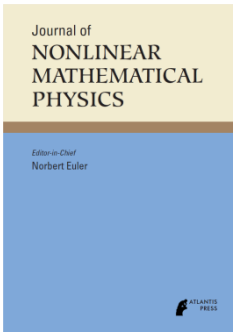

ATLANTIS PRESS

Journal of Nonlinear Mathematical Physics

ISSN (Online): 1776-0852 **ISSN (Print): 1402-9251**
Journal Home Page: https://www.atlantis-press.com/journals/jnmp

Preface to the Special Issue of Jnmp in Memory of F. A. Berezin

Dimitry Leites

To cite this article: Dimitry Leites (2010) Preface to the Special Issue of Jnmp in Memory of F. A. Berezin, Journal of Nonlinear Mathematical Physics 17:Supplement 1, v–viii, DOI: https://doi.org/10.1142/S1402925110000751

To link to this article: https://doi.org/10.1142/S1402925110000751

Published online: 04 January 2021

World Scientific
www.worldscientific.com

ATLANTIS PRESS

Journal of Nonlinear Mathematical Physics, Vol. 17, Suppl. 1 (2010) v–viii

DOI: 10.1142/S1402925110000751

PREFACE TO THE SPECIAL ISSUE OF JNMP IN MEMORY OF F. A. BEREZIN

DIMITRY LEITES
Dept. of Math., Stockholm University
Roslagsv. 101, Kräftriket hus 6
SE-106 91 Stockholm, Sweden
mleites@math.su.se

Felix Alexandrovich Berezin was born on April 25, 1931 and perished on July 14, 1980 (or several days before) under obscure circumstances while serving — during his summer holidays — as an auxiliary worker for a geological party near the Kolyma river.

Wikipedia currently lists several researchers as discoverers of what we now call *supersymmetry*. Interestingly, Berezin is not even mentioned in this Wikipedia article. However, none of the authors mentioned so far as "pioneers of supersymmetry" qualify as such: Before Wess and Zumino have explained the meaning and importance of supersymmetry none of the earlier discoverers did fully appreciate the meaning and value of their own works, and some of them do not realize its full meaning even now judging by their later works.

Contrariwise, Berezin definitely was the first to realize the real value of his discoveries, the fact that he **stands on the threshold of the new branch of mathematics, currently referred to as "supermathematics"** and guess its possible worthiness for physics, first consciously demonstrated by Wess and Zumino. In early 1960s, Berezin obtained a parallel description of the second quantization for Boze and Fermi particles summarized in his book [1] (where, among other things, Berezin introduces the Berezin integral, and gives a parallel description of the spinor and oscillator representations of, respectively, infinite dimensional orthogonal $\mathfrak{o}(2\infty)$ and symplectic Lie algebras $\mathfrak{sp}(2\infty)$, later rediscovered many times; for an overview of Berezin's works, see the "scientific part" of [5]).

In May 2007, World Scientific published the book "*FELIX BEREZIN: Life and Death of the Mastermind of Supermathematics*" edited by M. Shifman, [8]. The book consists of contributions of Berezin's friends, colleagues and students; recollections of Berezin as a person, and some scientific reviews of his results and ideas, their role in mathematical physics and their developments. The core of the book is the skillfully written heart-breaking story of Berezin's widow, Elena Karpel; and the two Berezin's letters — to the Rector of Moscow State University and the Moscow Mathematical Society — on the situation in these institutions and in mathematical community in the Soviet Union in general (the problems pointed at in these letters did not lose their timeliness today).

After the book had been published, it was difficult to continue to find excuses not to publish Karpel's article and these letters in the original Russian and, notwithstanding certain residual resistance of some of Berezin's colleagues, the publishing house of the Moscow Center of Continuous Mathematical Education[a] published a Russian version of the book [5] of recollections about Berezin. Together with Robert Minlos and with some help of the editors, Karpel compiled a collection about half of which constituted updated Russian originals of the book Shifman edited.

Shortly before MCCME published [5], MCCME also published Berezin's *Lectures on Statistical Physics* [2], the book which, in the opinion of those capable to appreciate its parallel description of Bose and Fermi particles, is still (more than 40 years after Berezin read these lectures) the most comprehensible book on the topic. The English version of the book is also available; it should be published soon by A. Salam School of Mathematical Sciences, Lahore.

The discussions preceding publication of these two books were unexpectedly (to me) heated, and evaluations of Berezin's lectures by different experts were totally opposite: From "the only understandable book on the topic I could find, the classical Landau-and-Lifshitz and all other sources are no match" to "purely standard material, of no interest except, perhaps, historical one". The opponents are definitely wrong: The first edition of [2] was an unedited preprint of meagre circulation, whereas the second edition which reproduced the preprint, a pirate one, immediately sold out.

Thus, Berezin's life was not easy (he was not even promoted to full professor at the time he perished; he was a senior researcher), his death violent, and even his legacy cannot be in peace yet, although being dead helps Berezin to make friends and holds some of the competing colleagues at bay and at leash.

These undercurrents of my editor's work on [5] forced me to speed up with my project [7], three decades overdue, and finish editing Berezin's posthumous book [3] — part of my contribution to the memory of Berezin. I was also encouraged to compile this memorial volume including in it papers Berezin might be interested in. This is my duty not only to Berezin but also to the authors waiting to see their results published while we all are still alive.

This collection, dedicated to Berezin's memory, contains the works of several of Berezin's students, and students of his students, or friends. Almost each paper has something unconventional, so I want to comment some of them in order to encourage the reader to work through them, and go further.[b]

(1) The date when Lie superalgebras were consciously identified as a new notion is unknown to me. This was not in 1970s in the works of physicists, as some think. I'd say this happened in 1930s, in the works by M. F. Bockstein or other topologists; definitely earlier than in 1940 which Wikipedia gives as the time when Schouten published a paper with a description of various Schouten brackets (one of them is now gained a great deal of currency as the *anti-bracket*). These first Lie superrings were either the sets of elements

[a]MCCME, and especially its publishing house, is no less famous now than the Independent University of Moscow, the two being inseparable.

[b]For example, the definition of the supermanifold AS BEREZIN WANTED IT (if one will ever be able to guess what was meant, same as with Leibniz's mysterious *Analysis Situs*) is not what I suggested in [6] and what is accepted now. For an explanation of this statement, see [7].

of homotopy groups under the Whitehead's product or the images of these sets under the Hurewicz homomorphism — the homology of manifolds with coefficients in $\mathbb{Z}/p$ (in modern terms, these are the $\mathbb{Z}/p$-points of certain supergroup schemes). These first Lie superrings (of functions on the supergroup schemes) or Lie superalgebras were defined over finite fields, i.e., the first discovered Lie superalgebras were examples of what is nowadays called *modular* Lie superalgebras.

Since Berezin was interested in topological invariants (see, e.g., his undeservingly neglected paper [4]) and Lie superalgebras, we include here our step towards solution of the super analog of the Kostrikin–Shafarevich conjecture — towards classification of simple finite dimensional Lie superalgebras over algebraically closed fields of characteristic $p = 2$. Related are "classical" questions neglected far too long, for example (all for $p = 2$, of course): What are equivalence classes of non-degenerate bilinear forms and Lie algebras preserving them? What are spinor representations of orthogonal Lie algebras? How many analogs of Lie algebras of Hamiltonian vector fields are there and what do they preserve? Are Poisson Lie algebras non-trivial central extensions of the Hamiltonian Lie algebras as is the case for $p \neq 2$? Several of these questions are answered in this issue, some are still open.

(2) The following topic is one more step in the direction of the above-mentioned classification. At the same time it might be of interest to physicists: Volichenko algebras — inhomogeneous with respect to parity subspaces of simple Lie superalgebras closed with respect to the bracket of their ambient — as analogs of Lie (super)algebras for particles more general than Bose and Fermi ones.

(3) With doubly extended (in a certain way) Dynkin diagrams associated are simple Lie algebras (we call them *almost affine*) which are being applied to the study of cosmic billiards. The classification of almost affine Lie superalgebras (hopefully needed to describe hidden supersymmetry of cosmic billiards and definitely related to Lorentz superalgebras studied by Borcherds as well as Gritsenko and Nikulin) was performed twice, both times with mistakes even in definitions. Here we publish a correct and unexpected result.

(4) Related is the description of the Hilbert–Poincaré series of the Weyl groups of the almost affine Lie algebras. This description is related to the famous Eneström's theorem and conjecturally points at a way to generalize it.

(5) The paper by Dynin (who recently claimed having solved the "Mass gap" problem) is, as all his papers, difficult to read but certainly contains interesting ideas. Perhaps the reader can make its results clearer, as was the case with other Dynin's papers, none of which was accused of having mistakes so far.

References

[1] F. A. Berezin, *The Method of Second Quantization* (Academic Press, 1966) (2nd edition, M. K. Polivanov (ed.), Nauka, Moscow, 1986, in Russian).

[2] F. A. Berezin, (3rd edn., corrected. Edited and with a foreword by D. Leites), *Lectures on Statistical Physics*. MCCME, 2008 (in Russian; for the English translation, see http://www.mis.mpg.de/publications/preprints/2006/prepr2006-157.html and ASSMS, Lahore, 2010; to appear).

[3] F. A Berezin (2nd edn., corrected. Edited and with a foreword by D. Leites) *Introduction to Supercalculus. With Appendices from Seminar on Supersymmetries*, Vol. $3\frac{1}{2}$ (MCCME, 2010) (in Russian).

[4] F. A. Berezin and V. Retakh, *Rep. Math. Phys.* **17**(3) (1980) 363–378.

[5] E. Karpel and R. Minlos, (compilers); D. Leites and I. Tyutin (eds.), *Recollections on Felix Aleksandrovich Berezin — the Founder of Supermathematics* (MCCME, 2009), (in Russian).
[6] D. Leites, Spectra of graded-commutative rings, *Uspehi Matem Nauk* **29**(3) (1974) 209–210 (in Russian).
[7] D. Leites (ed.), *Seminar on Supersymmetries*, Vol. 1 Algebra and Calculus: main facts; Vol. 2 Lie superalgebras; Vol. 3 Additional chapters (MCCME, 2010), (in Russian) ca 1000 pp.
[8] M. Shifman (ed.), *FELIX BEREZIN: Life and Death of the Mastermind of Supermathematics* (World Scientific, 2007).