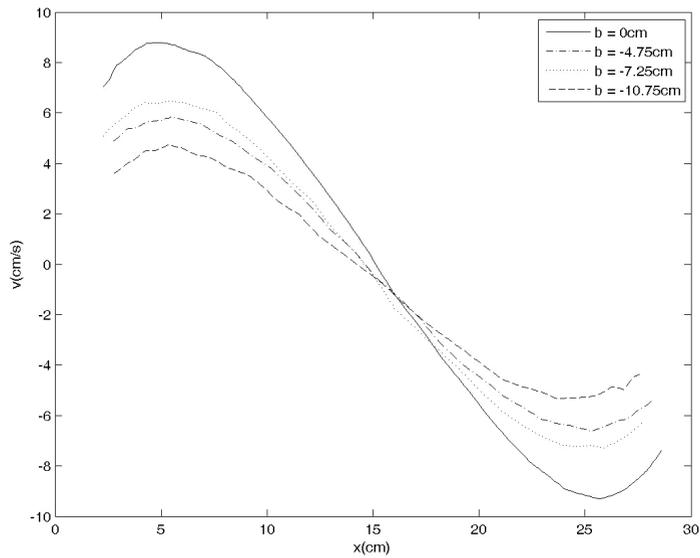


(a)

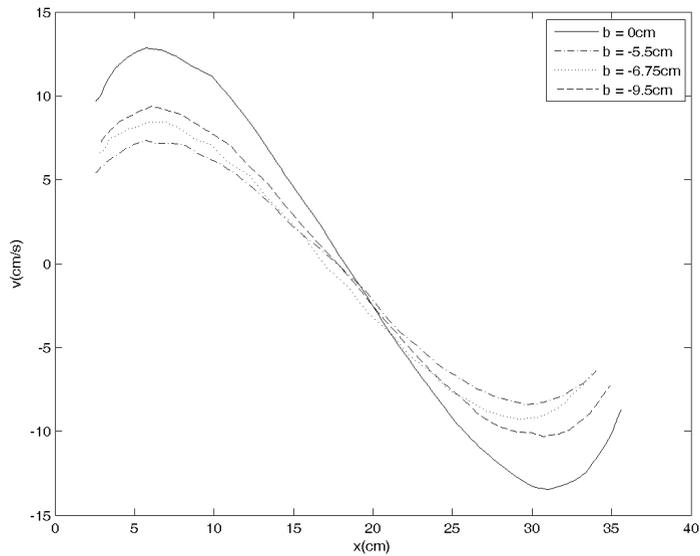


(b)

Fig. 6. The vertical distribution beneath solitary waves.



(c)



(d)

Fig. 6. (Continued)

is positive at any time and reaches maximum at the crest. The experimental results are in close agreement with the theoretical findings [9, 11, 12]. The vertical velocity  $v$  for surface and subsurface particles in Fig. 6 shows that  $v$  is non-monotonicity and has one maximum and one minimum during one complete solitary wave cycle along any horizontal line. The vertical velocity component is positive from the still water to the crest and is negative behind the crest. The horizontal and vertical velocity components in a solitary wave are largest at the free surface, and decrease at deeper levels. The experimental results shown in Fig. 6 remain to be demonstrated mathematically.

## Acknowledgment

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