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## NEW TYPE OF NONISOSPECTRAL KP EQUATION WITH SELF-CONSISTENT SOURCES AND ITS BILINEAR BÄCKLUND TRANSFORMATION

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A new type of the nonisospectral KP equation with self-consistent sources is constructed by using the source generation procedure. A new feature of the obtained nonisospectral system is that we allow  $y$ -dependence of the arbitrary constants in the determinantal solution for the nonisospectral KP equation. In order to further show integrability of the novel nonisospectral KP equation with self-consistent sources, we give a bilinear Bäcklund transformation.

*Keywords:* Nonisospectral KP equation; Bäcklund transformation; source generation procedure; soliton solution.

Mathematics Subject Classification: 35Q58

### 1. Introduction

Since the discovery of soliton, the soliton equations have been used to describe various nonlinear phenomena in many fields of natural science, such as the plasma physics, fluid dynamics, biology and so on [1, 2]. For example, the Kadomtsev–Petviashvili (KP) equation, a two-dimensional generalization of the well-known KdV equation, can model several significant situations such as ones arising from the plasma [3]. Recently, the nonisospectral KP equation have attracted much research attention [4–7]. The nonisospectral KP equation provide a description of surface waves in a more realistic situation than the KP equation itself. The nonisospectral KP equation can also describe the waves in a certain type of nonuniform media.

In this paper, we consider soliton equations with self-consistent sources, because soliton equations with self-consistent sources are an important class of integrable equations. Physically, the sources can result in solitary waves moving with a nonconstant velocity and therefore lead to a variety of dynamics of physical models. For applications, these kinds of systems are usually used to describe interaction between different solitary waves. One typical example is the KP equation with self-consistent sources [8]. Many methods have been developed to solve soliton equations with self-consistent sources, such as the inverse scattering transform [9], Darboux transformation [10–12], Hirota bilinear method [13–16], source generation procedure [17–19] and so on.

Very recently, Hu and Wang have proposed the source generation procedure to systematically construct and solve soliton equations with self-consistent sources [17, 18]. The source generation procedure consists of three steps:

- (1) express  $N$ -soliton solutions of a soliton equation without sources in the form of determinant or Pfaffian with some arbitrary constants;
- (2) construct corresponding determinant or Pfaffian with arbitrary functions of one variable;
- (3) seek coupled bilinear equations whose solutions are just these generalized determinants or Pfaffians.

It is noted that the success of step (3) heavily depends on the suitable choice of arbitrary functions involved in step (2). Some soliton equations with self-consistent sources found by the source generation procedure require time dependence of the arbitrary constants appearing in the determinantal or Pfaffian solutions for the equations without sources. For example, we have constructed the following nonisospectral KP equation with self-consistent sources (KPESCS) by allowing time dependence of the arbitrary constants in the determinantal solution for the nonisospectral KP equation [20].

$$4u_t + y(u_{xxx} + 6uu_x + 3\partial_x^{-1}u_{yy}) + 2xu_y + 4\partial_x^{-1}u_y + \sum_{j=1}^K (\Phi_j \Psi_j)_x = 0, \quad (1)$$

$$\Phi_{j,y} = \Phi_{j,xx} + u\Phi_j, \quad j = 1, 2, \dots, K, \quad (2)$$

$$\Psi_{j,y} = -\Psi_{j,xx} - u\Psi_j, \quad j = 1, 2, \dots, K, \quad (3)$$

The purpose of this paper is to apply the source generation procedure to the nonisospectral KP equation by allowing  $y$ -dependence of the arbitrary constants appearing in the determinantal or Pfaffian solutions for the nonisospectral KP equation. Consequently, a new type of the nonisospectral KPESCS is produced, which is quite different from the nonisospectral KPESCS (1)–(3). In order to further show integrability of the novel nonisospectral KPESCS, we propose a bilinear Bäcklund transformation for the new nonisospectral KPESCS.

This paper is organized as follows. In Sec. 2, a new type of nonisospectral KP equation with self-consistent sources is constructed via the source generation procedure by allowing  $y$ -dependence of the arbitrary constants appearing in the determinantal solutions for the nonisospectral KP equation. In Sec. 3, we present a bilinear Bäcklund transformation for the novel nonisospectral KPESCS. Finally, some conclusions and discussions are given.

### 2. New Type of Nonisospectral KP Equation with Self-Consistent Sources

In this section, we will apply the source generation procedure to the nonisospectral KP equation by allowing  $y$ -dependence of the arbitrary constants appearing in the determinantal solutions for the nonisospectral KP equation.

The nonisospectral KP equation can be written as [21]

$$4u_t + y(u_{xxx} + 6uu_x + 3\partial_x^{-1}u_{yy}) + 2xu_y + 4\partial_x^{-1}u_y = 0. \tag{4}$$

Its Lax pair is

$$\phi_y = \phi_{xx} + 2u\phi, \tag{5a}$$

$$\phi_t = y \left[ \phi_{xxx} + 3u\phi_x + \frac{3}{2}(\partial_x^{-1}u_y + u_x)\phi \right] + \frac{1}{2}x(\phi_{xx} + 2u\phi) + \frac{1}{2}\phi_x + \frac{1}{2}(\partial_x^{-1}u)\phi. \tag{5b}$$

Through the dependent variable transformation

$$u = 2(\ln \tau)_{xx},$$

Equation (4) can be transformed into the bilinear form

$$4D_x D_t \tau \cdot \tau + y(D_x^4 \tau \cdot \tau + 3D_y^2 \tau \cdot \tau) + 2x D_x D_y \tau \cdot \tau + 4\tau_y \tau = 0, \tag{6}$$

where the well-known Hirota bilinear operator  $D$  is defined by [22]

$$D_x^l D_y^m D_t^n a \cdot b = (\partial_x - \partial_{x'})^l (\partial_y - \partial_{y'})^m (\partial_t - \partial_{t'})^n a(x, y, t) b(x', y', t')|_{x'=x, y'=y, t'=t}.$$

The nonisospectral KP equation (6) has the following Grammian determinant solution [5]:

$$\tau = \det \left( \beta_{ij} + \int_{-\infty}^x f_i \tilde{f}_j dx \right)_{1 \leq i, j \leq N}, \quad \beta_{ij} = \text{constant}, \tag{7}$$

where  $f_i$  and  $\tilde{f}_j$  satisfy the following differential equations

$$f_{i,y} = f_{i,xx}, \quad \tilde{f}_{j,y} = -\tilde{f}_{j,xx}, \tag{8a}$$

$$f_{i,t} = -y f_{i,xxx} - \frac{1}{2} x f_{i,xx} - \frac{1}{2} f_{i,x}, \tag{8b}$$

$$\tilde{f}_{j,t} = -y \tilde{f}_{j,xxx} + \frac{1}{2} x \tilde{f}_{j,xx} + \frac{1}{2} \tilde{f}_{j,x}. \tag{8c}$$

In order to construct new type of nonisospectral KPESCS, according to the source generation procedure, we generalize  $\tau$  into the following new function:

$$f = \det(a_{ij})_{1 \leq i, j \leq N} = (1, 2, \dots, N, N^*, \dots, 2^*, 1^*) = (\bullet), \tag{9}$$

where Pfaffian elements are defined by

$$(i, j^*) = a_{ij} = \beta_{ij}(y) + \int_{-\infty}^x f_i \tilde{f}_j dx, \quad (i, j) = (i^*, j^*) = 0, \quad i, j = 1, 2, \dots, N,$$

with each function  $\beta_{ij}(y)$  satisfying

$$\beta_{ij}(y) \equiv \begin{cases} \beta_i(y), & i = j \text{ and } 1 \leq i \leq M \leq N, M, N \in \mathbb{Z}^+ \\ \beta_{ij}, & \text{otherwise,} \end{cases}$$

where  $\beta_i(y)$  is a function of the variable  $y$ . Then we get the following formulae through derivative formulae of Pfaffian:

$$f_y = (d_0, d_1^*, \bullet) - (d_1, d_0^*, \bullet) + \sum_{i=1}^M k_i, \tag{10}$$

$$\begin{aligned} f_{yy} &= (d_3, d_0^*, \bullet) - (d_1, d_2^*, \bullet) + (d_0, d_3^*, \bullet) - (d_2, d_1^*, \bullet) + 2(d_0, d_0^*, d_1, d_1^*, \bullet) \\ &+ \sum_{i=1}^M k_{i,y} + \sum_{i=1}^M \beta_i(y)[(d_0, d_1^*, 1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) \\ &- (d_1, d_0^*, 1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)], \end{aligned} \tag{11}$$

where the function  $k_i$  is defined by

$$k_i = \dot{\beta}_i(y)(1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*), \quad i = 1, 2, \dots, M, \tag{12}$$

new Pfaffian elements are defined by

$$\begin{aligned} (d_n^*, i) &= \frac{\partial^n}{\partial x^n} f_i, \quad (d_m, d_n^*) = 0, \quad (d_m^*, d_n^*) = 0, \quad (d_m, d_n) = 0, \\ (d_n, j^*) &= \frac{\partial^n}{\partial x^n} \tilde{f}_j, \quad (d_m^*, i^*) = 0, \quad (d_n, i) = 0, \quad m, n \in \mathbb{Z}, \end{aligned}$$

where  $\hat{\phantom{x}}$  indicates deletion of the letter under it, and the dot denotes the derivative of the function  $\beta_i(y)$  with respect to the variable  $y$ . By a direct computation, we find the function  $f$  will not satisfy Eq. (6) again. Therefore, we need to introduce other new functions defined by

$$g_i = \sqrt{\dot{\beta}_i(y)}(d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*), \quad i = 1, 2, \dots, M, \tag{13}$$

$$h_i = \sqrt{\dot{\beta}_i(y)}(d_0, 1, \dots, \hat{i}, \dots, N, N^*, \dots, 1^*), \quad i = 1, 2, \dots, M, \tag{14}$$

$$\begin{aligned} P_i &= \frac{\beta_i''(y)}{2\sqrt{\dot{\beta}_i(y)}}(d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) \\ &+ \sqrt{\dot{\beta}_i(y)} \left[ \sum_{1 \leq i < j \leq M} \dot{\beta}_i(y)(d_0^*, 1, \dots, \hat{j}, \dots, N, N^*, \dots, \hat{j}^*, \dots, \hat{i}^*, \dots, 1^*) \right. \\ &\left. - \sum_{1 \leq j < i \leq M} \dot{\beta}_i(y)(d_0^*, 1, \dots, \hat{j}, \dots, N, N^*, \dots, \hat{i}^*, \dots, \hat{j}^*, \dots, 1^*) \right], \end{aligned} \tag{15}$$

$$\begin{aligned}
 Q_i &= \frac{\beta_i''(y)}{2\sqrt{\dot{\beta}_i(y)}}(d_0, 1, \dots, \hat{i}, \dots, N, N^*, \dots, 1^*) \\
 &+ \sqrt{\dot{\beta}_i(y)} \left[ \sum_{1 \leq i < j \leq M} \dot{\beta}_i(y)(d_0, 1, \dots, \hat{i}, \dots, \hat{j}, \dots, N, N^*, \dots, \hat{j}^*, \dots, 1^*) \right. \\
 &\left. - \sum_{1 \leq j < i \leq M} \dot{\beta}_i(y)(d_0, 1, \dots, \hat{j}, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{j}^*, \dots, 1^*) \right], \tag{16}
 \end{aligned}$$

where "'' denotes the second-order derivative of the function  $\beta_i(y)$ .

We can show that the above new functions satisfy the following new bilinear equations:

$$\begin{aligned}
 &4D_x D_t f \cdot f + y(D_x^4 f \cdot f + 3D_y^2 f \cdot f) + 2xD_x D_y f \cdot f + 4f_y f \\
 &= 6y \sum_{i=1}^M (D_y k_i \cdot f - D_x g_i \cdot h_i) - 4x \sum_{i=1}^M g_i h_i + 4 \left( \sum_{i=1}^M k_i \right) f, \tag{17}
 \end{aligned}$$

$$D_x k_i \cdot f + g_i h_i = 0, \tag{18}$$

$$(D_y - D_x^2)g_i \cdot f = P_i f - g_i \left( \sum_{j=1}^M k_j \right), \tag{19}$$

$$(D_y - D_x^2)f \cdot h_j = h_i \left( \sum_{j=1}^M k_j \right) - f Q_i, \tag{20}$$

$$\begin{aligned}
 &4D_t g_i \cdot f + y(D_x^3 g_i \cdot f + 3D_x D_y g_i \cdot f) + 2xD_y g_i \cdot f \\
 &= 3y \left( D_x \left[ P_i \cdot f - g_i \cdot \left( \sum_{j=1}^M k_j \right) \right] \right) + 2x \left[ P_i f - g_i \left( \sum_{j=1}^M k_j \right) \right], \tag{21}
 \end{aligned}$$

$$\begin{aligned}
 &4D_t f \cdot h_i + y(D_x^3 f \cdot h_i + 3D_x D_y f \cdot h_i) + 2xD_y f \cdot h_i \\
 &= 3y \left( D_x \left[ \left( \sum_{j=1}^M k_j \right) \cdot h_i - f \cdot Q_i \right] \right) - 2x \left[ f Q_i - h_i \left( \sum_{j=1}^M k_j \right) \right]. \tag{22}
 \end{aligned}$$

In the following, we prove that those new functions so-defined are solutions of Eqs. (17)–(22). First, we calculate the following derivative formulae:

$$f = (1, 2, \dots, N, N^*, \dots, 2^*, 1^*) \equiv (\bullet), \quad f_x = (d_0, d_0^*, \bullet), \tag{23a}$$

$$f_{xx} = (d_0, d_1^*, \bullet) + (d_1, d_0^*, \bullet), \quad f_{xxx} = (d_0, d_2^*, \bullet) + (d_2, d_0^*, \bullet) + 2(d_1, d_1^*, \bullet), \tag{23b}$$

$$f_{xxxx} = (d_3, d_0^*, \bullet) + 3(d_2, d_1^*, \bullet) + 2(d_0, d_0^*, d_1, d_1^*, \bullet) + 3(d_1, d_2^*, \bullet) + (d_0, d_3^*, \bullet), \tag{23c}$$

$$f_{yx} = (d_0, d_2^*, \bullet) - (d_2, d_0^*, \bullet) + \sum_{i=1}^M \dot{\beta}_i(y)(d_0, d_0^*, 1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) \quad (24a)$$

$$f_t = -y[(d_0, d_2^*, \bullet) - (d_1, d_1^*, \bullet) + (d_2, d_0^*, \bullet)] - \frac{1}{2}x[(d_0, d_1^*, \bullet) - (d_1, d_0^*, \bullet)], \quad (24b)$$

$$f_{tx} = -y[(d_0, d_3^*, \bullet) - (d_0, d_0^*, d_1, d_1^*, \bullet) + (d_3, d_0^*, \bullet)] - \frac{1}{2}x[(d_0, d_2^*, \bullet) - (d_2, d_0^*, \bullet)] - \frac{1}{2}[(d_0, d_1^*, \bullet) - (d_1, d_0^*, \bullet)], \quad (24c)$$

$$g_{i,x} = \sqrt{\dot{\beta}_i(y)}(d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*), \quad (25a)$$

$$g_{i,xx} = \sqrt{\dot{\beta}_i(y)}[(d_2^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) + (d_0, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)], \quad (25b)$$

$$g_{i,y} = P_i + \sqrt{\dot{\beta}_i(y)}[(d_2^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) - (d_0, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)]. \quad (25c)$$

$$g_{i,xy} = P_{ix} + \sqrt{\dot{\beta}_i(y)}[(d_3^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) - (d_1, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)]. \quad (25d)$$

$$g_{i,t} = -y\sqrt{\dot{\beta}_i(y)}[(d_3^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) + (d_1, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) - (d_0, d_0^*, d_2^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)] - \frac{1}{2}x\sqrt{\dot{\beta}_i(y)}[(d_2^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) - (d_0, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)], \quad (25e)$$

$$g_{i,xxx} = \sqrt{\dot{\beta}_i(y)}[(d_3^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) + 2(d_0, d_0^*, d_2^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*) + (d_1, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)]. \quad (25f)$$

Substituting (9)–(11), (13) and (14) into Eq. (17) yields the sum of the determinant identities

$$24y[(d_0, d_0^*, d_1, d_1^*, \bullet)(\bullet) - (d_0, d_0^*, \bullet)(d_1, d_1^*, \bullet) + (d_0, d_1^*, \bullet)(d_1, d_0^*, \bullet)] + 6y \sum_{i=1}^M \dot{\beta}_i(y)[(d_0, d_1^*, 1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(\bullet) - (1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, d_1^*, \bullet)]$$

$$\begin{aligned}
 &+ (d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, 1, \dots, \hat{i}, \dots, N, N^*, \dots, 1^*) \\
 &- (d_1, d_0^*, 1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(\bullet) \\
 &+ (1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_1, d_0^*, \bullet) \\
 &- (d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_1, 1, \dots, \hat{i}, \dots, N, N^*, \dots, 1^*) \\
 &+ 4x \sum_{i=1}^M \dot{\beta}_i(y) [(d_0, d_0^*, 1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(\bullet) \\
 &\quad - (1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, d_0^*, \bullet) \\
 &\quad + (d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, 1, \dots, \hat{i}, \dots, N, N^*, \dots, 1^*)] = 0,
 \end{aligned}$$

which indicates that Eq. (17) holds. In the same way, substitution of (9), (12)–(14) into (18) leads to the Jacobi identity of determinants

$$\begin{aligned}
 &\dot{\beta}_i(y) [(d_0, d_0^*, 1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(\bullet) \\
 &\quad - (1, \dots, \hat{i}, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, d_0^*, \bullet) \\
 &\quad + (d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, 1, \dots, \hat{i}, \dots, N, N^*, \dots, 1^*)] = 0,
 \end{aligned}$$

then Eq. (18) holds. Substituting (23) and (25) into Eq. (19), we get the following determinant identity

$$\begin{aligned}
 &2\sqrt{\dot{\beta}_i(y)} [-(d_0, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(\bullet) \\
 &\quad + (d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, d_0^*, \bullet) \\
 &\quad - (d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, d_1^*, \bullet)] = 0,
 \end{aligned}$$

so Eq. (19) holds. Similarly, substitution of (9), (13), (15) and (23)–(25) into (21) leads to the determinant identity

$$\begin{aligned}
 &6y\sqrt{\dot{\beta}_i(y)} [(d_0, d_0^*, d_2^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(\bullet) \\
 &\quad - (d_2^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, d_0^*, \bullet) \\
 &\quad + (d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_0, d_2^*, \bullet) \\
 &\quad - (d_1, d_0^*, d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(\bullet) \\
 &\quad + (d_1^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_1, d_0^*, \bullet) \\
 &\quad - (d_0^*, 1, \dots, N, N^*, \dots, \hat{i}^*, \dots, 1^*)(d_1, d_1^*, \bullet)] = 0,
 \end{aligned}$$

then Eq. (21) holds. In the same way, we can prove that new functions in (9), (12) and (13)–(16) are determinant solutions of bilinear Eqs. (17)–(22). And Eqs. (17)–(22) just constitute a new type of nonisospectral KPESCS in the bilinear forms.



Using the dependent variable transformations

$$u = 2(\ln f)_{xx}, \quad \psi_i = g_i/f, \quad \varphi_i = h_i/f, \quad i = 1, 2, \dots, M, \tag{26a}$$

$$\chi_i = k_i/f, \quad \phi_{1,i} = P_i/f, \quad \phi_{2,i} = Q_i/f, \quad i = 1, 2, \dots, M, \tag{26b}$$

the bilinear equations (17)–(22) are transformed into the following nonlinear nonisospectral equations

$$\begin{aligned} 4u_t + y \left( u_{xxx} + 6uu_x + 3 \int_{-\infty}^x u_{yy} dx \right) + 2xu_y + 4 \int_{-\infty}^x u_y dx \\ = 6y \sum_{i=1}^M (\chi_{i,y} + \varphi_{i,x}\psi_i - \varphi_i\psi_{i,x})_x - 4x \sum_{i=1}^M (\varphi_i\psi_i)_x + 4 \sum_{i=1}^M \chi_{i,x}, \end{aligned} \tag{27a}$$

$$\chi_{i,x} + \varphi_i\psi_i = 0, \tag{27b}$$

$$\psi_{i,y} = \psi_{i,xx} + u\psi_i + \phi_{1,i} - \psi_i \sum_{j=1}^M \chi_j, \tag{27c}$$

$$\varphi_{i,y} = -\varphi_{i,xx} - u\varphi_i + \phi_{2,i} - \varphi_i \sum_{j=1}^M \chi_j, \tag{27d}$$

$$\begin{aligned} 4\psi_{i,t} + y \left( \psi_{i,xxx} + 3u\psi_{i,x} + 3\psi_{i,xy} + 3\psi_i \int_{-\infty}^x u_y dx \right) + 2x\psi_{i,y} \\ = 3y \left[ \phi_{1,i,x} - \sum_{j=1}^M (\psi_{i,x}\chi_j - \psi_i\chi_{j,x}) \right] + 2x \left( \phi_{1,i} - \psi_i \sum_{j=1}^M \chi_j \right), \end{aligned} \tag{27e}$$

$$\begin{aligned} 4\varphi_{i,t} + y \left( \varphi_{i,xxx} + 3u\varphi_{i,x} - 3\varphi_{i,xy} - 3\varphi_i \int_{-\infty}^x u_y dx \right) - 2x\varphi_{i,y} \\ = 3y \left[ -\phi_{2,i,x} - \sum_{j=1}^M (\varphi_{i,x}\chi_j - \varphi_i\chi_{j,x}) \right] - 2x \left( \phi_{2,i} - \varphi_i \sum_{j=1}^M \chi_j \right). \end{aligned} \tag{27f}$$

Equations (27) can be further simplified into the following nonlinear nonisospectral equations:

$$\begin{aligned} 4u_t + y \left( u_{xxx} + 6uu_x + 3 \int_{-\infty}^x u_{yy} dx \right) + 2xu_y + 4 \int_{-\infty}^x u_y dx \\ = 6y \sum_{i=1}^M [\varphi_{i,xx}\psi_i - \varphi_i\psi_{i,xx} - (\varphi_i\psi_i)_y] - 4x \sum_{i=1}^M (\varphi_i\psi_i)_x - 4 \sum_{i=1}^M \varphi_i\psi_i, \end{aligned} \tag{28a}$$

$$\begin{aligned}
 &4\psi_{i,t} + y \left( 4\psi_{i,xxx} + 6u\psi_{i,x} + 3u_x\psi_i + 3\psi_i \int_{-\infty}^x u_y dx \right) + 2x\psi_{i,y} \\
 &= -6y\psi_i \left( \sum_{j=1}^M \varphi_j\psi_j \right) + 2x(\psi_{i,y} - \psi_{i,xx} - u\psi_i),
 \end{aligned} \tag{28b}$$

$$\begin{aligned}
 &4\varphi_{i,t} + y \left( 4\varphi_{i,xxx} + 6u\varphi_{i,x} + 3u_x\varphi_i - 3\varphi_i \int_{-\infty}^x u_y dx \right) - 2x\varphi_{i,y} \\
 &= 6y\varphi_i \left( \sum_{j=1}^M \varphi_j\psi_j \right) - 2x(\varphi_{i,y} + \varphi_{i,xx} + u\varphi_i),
 \end{aligned} \tag{28c}$$

which is new type of the nonisospectral KPESCS.

Utilizing expressions (9), (12)–(16) and relation (26), we can give the  $N$ -soliton ( $N \geq M$ ) solution of the new type of nonisospectral KPESCS (28).

For example, when  $N = 1, M = 1$ , we take

$$\begin{aligned}
 \beta_1(y) &= \frac{e^{2\alpha(y)}}{m+n}, \quad m, n \in R, \\
 f_1 &= e^\xi = e^{p(t)x+p^2(t)y-l(t)}, \quad \tilde{f}_1 = e^\eta = e^{q(t)x-q^2(t)y+\gamma(t)}, \\
 p_t(t) &= -\frac{1}{2}p^2(t), \quad q_t(t) = \frac{1}{2}q^2(t), \quad p(t) = \frac{2}{2c_1+t}, \quad q(t) = \frac{2}{2c_2-t}, \\
 l(t) &= c_3 + \ln(2c_1+t), \quad \gamma(t) = c_4 + \ln(2c_2-t), \quad c_1, c_2, c_3, c_4 \in R,
 \end{aligned}$$

where  $\alpha(y)$  is an arbitrary function of the variable  $y$ ,  $p(t)$ ,  $q(t)$ ,  $l(t)$  and  $\gamma(t)$  are four arbitrary functions of the variable  $t$ . Then, the 1-soliton solution can be expressed in the following forms.

$$\begin{aligned}
 u &= 2\frac{\partial^2}{\partial x^2} \ln \left( 1 + \frac{m+n}{p(t)+q(t)} e^{\xi+\eta-2\alpha(y)} \right), \quad \psi = \frac{\sqrt{2(m+n)\dot{\alpha}(y)}e^{\xi-\alpha(y)}}{1 + \frac{m+n}{p(t)+q(t)} e^{\xi+\eta-2\alpha(y)}}, \\
 \varphi &= \frac{\sqrt{2(m+n)\dot{\alpha}(y)}e^{\eta-\alpha(y)}}{1 + \frac{m+n}{p(t)+q(t)} e^{\xi+\eta-2\alpha(y)}},
 \end{aligned}$$

when  $N = 2, M = 2$ , we take

$$\begin{aligned}
 \beta_i(y) &= \frac{e^{2\alpha_i(y)}}{m_i+n_i}, \quad m_i, n_i \in R, \quad \beta_{12} = \beta_{21} = 0, \quad i = 1, 2, \\
 f_i &= e^{\xi_i} = e^{p_i(t)x+p_i^2(t)y-l_i(t)}, \quad \tilde{f}_i = e^{\eta_i} = e^{q_i(t)x-q_i^2(t)y+\gamma_i(t)}, \quad i = 1, 2, \\
 p_{i,t}(t) &= -\frac{1}{2}p_i^2(t), \quad q_{i,t}(t) = \frac{1}{2}q_i^2(t), \quad i = 1, 2, \\
 p_i(t) &= \frac{2}{2c_{1i}+t}, \quad q_i(t) = \frac{2}{2c_{2i}-t}, \quad c_{1i}, c_{2i} \in R, \quad i = 1, 2, \\
 l_i(t) &= c_{3i} + \ln(2c_{1i}+t), \quad \gamma_i(t) = c_{4i} + \ln(2c_{2i}-t), \quad c_{3i}, c_{4i} \in R.
 \end{aligned}$$

Then the 2-soliton solution of the system (28) has the following form:

$$u = 2 \frac{\partial^2}{\partial x^2} \ln [1 + A_1 e^{\xi_1 + \eta_1 - 2\alpha_1(y)} + A_2 e^{\xi_2 + \eta_2 - 2\alpha_2(y)} + A_3 e^{\xi_1 + \eta_1 + \xi_2 + \eta_2 - 2\alpha_1(y) - 2\alpha_2(y)}],$$

$$\psi_1 = \frac{\sqrt{2(m_1 + n_1)\dot{\alpha}_1(y)} e^{\xi_1 - \alpha_1(y)} (1 + a_1 e^{\xi_2 + \eta_2 - 2\alpha_2(y)})}{1 + A_1 e^{\xi_1 + \eta_1 - 2\alpha_1(y)} + A_2 e^{\xi_2 + \eta_2 - 2\alpha_2(y)} + A_3 e^{\xi_1 + \eta_1 + \xi_2 + \eta_2 - 2\alpha_1(y) - 2\alpha_2(y)}},$$

$$\psi_2 = -\frac{\sqrt{2(m_2 + n_2)\dot{\alpha}_2(y)} e^{\xi_2 - \alpha_2(y)} (1 + a_2 e^{\xi_1 + \eta_1 - 2\alpha_1(y)})}{1 + A_1 e^{\xi_1 + \eta_1 - 2\alpha_1(y)} + A_2 e^{\xi_2 + \eta_2 - 2\alpha_2(y)} + A_3 e^{\xi_1 + \eta_1 + \xi_2 + \eta_2 - 2\alpha_1(y) - 2\alpha_2(y)}},$$

$$\varphi_1 = \frac{\sqrt{2(m_1 + n_1)\dot{\alpha}_1(y)} e^{\eta_1 - \alpha_1(y)} (1 + b_1 e^{\xi_2 + \eta_2 - 2\alpha_2(y)})}{1 + A_1 e^{\xi_1 + \eta_1 - 2\alpha_1(y)} + A_2 e^{\xi_2 + \eta_2 - 2\alpha_2(y)} + A_3 e^{\xi_1 + \eta_1 + \xi_2 + \eta_2 - 2\alpha_1(y) - 2\alpha_2(y)}},$$

$$\varphi_2 = -\frac{\sqrt{2(m_2 + n_2)\dot{\alpha}_2(y)} e^{\eta_2 - \alpha_2(y)} (1 + b_2 e^{\xi_1 + \eta_1 - 2\alpha_1(y)})}{1 + A_1 e^{\xi_1 + \eta_1 - 2\alpha_1(y)} + A_2 e^{\xi_2 + \eta_2 - 2\alpha_2(y)} + A_3 e^{\xi_1 + \eta_1 + \xi_2 + \eta_2 - 2\alpha_1(y) - 2\alpha_2(y)}},$$

where

$$\begin{aligned} A_1 &= \frac{m_1 + n_1}{p_1(t) + q_1(t)}, & A_2 &= \frac{m_2 + n_2}{p_2(t) + q_2(t)}, & A_3 &= -\frac{(m_1 + n_1)(m_2 + n_2)}{(p_1(t) + q_2(t))(p_2(t) + q_1(t))}, \\ a_1 &= \frac{(m_2 + n_2)(p_1(t) - p_2(t))}{(p_1(t) + q_2(t))(p_2(t) + q_2(t))}, & a_2 &= \frac{(m_1 + n_1)(p_2(t) - p_1(t))}{(p_1(t) + q_1(t))(p_2(t) + q_1(t))}, \\ b_1 &= \frac{(m_2 + n_2)(q_1(t) - q_2(t))}{(p_2(t) + q_2(t))(p_2(t) + q_1(t))}, & b_2 &= \frac{(m_1 + n_1)(q_2(t) - q_1(t))}{(p_1(t) + q_1(t))(p_1(t) + q_2(t))}. \end{aligned}$$

From the expressions of the above solutions, we can find these solutions of new non-isospectral KPESCS (28) include arbitrary function of the spatial variable  $y$ , which are different from the solutions of previous nonisospectral KPESCS (1)–(3) which are related to arbitrary functions of the temporal variable  $t$ .

### 3. The Bilinear Bäcklund Transformation

In this section, we will present a bilinear Bäcklund transformation for the new type of nonisospectral KPESCS (17)–(22).

**Proposition 1.** *The bilinear nonisospectral KPESCS (17)–(22) has the following bilinear Bäcklund transformation*

$$(4D_t + yD_x^3 - 3yD_xD_y + 2xD_y - 4D_x)f \cdot f' = -3y \sum_{i=1}^M D_x g_i \cdot h'_i - 2x \sum_{i=1}^M g_i h'_i, \quad (29)$$

$$(4D_t + yD_x^3 - 3yD_xD_y - 2xD_x^2)g_i \cdot g'_i = -3yD_x(P_i \cdot g'_i - g_i \cdot P'_i), \quad (30)$$

$$(4D_t + yD_x^3 - 3yD_xD_y - 2xD_x^2)h_i \cdot h'_i = -3yD_x(Q_i \cdot g'_i - g_i \cdot Q'_i), \quad (31)$$

$$(D_y + D_x^2)f \cdot f' = - \sum_{i=1}^M g_i h'_i, \tag{32}$$

$$(D_y + D_x^2)g_i \cdot g'_i = P_i g'_i - g_i P'_i, \tag{33}$$

$$(D_y + D_x^2)h_i \cdot h'_i = Q_i g'_i - g_i Q'_i, \tag{34}$$

$$(4D_t + yD_x^3)g_i \cdot f' - 3yD_x^2 f \cdot g'_i = 0, \tag{35}$$

$$(4D_t + yD_x^3)f \cdot h'_i - 3yD_x^2 h_i \cdot f' = 0, \tag{36}$$

$$D_x g_i \cdot h'_i - D_x k_i \cdot f' + D_x f \cdot k'_i = 0. \tag{37}$$

$$f k'_i - k_i f' = g_i h'_i, \tag{38}$$

$$D_x g_i \cdot f' + f g'_i = 0, \tag{39}$$

$$D_x f \cdot h'_i + f' h_i = 0. \tag{40}$$

**Proof.** Let  $(f, g_i, h_i, k_i, P_i, Q_i)$  be a solution of Eqs. (17)–(22) and  $(f', g'_i, h'_i, k'_i, P'_i, Q'_i)$  satisfies relations (29)–(40). What we need to prove is that  $(f', g'_i, h'_i, k'_i, P'_i, Q'_i)$  is also a solution of Eqs. (17)–(22). In fact, through relations (29)–(40) and the bilinear operator identities in Appendix A, we have

$$\begin{aligned} P_1 &= \left[ 4D_x D_t f \cdot f + y(D_x^4 f \cdot f + 3D_y^2 f \cdot f) + 2xD_x D_y f \cdot f + 4f_y f \right. \\ &\quad \left. - 6y \sum_{i=1}^M (D_y k_i \cdot f - D_x g_i \cdot h_i) + 4x \sum_{i=1}^M g_i h_i - 4 \left( \sum_{i=1}^M k_i \right) f \right] (f')^2 \\ &\quad - f^2 \left[ 4D_x D_t f' \cdot f' + y(D_x^4 f' \cdot f' + 3D_y^2 f' \cdot f') + 2xD_x D_y f' \cdot f' + 4f'_y f' \right. \\ &\quad \left. - 6y \sum_{i=1}^M (D_y k'_i \cdot f' - D_x g'_i \cdot h'_i) + 4x \sum_{i=1}^M g'_i h'_i - 4 \left( \sum_{i=1}^M k'_i \right) f' \right] \\ &= 8D_x (D_t f \cdot f') \cdot f f' + 2yD_x (D_x^3 f \cdot f') \cdot f f' - 6yD_x (D_x^2 f \cdot f') \cdot (D_x f \cdot f') \\ &\quad + 6yD_y (D_y f \cdot f') \cdot f f' - 6yD_y \sum_{i=1}^M (k_i f' - f k'_i) \cdot f f' \\ &\quad + 6y \sum_{i=1}^M D_x (g_i f' \cdot h_i f' - f g'_i \cdot f h'_i) \\ &\quad + 4xD_x (D_y f \cdot f') \cdot f f' + 4x \sum_{i=1}^M D_x (g_i h'_i) \cdot f f' \\ &\quad + 4f f' (D_y f \cdot f') + 4f f' \sum_{i=1}^M (k'_i f - f' k_i) \end{aligned}$$

$$\begin{aligned}
 &= 2D_x[(4D_t + yD_x^3)f \cdot f'] \cdot ff' - 6yD_x(D_x^2f \cdot f') \cdot (D_xf \cdot f') \\
 &\quad - 6yD_y(D_x^2f \cdot f') \cdot ff' + 6y \sum_{i=1}^M D_x(g_i f' \cdot h_i f' - f g'_i \cdot f h'_i) \\
 &\quad + 4x D_x(D_y f \cdot f') \cdot ff' + 4x \sum_{i=1}^M D_x(g_i h'_i) \cdot ff' \\
 &\quad + 4ff'(D_y f \cdot f') + 4ff' \sum_{i=1}^M (k'_i f - f' k_i) \\
 &= -6y \sum_{i=1}^M D_x[(D_x f \cdot h'_i) \cdot g_i f' + (D_x g_i \cdot f') \cdot f h'_i] \\
 &\quad + 6y \sum_{i=1}^M D_x(g_i f' \cdot h_i f' - f g'_i \cdot f h'_i) \equiv 0.
 \end{aligned}$$

$$\begin{aligned}
 P_{2,i} &= (D_x k_i \cdot f + g_i h_i)(f')^2 - f^2(D_x k'_i \cdot f' + g'_i h'_i) \\
 &= D_x(k_i f' - f k'_i) \cdot ff' + g_i h_i (f')^2 - f^2 g'_i h'_i \\
 &= -D_x(g_i h'_i) \cdot ff' + g_i h_i (f')^2 - f^2 g'_i h'_i \\
 &= g_i f'(D_x f \cdot h'_i + h_i f) - (D_x g_i \cdot f' + f g'_i) f h'_i \equiv 0.
 \end{aligned}$$

$$\begin{aligned}
 P_{3,i} &= \left[ (D_y - D_x^2)g_i \cdot f - P_i f + g_i \left( \sum_{j=1}^M k_j \right) \right] g'_i f' \\
 &\quad - g_i f \left[ (D_y - D_x^2)g'_i \cdot f' - P'_i f' + g'_i \left( \sum_{j=1}^M k'_j \right) \right] \\
 &= [(D_y + D_x^2)g_i \cdot g'_i] f f' - g_i g'_i [(D_y + D_x^2)f \cdot f'] + g_i g'_i \left( \sum_{j=1}^M k_j \right) f' \\
 &\quad - 2D_x(D_x g_i \cdot f') \cdot f g'_i - P_i g'_i f f' + P'_i g_i f f' - g_i f g'_i \left( \sum_{j=1}^M k'_j \right) \\
 &= [(D_y + D_x^2)g_i \cdot g'_i - P_i g'_i + P'_i g_i] f f' - g_i g'_i [(D_y + D_x^2)f \cdot f' + \sum_{j=1}^M (f k'_j - k_j f')] \equiv 0.
 \end{aligned}$$

The above results indicate that  $(f', g'_i, h'_i, k'_i, P'_i, Q'_i)$  satisfies Eqs. (17)–(19). Similarly, we can show that  $(f', g'_i, h'_i, k'_i, P'_i, Q'_i)$  satisfies Eqs. (20)–(22). So,  $(f', g'_i, h'_i, k'_i, P'_i, Q'_i)$  is a solution of new type of nonisospectral KPESCS (17)–(22). Thus, we complete the proof.  $\square$

#### 4. Conclusions and Discussions

In this paper, we have obtained a new type of nonisospectral KPESCS and its Grammian determinant solutions. Moreover, we gave 1-soliton solution and 2-soliton solution for the novel nonisospectral KPESCS. Furthermore, a bilinear Bäcklund transformation for the new type of nonisospectral KPESCS is presented. If we set each arbitrary function  $\beta_i(y)$  a constant, the new nonisospectral KPESCS is reduced to the nonisospectral KP equation, and its determinant solutions (9) and (12)–(16) are transformed into the solution of the nonisospectral KP equation.

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#### Appendix A. Hirota’s Bilinear Operator Identities

The following bilinear operator identities hold for arbitrary functions  $a, b, a', b', c$  and  $d$ .

$$(D_z a \cdot b)cd - ab(D_z c \cdot d) = (D_z a \cdot c)bd - ac(D_z b \cdot d); \tag{A.1}$$

$$(D_z^2 a \cdot b)cd - ab(D_z^2 c \cdot d) = D_z[(D_z a \cdot d)cb - ad(D_z c \cdot b)]; \tag{A.2}$$

$$D_x(D_x D_y a \cdot b) \cdot ab = D_y(D_x^2 a \cdot b) \cdot ab - D_x(D_y a \cdot b) \cdot (D_x a \cdot b); \tag{A.3}$$

$$(D_x D_t a \cdot a)b^2 - a^2(D_x D_t b \cdot b) = 2D_t(D_x a \cdot b) \cdot ba = 2D_x(D_t a \cdot b) \cdot ba; \tag{A.4}$$

$$(D_x^4 a \cdot a)b^2 - a^2(D_x^4 b \cdot b) = 2D_x(D_x^3 a \cdot b) \cdot ba - 6D_x(D_x^2 a \cdot b) \cdot (D_x a \cdot b); \tag{A.5}$$

$$\begin{aligned} &(D_x^3 a \cdot b)a'b' - ab(D_x^3 a' \cdot b') - 3(D_x^2 a \cdot b)(D_x a' \cdot b') + 3(D_x a \cdot b)(D_x^2 a' \cdot b') \\ &= (D_x^3 a \cdot a')bb' - aa'(D_x^3 b \cdot b') - 3(D_x^2 a \cdot a')(D_x b \cdot b') + 3(D_x a \cdot a')(D_x^2 b \cdot b'); \end{aligned} \tag{A.6}$$

$$D_x[(D_y a \cdot b) \cdot cd + (D_y c \cdot d) \cdot ab] = D_y[(D_x a \cdot d) \cdot cb - ad(D_x c \cdot b)]; \tag{A.7}$$

$$\begin{aligned} &D_x[(D_y a \cdot b) \cdot cd + (D_y c \cdot d) \cdot ab] + (D_x D_y a \cdot b) \cdot cd - (D_x D_y c \cdot d) \cdot ab \\ &= (D_x a \cdot b)(D_y c \cdot d) - (D_y a \cdot b)(D_x c \cdot d) + D_y[(D_x a \cdot d) \cdot cb]. \end{aligned} \tag{A.8}$$

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