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ON THE PROPAGATION OF BINARY SIGNALS IN A TWO-DIMENSIONAL NONLINEAR LATTICE WITH NEAREST-NEIGHBOR INTERACTIONS

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In this work, we use a computational technique with multiple properties of consistency in order to approximate solutions of a bounded β -Fermi–Pasta–Ulam lattice in two space dimensions subject to harmonic driving in two adjacent boundaries. The processes of nonlinear supratransmission and infratransmission are employed in order to propagate binary signals in the medium by means of a suitable modulation of the driving amplitude.

Keywords: Two-dimensional lattice; Fermi–Pasta–Ulam system; finite difference scheme; harmonic boundary perturbation; wave propagation; binary signals.

1. Introduction

The process of supratransmission consists in the sudden increase in the amplitude of wave signals transmitted into a nonlinear medium perturbed by a harmonic source, irradiating at a frequency in the forbidden band gap. The process was discovered in discrete chains of harmonic oscillators coupled by springs satisfying Hooke's law [9], where the governing law consists of a system of coupled sine-Gordon equations with Dirichlet boundary data. Later on, it was shown that the process is likewise present in superconductivity problems, particularly, in discrete Josephson-junction arrays coupled through superconducting wires and perturbed sinusoidally by an external source [5], where the regime is again a system of coupled sine-Gordon equations, this time with a Neumann boundary condition.

Nonlinear supratransmission has been investigated in other $(1+1)$ -dimensional physical models. For instance, it has been shown that supratransmission is present in models consisting of discrete Klein–Gordon equations [21, 9], double sine-Gordon equations [10], Bragg media in the nonlinear Kerr regime [23], and even in continuous problems governed by sine-Gordon equations subject to Dirichlet [26] or Neumann boundary data [5]. Moreover, this nonlinear phenomenon is present in several $(2+1)$ -dimensional systems [17, 16, 15], and several applications have been proposed for one-dimensional scenarios, including applications to the transmission of binary signals in nonlinear media [18–20], as well as for two-dimensional [16] and three-dimensional cases [15].

Supratransmission in $(1+1)$ -dimensional chains of anharmonic oscillators has also been investigated in the literature [27]. The existence of this process together with regions of bistability where conducting and insulating regimes coexist has been approximated analytically and numerically, as it was previously done for the continuous sine-Gordon equation [26]. More precisely, it has been established that the classical β -Fermi–Pasta–Ulam chains studied in [7] (a model that has been studied extensively in more general settings [4, 8]) are capable of transmitting energy in the form of localized nonlinear modes [22] when the boundary is perturbed harmonically at a frequency in the forbidden band-gap. Moreover, the existence of a bistable regime within the forbidden band-gap of the model was approximated both analytically and computationally.

In this work, we present computational evidence of the possibility to transmit binary information in a generalization of the $(2+1)$ -dimensional version of the problem studied in [27], using nonlinear bistability. There exist many possible generalizations of the original model studied by Fermi, Pasta and Ulam in 1955, including generalizations to square [13] and hexagonal [14] lattices; however, in this paper, we study the simplest generalization of the β -Fermi–Pasta–Ulam chain, namely, the generalization studied in [29]. Our results are based on a numerical method with consistency properties in the domains of both solution and energy, and they establish the feasibility to transmit binary bits in the medium under study by employing the phenomena of nonlinear supratransmission and infratransmission.

2. Mathematical Model

Let N be a positive integer, and for every $m, n = 0, 1, \dots, N$, let $u_{m,n}(t)$ be a function of time $t \geq 0$. Let γ be a nonnegative constant, and let β and c be positive real numbers. In this paper we consider the following initial-value problem with boundary data, for $m, n = 1, 2, \dots, N-1$:

$$\begin{aligned} \frac{d^2 u_{m,n}}{dt^2} &= -\gamma \frac{du_{m,n}}{dt} + c^2(\delta_x^2 + \delta_y^2)u_{m,n} + \beta[(\delta_x u_{m,n})^3 \\ &\quad - (\delta_x u_{m,n-1})^3 + (\delta_y u_{m,n})^3 - (\delta_y u_{m,n-1})^3], \\ \text{s.t. } \begin{cases} \frac{du_{m,n}(0)}{dt} = u_{m,n}(0) = 0, \\ u_{m,0}(t) = u_{0,n}(t) = \phi(t), & t \geq 0, \\ \delta_x u_{N-1,n}(t) = \delta_y u_{m,N-1}(t) = 0, & t \geq 0. \end{cases} \end{aligned} \quad (2.1)$$

Clearly, the constant γ assumes the role of damping, while c and β are called the coefficients of coupling and nonlinearity, respectively. For practical reasons, the function ϕ

may be thought of as a sinusoidal perturbation at the boundary, namely,

$$\phi(t) = A \sin(\Omega t), \quad (2.2)$$

where $\Omega > 2$, following [27]. Meanwhile, the difference operators employed in (2.1) are given by

$$\begin{aligned} \delta_x u_{m,n} &= u_{m+1,n} - u_{m,n}, \\ \delta_y u_{m,n} &= u_{m,n+1} - u_{m,n}, \\ \delta_x^2 u_{m,n} &= u_{m+1,n} - 2u_{m,n} + u_{m-1,n}, \\ \delta_y^2 u_{m,n} &= u_{m,n+1} - 2u_{m,n} + u_{m,n-1}. \end{aligned} \quad (2.3)$$

These operators are assumed to be linear, and their compositions are defined in the usual way.

Obviously, (2.1) is a generalization to the $(2 + 1)$ -dimensional scenario, of the β -Fermi–Pasta–Ulam model studied in [7], which is a model that possesses many applications [12, 30, 24, 31, 2, 11, 25], and which has been extensively studied in the literature of nonlinear science [28, 6, 1, 3]. It is the simplest generalization to two space dimensions [29]. It describes a discrete, two-dimensional system of particles, each of them governed by the corresponding ordinary differential equation of (2.1), initially at rest, and where the boundary is harmonically perturbed with a frequency Ω and an amplitude A .

It is not difficult to check that the Hamiltonian of system (2.1) is given by

$$H_{m,n} = \frac{1}{2} \left(\frac{du_{m,n}}{dt} \right)^2 + \frac{c^2}{2} [(\delta_x u_{m,n})^2 + (\delta_y u_{m,n})^2] + \frac{\beta}{4} [(\delta_x u_{m,n})^4 + (\delta_y u_{m,n})^4] \quad (2.4)$$

for every $m, n = 1, 2, \dots, N - 1$. After including the potential functions between the couplings in the boundaries of the system, it can be checked that the total energy of the medium at any time is given by

$$\begin{aligned} E &= \sum_{m=1}^{N-1} \sum_{n=1}^{N-1} H_{m,n} + \frac{c^2}{2} \left[\sum_{m=1}^{N-1} (\delta_y u_{m,0})^2 + \sum_{n=1}^{N-1} (\delta_x u_{0,n})^2 \right] \\ &\quad + \frac{\beta}{4} \left[\sum_{m=1}^{N-1} (\delta_y u_{m,0})^4 + \sum_{n=1}^{N-1} (\delta_x u_{0,n})^4 \right]. \end{aligned} \quad (2.5)$$

Moreover, it turns out that the rate of change of the total energy of the system with respect to time is given by the expression

$$\begin{aligned} \frac{dE}{dt} &= -\gamma \sum_{m=1}^{N-1} \sum_{n=1}^{N-1} \left(\frac{du_{m,n}}{dt} \right)^2 - \sum_{m=1}^{N-1} (u_{m,1} - u_{m,0}) [c^2 + \beta(u_{m,1} - u_{m,0})^2] \frac{du_{m,0}}{dt} \\ &\quad - \sum_{n=1}^{N-1} (u_{1,n} - u_{0,n}) [c^2 + \beta(u_{1,n} - u_{0,n})^2] \frac{du_{0,n}}{dt}. \end{aligned} \quad (2.6)$$

3. Numerical Method

Let T be a positive real number. In order to approximate solutions to (2.1) on the time interval $[0, T]$, we fix a regular partition $0 = t_0 < t_1 < \dots < t_M = T$ with norm equal

to $\Delta t = T/M$, and let $u_{m,n}^k$ represent the approximation provided by our method to the actual value of the solution $u_{m,n}$ of problem (2.1) at time t_k , for every $k = 0, 1, \dots, M$. We introduce here the linear operators

$$\begin{aligned}\delta_t u_{m,n}^k &= u_{m,n}^{k+1} - u_{m,n}^k, \\ \delta_t^1 u_{m,n}^k &= u_{m,n}^{k+1} - u_{m,n}^{k-1}, \\ \delta_t^2 u_{m,n}^k &= u_{m,n}^{k+1} - 2u_{m,n}^k + u_{m,n}^{k-1},\end{aligned}\tag{3.1}$$

for every $m, n = 0, 1, \dots, N$ and $k = 1, 2, \dots, N-1$. Moreover, we introduce the nonstandard operators

$$\begin{aligned}\sigma_x u_{m,n} &= \sum_{j=0}^3 (\delta_x u_{m,n}^{k+1})^j (\delta_x u_{m,n}^k)^{3-j}, \\ \sigma_y u_{m,n} &= \sum_{j=0}^3 (\delta_y u_{m,n}^{k+1})^j (\delta_y u_{m,n}^k)^{3-j}.\end{aligned}\tag{3.2}$$

The finite-difference scheme proposed in this paper to approximate solutions of initial-value problem with boundary data (2.1) is given by the following system of difference equations, for $m, n = 1, 2, \dots, N-1$ and $k = 2, 3, \dots, M-2$:

$$\begin{aligned}\frac{\delta_t^2 u_{m,n}^{k+1} + \delta_t^2 u_{m,n}^k}{2\Delta t} &= -\gamma \frac{\delta_t^1 u_{m,n}^k}{2\Delta t} + c^2 \frac{\delta_x^2 u_{m,n}^{k+1} + \delta_x^2 u_{m,n}^k}{2} + c^2 \frac{\delta_y^2 u_{m,n}^{k+1} + \delta_y^2 u_{m,n}^k}{2} \\ &\quad + \beta \frac{\sigma_x u_{m,n}^k + \sigma_x u_{m-1,n}^k}{4} + \beta \frac{\sigma_y u_{m,n}^k + \sigma_y u_{m,n-1}^k}{4},\end{aligned}\tag{3.3}$$

$$\text{s.t. } \begin{cases} u_{m,n}^1 = u_{m,n}^0 = 0, \\ u_{m,0}^k = u_{0,n}^k = \phi(t_k), & k = 0, 1, \dots, M, \\ \delta_x u_{N-1,n}^k = \delta_y u_{m,N-1}^k = 0, & k = 0, 1, \dots, M. \end{cases}$$

It is readily checked that (3.3) provides consistent approximations of the solutions of (2.1). Moreover, the local energy density in the node (m, n) of our problem is also consistently approximated using the discrete expression

$$\begin{aligned}H_{m,n}^k &= \frac{(\delta_t u_{m,n}^k)(\delta_t u_{m,n}^{k-1})}{2(\Delta t)^2} + c^2 \frac{(\delta_x u_{m,n}^k)^2 + (\delta_x u_{m-1,n}^k)^2}{4} + c^2 \frac{(\delta_y u_{m,n}^k)^2 + (\delta_y u_{m,n-1}^k)^2}{4} \\ &\quad + \beta \frac{(\delta_x u_{m,n}^k)^4 + (\delta_x u_{m-1,n}^k)^4}{8} + \beta \frac{(\delta_y u_{m,n}^k)^4 + (\delta_y u_{m,n-1}^k)^4}{8}.\end{aligned}\tag{3.4}$$

In turn, the total energy E^k of the system at time t_k is calculated by adding all the discrete Hamiltonians to the potentials of the couplings to the nodes of the boundaries. In other words, the discrete energy associated to the finite-difference scheme (3.3) is given by

$$\begin{aligned}E^k &= \sum_{m=1}^{N-1} \sum_{n=1}^{N-1} H_{m,n}^k + \frac{c^2}{4} \left[\sum_{m=1}^{N-1} (\delta_y u_{m,0}^k)^2 + \sum_{n=1}^{N-1} (\delta_x u_{0,n}^k)^2 \right] \\ &\quad + \frac{\beta}{8} \left[\sum_{m=1}^{N-1} (\delta_y u_{m,0}^k)^4 + \sum_{n=1}^{N-1} (\delta_x u_{0,n}^k)^4 \right],\end{aligned}\tag{3.5}$$

which also provides a consistent approximation of the equation of total energy (2.5). Furthermore, it is easy to check that the discrete rate of change of energy provided by our method is given by the equation

$$\begin{aligned} \frac{\delta_t E^k}{\Delta t} = & -\gamma \sum_{m=1}^{N-1} \sum_{n=1}^{N-1} \left(\frac{\delta_t u_{m,n}^k}{\Delta t} \right) \left(\frac{\delta_t^1 u_{m,n}^k}{2\Delta t} \right) - c^2 \sum_{m=1}^{N-1} \left(\frac{\delta_y u_{m,0}^{k+1} + \delta_y u_{m,0}^k}{2} \right) \frac{\delta_t u_{m,0}^k}{\Delta t} \\ & - c^2 \sum_{n=1}^{N-1} \left(\frac{\delta_x u_{0,n}^{k+1} + \delta_x u_{0,n}^k}{2} \right) \frac{\delta_t u_{0,n}^k}{\Delta t} - \beta \sum_{m=1}^{N-1} \frac{\sigma_y u_{m,0}^k}{4} \frac{\delta_t u_{m,0}^k}{\Delta t} \\ & - \beta \sum_{n=1}^{N-1} \frac{\sigma_x u_{0,n}^k}{4} \frac{\delta_t u_{0,n}^k}{\Delta t}, \end{aligned} \quad (3.6)$$

which is a consistent approximation of the derivative of the total energy of the system (2.1) with respect to time.

4. Simulations

Let us consider a lattice consisting of $N \times N$ sites with $N = 25$. We fix a constant damping coefficient equal to 0.01, a coefficient of nonlinearity of 1, a coupling factor equal to 5, and a driving frequency in the forbidden band gap equal to 2.5. Computationally, we fixed a time step of 0.05, and a time period of length 1000.

Fig. 1. Graph of total energy of a system governed by (2.1) versus driving amplitude, initially at rest with each site in resting position, with parameters $T = 1000$, $N = 25$, $c = 5$, $\beta = 1$, $\gamma = 0.01$, $\Omega = 2.5$, and $\Delta t = 0.05$. The system was harmonically perturbed in two boundaries by (2.2), and void Neumann conditions were imposed on the other two. The vertical dashed lines represent the critical thresholds of infratransmission and supratransmission (A_i and A_s , respectively), and the arrows represent the direction of increase or decrease of the driving amplitude.

Fig. 2. Graphs of local energy versus time of the node located in site (18, 18), of a system governed by (2.1) with $N = 25$. The following parametric and computational values were employed in the simulations: $\Omega = 2.5$, $\gamma = 0.01$, $\beta = 1$, $c = 5$, $T = 1000$, and $\Delta t = 0.05$. In the top row, the driving amplitude is increased in finite time from a value of zero to the value A given in each case. Meanwhile, in the bottom row, a system which has already reached supratransmission is considered, and the value of the driving amplitude is decreased from the supratransmission threshold 0.845 to the actual given value A .

The boundaries of the system were perturbed using the sinusoidal function

$$\varphi(t) = A(t) \sin(\Omega t), \quad (4.1)$$

where the nonnegative function $A(t)$ modulates the amplitude, increasing it progressively from 0 to a fixed constant A' , in order to avoid the possible generation of shock waves. In our case, $A(t)$ was a linear function, and the value A' was reached after 50 time units. We employed our method for increasing values of A' , and we kept track of the total energy of the system until a sudden and massive peak of energy was registered, reaching thus the supratransmission threshold (A_s). Next, we carried out simulations but now considering a system which was already transmitting energy from the driving boundary. Clearly, the values given to A' on the second set of simulations decreased from the supratransmission threshold, and so $A(t)$ likewise decreased from A_s down to A' for the first 50 units of time. A significant drop in the total energy marked the presence of the infratransmission threshold (A_i). In fact Fig. 1 presents the results of our simulations, together with the directions of increase or decrease of the driving amplitude.

According to our simulations, we have found that $A_s \approx 0.835$ and $A_i \approx 0.310$. We wish to approximate these values more accurately by studying the dynamics of a specific node of the lattice; particularly, we fixed the node located in the site (18,18), and took two values for the driving amplitudes: one right before supratransmission starts, and the other right after the supratransmission threshold. In similar fashion, we proceeded systematically to determine the infratransmission threshold to an accuracy of within 0.01. The first and second rows of Fig. 2 show that the actual values for A_s and A_i are 0.845 and 0.415, respectively.

5. Application

In the present section, we employ our model to send binary signals taking advantage of the simplicity to pass from a state of supratransmission to one of infratransmission and vice versa. This was done by using a continuous function to regulate the amplitude according to the corresponding bit that we intended to send. We took $A_s = 0.85$ and $A_i = 0.13$ in order to facilitate the transition from one state to another.

Let $S = b_1 b_2 \cdots b_n$ be the chain consisting of binary bits to be transmitted into the system. We employed the following function to control the amplitude

$$A(t_k, b_i) = \begin{cases} 0.36 \sin \left[\frac{\pi}{100} (t + 150) \right] + 0.49, & b_i = 1, \\ 0.1, & b_i = 0, \end{cases} \quad (5.1)$$

$$P_{k-1} \leq t_k \leq P_k$$

where k is a positive integer and $P_k = 200k$, that is, the period of the function when $b_i = 1$ multiplied by k . We sent a 10-bit message given by $S = 1101010011$ using the amplitude

Fig. 3. Graphs of local energy versus time of the node located in site (18, 18), of a system governed by (2.1) with $N = 25$. The following parametric and computational values were employed in the simulations: $\Omega = 2.5$, $\gamma = 0.01$, $\beta = 1$, $c = 5$, $T = 1000$, and $\Delta t = 0.05$. The graph is the result of transmitting the binary code “1101010011” when perturbing the boundary by the amplitude modulation function presented in the inset.

Fig. 4. Graphs of local energy versus time of the node located in site (18, 18), of a system governed by (2.1) with $N = 25$. The following parametric and computational values were employed in the simulations: $\Omega = 2.5$, $\gamma = 0.01$, $\beta = 1$, $c = 5$, $T = 1000$, and $\Delta t = 0.05$. The graph is the result of transmitting the binary code “10111001011011101001” when perturbing the boundary.

function described above. The results are shown in Fig. 3 as a graph of the local energy at site (18, 18) versus time, and they evidence the fact the binary information is transmitted accurately from the driving boundary.

Having seen that the information was transmitted without any difficulties we send a message twice as long as the first, in order to verify that the reliability of the process is independent of the length of the signal. We proceeded to propagate the binary message “10111001011011101001”, and Fig. 4 summarizes the result of our simulation, confirming that no distortion of information was generated during the process.

6. Conclusion

A β -Fermi–Pasta–Ulam lattice in two space dimensions can effectively be used to send binary data by modulating the driving amplitude of two adjacent boundaries between the supra- and infratransmission thresholds; a sinusoidal function fulfills this task in a very efficient way. This method for transmitting binary signals is reliable and “economical” in view of the fact that the system never completely ceases to irradiate energy; it simply stays idle until a proper variation of the driving amplitude changes it between conducting and insulating regimes.

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