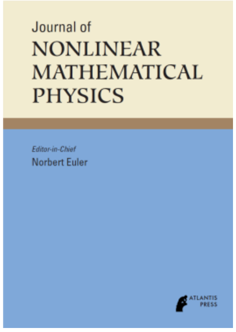

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EXACT SOLUTIONS OF THE MODIFIED GROSS–PITAEVSKII EQUATION IN "SMART" PERIODIC POTENTIALS IN THE PRESENCE OF EXTERNAL SOURCE

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We report wide class of exact solutions of the modified Gross–Pitaevskii equation (GPE) in "smart" Jacobian elliptic potentials, in the presence of external source. Solitonlike solutions, singular solutions, and periodic solutions are found using a recently developed fractional transform in which all the amplitude parameters are *nonzero*. These results generalize those contained in (T. Paul, K. Richter and P. Schlagheck, *Phys. Rev. Lett.* **94** (2005) 020404.) for nonzero trapping potential.

Keywords: Modified Gross–Pitaevskii equation; Jacobian elliptic potentials; external source; fractional transform; solitonlike solutions.

1. Introduction

In a mean-field approximation, the dynamics of a dilute-gas Bose–Einstein condensate (BEC) can be captured by the cubic nonlinear Schrödinger equation (NLSE) with a trapping potential [1–3]. The various traps which are used to contain the BEC have spurred the solutions of the NLSE with new potentials [4, 5]. We consider the mean-field model of a quasi-one-dimensional BEC trapped in a "smart" potential in the presence of an external source [6]

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(\xi) + g|\psi|^2\right]\psi + K\exp(i\chi(\xi) - i\omega t), \tag{1.1}$$

where $\psi(x,t)$ represents the macroscopic wave function of the condensate and $V(\xi)$ is an experimentally generated macroscopic potential. The parameter g indicates the strength of atom-atom interactions and it alone decides whether Eq. (1.1) is attractive ($g = -1$, focussing nonlinearity) or repulsive ($g = 1$, defocusing nonlinearity). Here, K and ω are real constants related to the source amplitude and the chemical potential, respectively, and $\chi(\xi)$ is a real function of $\xi = \alpha(x - vt)$, α and v being two real parameters. In the field of nonlinear optics, Eq. (1.1) may describe the evolution of the local amplitude of an electromagnetic wave in the spatial domain, in a two-dimensional waveguide where t becomes the propagation distance and x becomes the retarded time, and the system is driven by an external plane pump wave. The Jacobian elliptic potential may describe a transverse modulation of the refractive index in the waveguide.

As is well known, Eq. (1.1) is not integrable if $KV_0 \neq 0$, where V_0 indicates the height of the potential barrier. And only small classes of explicit solutions can most likely exist for this case. For $V(\xi) = 0$, Eq. (1.1) is a cubic NLSE with a source, and exact rational solutions using a fractional transform are found in [7]. And in [11] periodic solutions of Eq. (1.1), without source have been reported. More recently, in [8], a class of exact solutions of Eq. (1.1) for $V(\xi) = -V_0\,\mathrm{sn}^2(\xi, m)$ have been reported. In particular, the rational solutions of the fractional transform: $\rho(\xi) = \frac{A+Bf^2}{1+Df^2}$, where $f = \mathrm{sn}$ have been reported for $B = 0$. This is due to the form of the trapping potential. Nonetheless, in the present paper we find rational solutions of the type $\rho(\xi) = \frac{A+Bf^2}{1+Df}$, where f being the respective Jacobian elliptic function with all the amplitude parameters A, B, and D *nonzero*. These results generalize those contained in [6] for nonzero trapping potential. However, there is an important difference between the source term considered in [6] and ours. The former one is spatially localized indicating injection of atoms into the waveguide at one specific point in space, while the latter one is a spatially homogeneous source. The choice of a smart potential $V(\xi)$ allows one to construct a large class of exact solutions, as done in a number of works for the cubic GP equations [9–11]. In the present work, we consider three different potentials in the GP equation: $V(\xi) = -V_0\,\mathrm{sn}(\xi, \mathrm{m})$, $V(\xi) = -V_0\,\mathrm{cn}(\xi, \mathrm{m})$, and $V(\xi) = -V_0\,\mathrm{dn}(\xi, \mathrm{m})$ in the presence of external source, and find exact travelling wave solutions of Eq. (1.1) with $K \neq 0$. The choice of these three different "smart" potentials is motivated by the following facts. Firstly, the potential $V(\xi) = -V_0\,\mathrm{sn}(\xi, \mathrm{m})$ in the limit $m \to 0$ is $V(\xi) = -V_0 \sin(\xi)$ which is similar to the standard optical lattice potential [12, 13]. Secondly, the choice of the potential $V(\xi) = -V_0\,\mathrm{dn}(\xi, \mathrm{m})$ in GP equation mimics [14] the harmonic potential that was used to achieve BEC experimentally. The third potential, we hope it is relevant to the available experimental conditions to achieve BEC.

2. Exact Solutions of the GPE in "Smart" Periodic Potential with Source

The traveling wave solutions of Eq. (1.1) with potential $V(\xi)$ are taken to be of the form $\psi(x,t) = \rho(\xi)e^{i\chi[\alpha(x-vt)]-i\omega t}$. Inserting this expression for $\psi(x,t)$ in Eq. (1.1) and separating the real and imaginary parts, and integrating the imaginary part, one gets

$$\chi' = \frac{v}{\alpha} + \frac{C}{\alpha\rho^2}, \tag{2.1}$$

where C is a constant of integration. In order that the external phase be independent of ψ, we consider only solutions with $C = 0$, to obtain

$$\rho'' + \left(\frac{v^2 + 2\omega}{\alpha^2}\right)\rho - \frac{2g}{\alpha^2}\rho^3 - \frac{2V(\xi)}{\alpha^2}\rho - \frac{2K}{\alpha^2} = 0. \tag{2.2}$$

Below we consider three different "smart" Jacobian elliptic potentials [15] and find exact solutions.

Case (I):- $V(\xi) = -V_0\,\mathrm{sn}(\xi, m)$. Equation (2.2) reads as

$$\rho'' + \left(\frac{v^2 + 2\omega}{\alpha^2}\right)\rho - \frac{2g}{\alpha^2}\rho^3 + \frac{2V_0}{\alpha^2}\,\mathrm{sn}\rho - \frac{2K}{\alpha^2} = 0. \tag{2.3}$$

Substituting

$$\rho(\xi) = A_1 + B_1\mathrm{sn}(\xi, m) \tag{2.4}$$

in Eq. (2.3) and equating the coefficients of equal powers of $\mathrm{sn}(\xi, m)$ result in relations among the solution parameters A_1, and B_1, and the equation parameters V_0, g, K, α, and ω. We find that

$$\alpha^2 = \frac{2\omega + v^2}{1 + m}, \tag{2.5}$$

$$A_1 = \frac{V_0}{3\alpha\sqrt{gm}}, \quad B_1 = \sqrt{\frac{m\alpha^2}{g}}. \tag{2.6}$$

From Eq. (2.6) it follows that $V_0 > 0$ and $g > 0$ implying the GPE with repulsive nonlinearity.

Case (II):- $V(\xi) = -V_0\,\mathrm{cn}(\xi, m)$. Equation (2.2) reads as

$$\rho'' + \left(\frac{v^2 + 2\omega}{\alpha^2}\right)\rho - \frac{2g}{\alpha^2}\rho^3 + \frac{2V_0}{\alpha^2}\,\mathrm{cn}\rho - \frac{2K}{\alpha^2} = 0. \tag{2.7}$$

Substituting

$$\rho(\xi) = A_2 + B_2\mathrm{cn}(\xi, m) \tag{2.8}$$

in Eq. (2.7) and equating the coefficients of equal powers of $\mathrm{scn}(\xi, m)$ result in relations among the solution parameters A_2, and B_2, and the equation parameters V_0, g, K, α, and ω. We find that

$$\alpha^2 = \frac{2\omega + v^2}{1 - 2m}, \tag{2.9}$$

$$A_2 = \frac{V_0}{3\alpha\sqrt{-gm}}, \quad B_1 = \sqrt{\frac{-m\alpha^2}{g}}. \tag{2.10}$$

From the positivity of α^2 we conclude that cn solutions exist for $V_0 > 0$ and $g < 0$. The condition $g < 0$ corresponds to the GPE with attractive nonlinearity.

Case (III):- $V(\xi) = -V_0\,\mathrm{dn}(\xi, m)$. Equation (2.2) reads as

$$\rho'' + \left(\frac{v^2 + 2\omega}{\alpha^2}\right)\rho - \frac{2g}{\alpha^2}\rho^3 + \frac{2V_0}{\alpha^2}\,\mathrm{dn}\rho - \frac{2K}{\alpha^2} = 0. \tag{2.11}$$

Substituting

$$\rho(\xi) = A_3 + B_3\,\mathrm{dn}(\xi, m) \tag{2.12}$$

in Eq. (2.11) and equating the coefficients of equal powers of $\mathrm{dn}(\xi, m)$ result in relations among the solution parameters A_3, and B_3, and the equation parameters V_0, g, K, α, and ω. We find that

$$\alpha^2 = \frac{2\omega + v^2}{m-2}, \tag{2.13}$$

$$A_3 = \frac{V_0}{3\alpha\sqrt{-g}}, \quad B_3 = \sqrt{\frac{-\alpha^2}{g}}. \tag{2.14}$$

Here we conclude that dn solutions exist only for $V_0 > 0$ and $g < 0$.

3. Rational Solutions

In order to obtain Lorentzian-type of solutions of Eq. (2.2) we use a fractional transform

$$\rho(\xi) = \frac{A + Bf^2}{1 + Df} \tag{3.1}$$

where f is the respective Jacobian elliptic functions. Again we obtain the Lorentzian-type of solutions of Eq. (2.2) for three different "smart" potentials.

Case (I): $-V(\xi) = -V_0\,\mathrm{sn}(\xi, m)$. Equation (2.2) reads as

$$\rho'' + \left(\frac{v^2 + 2\omega}{\alpha^2}\right)\rho - \frac{2g}{\alpha^2}\rho^3 + \frac{2V_0}{\alpha^2}\,\mathrm{sn}\rho - \frac{2K}{\alpha^2} = 0. \tag{3.2}$$

Substituting

$$\rho(\xi) = \frac{A_4 + B_4\,\mathrm{sn}^2(\xi, m)}{1 + D_1\,\mathrm{sn}(\xi, m)} \tag{3.3}$$

in Eq. (3.2) and equating the coefficients of equal powers of $\mathrm{sn}(\xi, m)$ will yield the following consistency conditions.

$$2B_4 + 2A_4D_1^2 + \Gamma A_4 - \frac{2g}{\alpha^2}A_4^3 - \frac{2K}{\alpha^2} = 0, \tag{3.4}$$

$$2mB_4D_1^2 - \frac{2g}{\alpha^2}B_4^3 = 0, \tag{3.5}$$

$$6mB_4D_1 + \frac{2V_0}{\alpha^2}B_4D_1^2 = 0, \tag{3.6}$$

$$6mB_4 + B_4D_1^2(\Gamma - m - 1) - \frac{6g}{\alpha^2}A_4B_4^2 + \frac{4V_0}{\alpha^2}B_4D_1 = 0, \tag{3.7}$$

$$-2mA_4D_1 + B_4D_1(2\Gamma - 3m - 3) + \frac{2V_0}{\alpha^2}A_4D_1^2\frac{2V_0}{\alpha^2}B - \frac{2K}{\alpha^2}D_1^3 = 0, \tag{3.8}$$

$$-4B_4(1+m) + A_4D_1^2(\Gamma - m - 1) + \Gamma B_4 - \frac{6g}{\alpha^2}A_4^2B_4 + \frac{4V_0}{\alpha^2}A_4D_1 - \frac{6K}{\alpha^2}D_1^2 = 0, \tag{3.9}$$

$$A_4D_1(2\Gamma + m + 1) + \frac{2V_0}{\alpha^2}A_4 - \frac{6K}{\alpha^2}D_1 = 0. \tag{3.10}$$

From the above consistency conditions we obtain the following relations.

$$A_4 = \frac{18mK}{3m(m+1)\alpha^2 + 6m\Gamma\alpha^2 - \frac{2V_0^2}{\alpha^2}}, \tag{3.11}$$

$$B_4 = -\frac{3\alpha^3 m^{3/2}}{V_0 g^{1/2}}, \quad D_1 = \frac{-3m\alpha^2}{V_0}, \tag{3.12}$$

where $\Gamma = \frac{v^2+2\omega}{\alpha^2}$. Here, we would like to emphasize that these results generalize those contained in [6], for nonzero trapping potential. This stems from the fact that the constant B_4 in expression (3.12) is nonzero, which follows from the choice of our "smart" potential in Eq. (1.1).

Case (II): $-V(\xi) = -V_0\,\mathrm{cn}(\xi, m)$. Equation (2.2) reads as

$$\rho'' + \left(\frac{v^2+2\omega}{\alpha^2}\right)\rho - \frac{2g}{\alpha^2}\rho^3 + \frac{2V_0}{\alpha^2}\,\mathrm{cn}\rho - \frac{2K}{\alpha^2} = 0. \tag{3.13}$$

Substituting

$$\rho(\xi) = \frac{A_5 + B_5\,\mathrm{cn}^2(\xi, m)}{1 + D_2\,\mathrm{cn}(\xi, m)} \tag{3.14}$$

in Eq. (3.13) and equating the coefficients of equal powers of $\mathrm{cn}(\xi, m)$ will yield the following consistency conditions.

$$2B_5(1-m) + 2A_5D_2^2(1-m) + \Gamma A_5 - \frac{2g}{\alpha^2}A_5^3 - \frac{2K}{\alpha^2} = 0, \tag{3.15}$$

$$-2mB_{45}D_2^2 - \frac{2g}{\alpha^2}B_5^3 = 0, \tag{3.16}$$

$$-6mB_5D_2 + \frac{2V_0}{\alpha^2}B_5D_2^2 = 0, \tag{3.17}$$

$$-6mB_5 + B_5D_2^2(\Gamma + 2m - 1) - \frac{6g}{\alpha^2}A_5B_5^2 + \frac{4V_0}{\alpha^2}B_5D_2 = 0, \tag{3.18}$$

$$2mA_5D_2 + B_5D_2(2\Gamma + 6m - 3) + \frac{2V_0}{\alpha^2}A_5D_2^2 + \frac{2V_0}{\alpha^2}B_5 - \frac{2K}{\alpha^2}D_2^3 = 0, \tag{3.19}$$

$$-4B_5(1-2m) + A_5D_2^2(\Gamma + 2m - 1) + \Gamma B_5 - \frac{6g}{\alpha^2}A_5^2B_5 + \frac{4V_0}{\alpha^2}A_5D_2 - \frac{6K}{\alpha^2}D_2^2 = 0, \tag{3.20}$$

$$A_5D_2(2\Gamma - 2m + 1) + \frac{2V_0}{\alpha^2}A_5 - \frac{6K}{\alpha^2}D_2 = 0. \tag{3.21}$$

From the above consistency conditions we obtain the following relations

$$A_5 = \frac{18mK}{3m(1-2m)\alpha^2 + 6m\Gamma\alpha^2 + \frac{2V_0^2}{\alpha^2}}, \tag{3.22}$$

$$B_5 = \frac{3\alpha^3 m}{V_0}\sqrt{-\frac{m}{g}}, \quad D_2 = \frac{3m\alpha^2}{V_0}. \tag{3.23}$$

Case (III):- $V(\xi) = -V_0\,\mathrm{dn}(\xi, m)$. Equation (2.2) reads as

$$\rho'' + \left(\frac{v^2 + 2\omega}{\alpha^2}\right)\rho - \frac{2g}{\alpha^2}\rho^3 + \frac{2V_0}{\alpha^2}\,\mathrm{dn}\rho - \frac{2K}{\alpha^2} = 0. \tag{3.24}$$

Substituting

$$\rho(\xi) = \frac{A_6 + B_6\,\mathrm{dn}^2(\xi, m)}{1 + D_3\,\mathrm{dn}(\xi, m)} \tag{3.25}$$

in Eq. (3.24) and equating the coefficients of equal powers of $\mathrm{dn}(\xi, m)$ will yield the following consistency conditions.

$$2A_6 D_3^2(m-1) + \Gamma A_6 - \frac{2g}{\alpha^2}A_6^3 - \frac{2K}{\alpha^2} = 0, \tag{3.26}$$

$$-2B_6 D_3^2 - \frac{2g}{\alpha^2}B_6^3 = 0, \tag{3.27}$$

$$-4B_6 D_3 + \frac{2V_0}{\alpha^2}B_6 D_3^2 = 0, \tag{3.28}$$

$$-4B_6 - 2B_6 D_3 + B_6 D_3^2(\Gamma - m + 2) - \frac{6g}{\alpha^2}A_6 B_6^2 + \frac{4V_0}{\alpha^2}B_6 D_3 = 0, \tag{3.29}$$

$$2A_6 D_3 + B_6 D_3(2\Gamma - 3m + 4) + \frac{2V_0}{\alpha^2}A_6 D_3^2 + \frac{2V_0}{\alpha^2}B_6 - \frac{2K}{\alpha^2}D_3^3 = 0, \tag{3.30}$$

$$\begin{aligned} -2B_6(m-2) + 2B_6 D_3 + A_6 D_3^2(\Gamma - m + 2) + \Gamma B_6 - \frac{6g}{\alpha^2}A_6^2 B_6 \\ + \frac{4V_0}{\alpha^2}A_6 D_3 - \frac{6K}{\alpha^2}D_3^2 = 0, \end{aligned} \tag{3.31}$$

$$A_6 D_3(2\Gamma + m - 2) + \frac{2V_0}{\alpha^2}A_6 - \frac{6K}{\alpha^2}D_3 = 0. \tag{3.32}$$

From the above consistency conditions we obtain the following relations

$$A_6 = \frac{6K}{\alpha^2(2\Gamma + m - 2) + \frac{2V_0^2}{\alpha^2}}, \tag{3.33}$$

$$B_6 = \frac{2\alpha^2}{V_0}\sqrt{-\frac{\alpha^2}{g}}, \quad D_2 = \frac{2\alpha^2}{V_0}. \tag{3.34}$$

3.1. *Trigonometric solutions*

From the consistency conditions that arise from the first two "smart" periodic potentials, we conclude that the limit $m = 0$ is forbidden as the amplitude parameters A, B, and D will be zero. On the other hand, for $V(\xi) = -V_0\,\mathrm{dn}(\xi, m)$ case, only flat background solutions will be possible for $m = 0$ limit.

3.2. *Solitonlike solutions*

Here, in this subsection, we describe the solitonlike solutions that are obtained from the solutions in $\mathrm{sn}(\xi, m)$ and $\mathrm{cn}(\xi, m)$ in the limit $m = 1$, in detail. In the limit $m = 1$, $V(\xi)$ becomes an array of well separated kink-type of potential barriers: $V(\xi) = -V_0 \tanh(\xi)$. Then we have the following relations

$$A_4 = \frac{18K}{6(\Gamma+1)\alpha^2 - \frac{2V_0^2}{\alpha^2}}, \tag{3.35}$$

$$B_4 = \frac{3\alpha^3}{V_0 g^{1/2}}, \quad D_1 = \frac{-3\alpha^2}{V_0} \tag{3.36}$$

and the strength of the source is

$$K = \frac{V_0[6(\Gamma+1)\alpha^4 - 2V_0^2]}{108\alpha^3 g^{1/2}}\left(\frac{63(\Gamma-2)\alpha^4 - 2V_0^2}{V_0^2}\right).$$

As a special case, if we set $\alpha = 1$ and $V_0 = 1$, then we get $A_4 = \frac{9K}{3\Gamma+2}$, $B_4 = 3/\sqrt{g}$, and $D_1 = -3$. This results in a solitonlike solution

$$\rho(\xi) = \frac{A_4 + B_4 \tanh^2(\xi)}{1 - 3\tanh(\xi)}. \tag{3.37}$$

This set corresponds to the singular solution for repulsive case i.e., $g > 0$. The singularity of the pulse profile may correspond to the beam power exceeding material breakdown due to self-focusing [16–19]. Figure 1 depicts a surface plot of this solution for the parameter values given in the figure caption.

Another interesting solitonlike solution is obtained from the solution in $\mathrm{cn}(\xi, m)$ for $m = 1$. In the limit $m = 1, V(\xi)$ becomes an array of well separated secant hyperbolic potential barriers: $V(\xi) = -V_0\,\mathrm{sech}(\xi)$. Then we have the following relations

$$A_5 = \frac{18K}{3(2\Gamma-1)\alpha^2 + \frac{2V_0^2}{\alpha^2}}, \tag{3.38}$$

$$B_5 = \frac{3\alpha^3}{V_0}\sqrt{-1/g}, \quad D_2 = \frac{3\alpha^2}{V_0}. \tag{3.39}$$

As a special case, if we set $\alpha = 1, V_0 = -6, g = -1$ and $K = 1/2$ then we get $A_5 = \frac{9}{6\Gamma+69}$, $B_5 = -(1/2)$, and $D_1 = -(1/2)$. This results in a solitonlike solution

$$\rho(\xi) = \frac{A_5 + B_5\,\mathrm{sech}^2(\xi)}{1 - 0.5\,\mathrm{sech}(\xi)}. \tag{3.40}$$

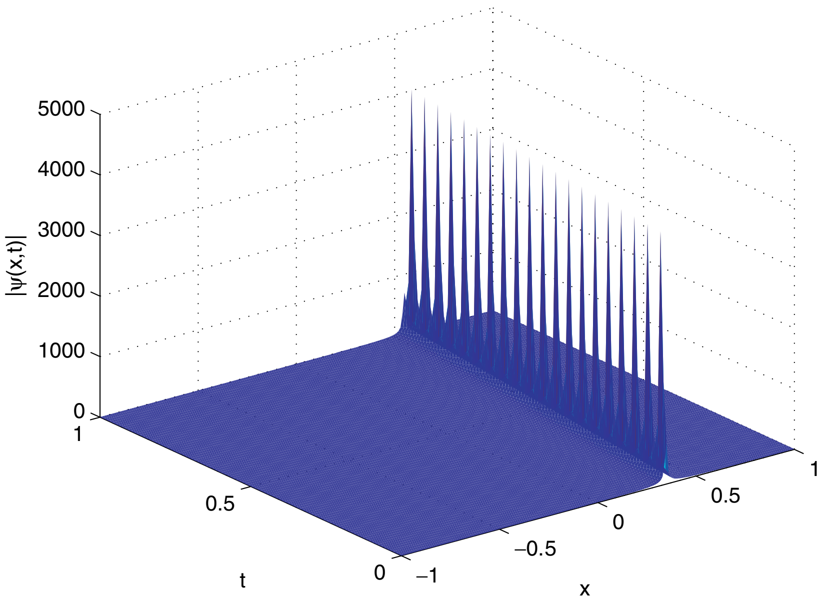

Fig. 1. Singular solitary wave solution for $\alpha = 1$ and $V_0 = 1$, and $g = 1$.

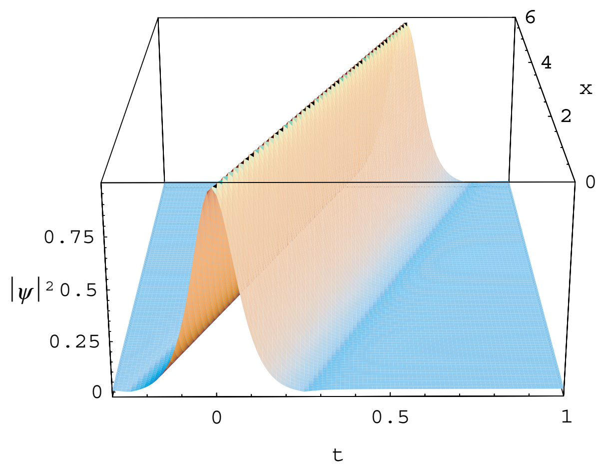

Fig. 2. Nonsingular solitary wave solution for $\alpha = 1$, $V_0 = -6$, $g = -1$ and $K = 1/2$.

This set corresponds to the non-singular solution for attractive case i.e., $g < 0$. The same has been depicted in Fig. 2 for the parameter values given in in the figure caption.

4. Conclusions

In conclusion, we have shown the existence of wide class of exact solutions for the modified GP equation in "smart" periodic potentials with an external source. The Lorentzian-type of solitons are obtained with the aid of a fractional transform. Our analysis applies to both attractive and repulsive cases of GP equation. Furthermore, our rational solutions generalize those contained in [6], for nonzero trapping potential, because of our choice of "smart" potential. We hope that these potentials may be experimentally realizable, to achieve BEC. Although not presented here, the stability of these wide class of solutions can be checked using a semi-implicit Crank–Nicolson finite difference method [7], as the much used numerical techniques based on fast Fourier transform (FFT) requires the FFT of the source, which is costly.

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T. S. R. would like to dedicate this paper to the fond memory of his father Shri. Thokala Ratna Raju, for his love and encouragement.

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