

Research Article

An Intuitionistic Fuzzy Time Series Model Based on New Data Transformation Method

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ABSTRACT

Traditional time series methods can predict seasonal problems, but not problems with transferred linguistic data. Thus, a forecasting method for such problems is required. However, existing intuitionistic fuzzy time series forecasting methods lack persuasiveness in determining the degree of hesitation and the lengths of intervals. Hence, this research is mainly to explore how to decide the degree of hesitation for each interval for intuitionistic fuzzy time series. This paper proposes the weighted intuitionistic fuzzy time series model based on the Nth quantile discretization approach (NQDA). The proposed model can decide the appropriate number, interval length, degree of hesitation, and membership and nonmembership functions of linguistic values on the basis of the training data. In the experimental section, the forecasts of several data sets are made for model validation. Results indicate that the proposed model can be used to obtain forecasts for other time-related data sets.

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1. INTRODUCTION

Trend forecasting plays an important role in many fields, such as decision-making, health, economics, and commerce [1]. Therefore, numerous fuzzy time series (FTS) models have been proposed to deal with forecasting problems [2–5]. In these studies, granular computing plays an important role in data fuzzification process [6,7]. Previously proposed methods usually lack persuasiveness in determining the universe of discourse and the lengths of intervals [2,4,8]. Moreover, these methods do not consider the strength of the fuzzy relation when making forecasts [9].

Huang [10] pointed out that the length of the interval affects the prediction performance. Chen [11] proposed the high-order FTS model, focusing on three main issues: (1) fuzzification, (2) fuzzy logic relations, and (3) defuzzification. In the study proposed by Mahua and Kalyani [12], the forecasting accuracy depends on two key points: (1) effective division of data and (2) establishment of fuzzy logic relationships and use in forecast [13]. On the basis of the above research, the width and number of intervals and the method of fuzzification influence the prediction performance.

Most previous experiments of fuzzy sets used a single value to express human opinions. However, ambiguous blurring is hard to express in a single value. In 1986, Atanassov [14] extended the fuzzy

theory and proved that the intuitionistic fuzzy set (IFS) theory can improve the defect of a single value. The IFS is an extension of the fuzzy set, which has a certain effect on handling uncertain problems. Castillo *et al.* [15] incorporated the IFS theory into time series analysis. Its main purpose is to improve the prediction accuracy of the model. Their experimental results confirmed that the intuitionistic fuzzy time series (IFTS) is another way to make forecasts.

After the work by Castillo *et al.* [15], an increasing number of studies in recent years have extended FTS to IFTS [16–20]. However, most of these studies focused on improving prediction results and did not consider uncertainty or hesitation when facing forecasting problems. For example, Egrioglu *et al.* [19] integrated particle swarm optimization into the process of IFTS to improve the weights of rules. In 2018, Danish Lohani *et al.* [18] adopted the intuitionistic fuzzy C-means to obtain reasonable IFS. Luo *et al.* [20] proposed the integration with IFS and neural networks to enhance prediction performance. Most of these studies also prove that IFTS has better prediction results than traditional FTS.

Most IFTS models only consider the influence of the degree of attribution on the predicted value and ignore the degree of non-attribution and the degree of hesitation. In addition, most studies used subjective opinions to calculate the degree of hesitation, and some of them set the degree of hesitation to “0.” Moreover, even though several approaches did not set the degree of hesitation to “0,” they set a default value and a consistent degree of hesitation of each

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interval. It will let the degree of hesitation having no effect when establishing the intuitionistic fuzzy rules.

To solve the aforementioned limitations for previous models, this study proposes an IFTS model based on the N th quantile discretization approach (NQDA) [8]. The functions of the proposed model are as follows: (1) it can obtain IFS in accordance with the extension from fuzzy sets. The degrees of memberships and nonmembership of IFS are considered in calculating the average membership values and minimum degrees of training data. (2) The degrees of memberships and nonmembership of IFS are also integrated when obtaining intuitionistic fuzzy rules and then obtain reasonable and precise forecasting results. (3) In the determination of interval length, the logarithm value of the number of data is calculated for obtaining the number of intervals. The coverage of granular computing is reasonable, and the numbers of data of the linguistic values are similar. (4) The transformation equation from fuzzy sets to IFSs is developed for the forecasting of time series data. NQDA calculates the cut point by the observation value of each interval and thus is more objective than the conventional method.

In the experimental section, the yearly data on enrollments at the University of Alabama comprise the first dataset. The seasonal data of Taiwan's total electricity consumption for the past 22 years are used and compared with the previously proposed FTS to verify the advantages and disadvantages of the improved model. The 2000–2004 data of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) are used to evaluate the performance of the proposed IFTS model. The expenditure in information technology maintenance by an optoelectronics company in Taiwan is also adopted for verification.

The remainder of this paper is organized as follows: The related literature is briefly reviewed in Section 2. The IFTS model and its step-by-step procedure are presented in Section 3 and the experiments are described in Section 4. The conclusions, implications, and limitations of this research are presented in Section 5.

2. RELATED LITERATURE

In this section, the literature related to IFTS and NQDA is briefly reviewed.

2.1. IFS and IFTS

Zadeh [21] proposed fuzzy sets to describe situations in which the data are imprecise or vague. Fuzzy set theory is a useful tool to handle such situations by attributing a degree to which a certain object belongs to a set. In real life, a person may assume that an object belongs to a set to a certain degree, but he may be unsure about it. In other words, the membership degree of in A may have hesitation or uncertainty. Fuzzy set theory does not consider hesitation in the membership degrees [22]. A possible solution is to use IFSs, defined by Atanassov in 1983 [23]. IFSs use a degree of truth membership function $u_A(x)$ and one of falsity membership function $v_A(x)$ to represent lower bound ($u_A(x)$) and upper bound ($1 - v_A(x)$) such that $u_A(x) + v_A(x) \leq 1$. The interval $[u_A(x), 1 - v_A(x)]$ can extend the fuzzy set of membership function by complementing the membership degree with a nonmembership degree that expresses to what extent the element does not belong to the IFS. The uncertainty

or hesitation of x can be quantified for each x in A by the length of the interval $\pi_A(x) = 1 - v_A(x) - u_A(x)$. If the $\pi_A(x)$ is little, it represent we are more certainly about x . If $\pi_A(x)$ is great, it represent we are more uncertainly about x . When $u_A(x) = 1 - v_A(x)$ for all elements of the universe, the traditional fuzzy set concept is recovered. Figure 1 shows an IFS of real number R [24].

Definition 2.1. Assume A is the subset of universal of discourse. Then, an IFS A is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\} \quad (1)$$

where $u_A(x)$ is a degree of truth membership function and $v_A(x)$ is degree of falsity membership and such that $u_A(x) + v_A(x) \leq 1$.

Definition 2.2. Let x and y are two intuitionistic fuzzy sets over U and V , respectively. Then, the intuitionistic fuzzy relation between x and y is defined as

$$R = \{\langle (x, y), \mu_R(x, y), \nu_R(x, y) \rangle \mid (x, y) \in U \times V\} \quad (2)$$

Song and Chissom [25,26] proposed a FTS model that uses linguistic values for forecasting data. They executed experiments on the enrollment data of the University of Alabama to determine the differences between traditional time series and FTS models. Chen [27] proposed a simplified arithmetic operations model with max-min composition operations to improve the approach of Song and Chissom [28]. In both aforementioned studies, the enrollment data of the University of Alabama were used to make forecasts.

Based on the concepts of FTS, Castillo *et al.* [15] proposed the IFTS model to improve the prediction accuracy and experimental results of the study. Tian and Wang [29] also proposed their definition of IFTS and improved the FTS prediction model, which included a weighting function to make the model applicable for the IFS. In the following, some basic concepts of IFTS are briefly reviewed.

Definition 2.3. Assume that $Y(t)$ ($t = \dots, 0, 1, 2, \dots$), is a subset of R . Let $Y(t)$ be the universe of discourse defined by intuitionistic fuzzy set $f_i(t)$. If $F(t)$ consists of $f_i(t)$ ($t = 1, 2, \dots$), $F(t)$ is defined as a IFTS on $Y(t)$ ($t = \dots, 0, 1, 2, \dots$). Then,

$$F(t) = \frac{\langle u_1(Y(t)), v_1(Y(t)) \rangle}{w_1} + \frac{\langle u_2(Y(t)), v_2(Y(t)) \rangle}{w_2} + \dots + \frac{\langle u_n(Y(t)), v_n(Y(t)) \rangle}{w_n} \quad (3)$$

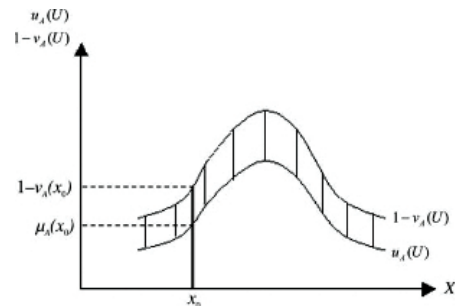


Figure 1 | Intuitionistic fuzzy sets of real number R [24].

Definition 2.4. If there is a intuitionistic fuzzy relationship $R(t-1, t)$, such that $F(t) = F(t-1) \times R(t-1, t)$. Where \times represents an operator, then $F(t)$ is said to be caused by $F(t-1)$. (Note that the operator can be another arithmetic operator.)

When $F(t-1) = A_i$ and $F(t) = A_j$, the relationship between $F(t-1)$ and $F(t)$ (called a intuitionistic fuzzy logical relationship (IFLR) is denoted by $A_i \rightarrow A_j$ where A_i is called the left-hand side (LHS) and A_j the right-hand side (RHS) of the IFLR.

Many studies about IFTS have been focused on dealing with the time series problems. Xu [30] defined the score and accuracy functions of interval-valued intuitionistic fuzzy number. A method for ranking interval-valued intuitionistic fuzzy numbers is presented based on these two functions, and then the method is applied for forecasting temperature. Kumar and Gangwar [31] proposed a novel IFTS model to forecast the enrollment of Alabama University. They also adopted the model to make forecasts about the stock index of SBI. Abhishekh and Kumar [16] proposed a new high-order IFTS model by transforming FTS data into intuitionistic FTS data by defining their appropriate membership and nonmembership grades. The fuzzification of time series data is intuitionistic fuzzification, which is based on the maximum score degree of intuitionistic fuzzy numbers. This method has been implemented on the prediction for historical data of rice production.

2.2. Nth Quantile Discretization Approach: NQDA [8]

In recent years, several FTS models that integrate new fuzzified methods have been proposed to improve forecasting accuracy levels. In this study, we adopt a relative objective method, namely NQDA. The detailed description of this method is provided in the following text:

- Constructing the length of interval using the NQDA

Step 1. Define the universe of discourse U . Let $U = [D_{min} - D_1, D_{max} + D_2]$ where D_{min} and D_{max} denote the minimum and maximum values of the historical data, respectively, and D_1 and D_2 denote two feasible values.

Step 2. Assume we want to partition the universe of discourse into m linguistic values. Then, we need to calculate $m - 1$ intervals for these linguistic values. The average observation quantity of each interval (Ave) is computed by

$$Ave = \frac{m}{l-1} \quad (4)$$

where m denotes the total number of linguistic values. The cut point is computed by

$$\text{Cut_Point}(i) = \begin{cases} \text{Value}(Q(i)), & \text{If } Q(i) \in N \\ \frac{\text{Value}(Q(i)) + \text{Value}(Q(i+1))}{2}, & \text{If } Q(i) \notin N \end{cases} \quad (5)$$

$i = 2, 3, \dots, (m-1)$

$$Q(i) = \lfloor Ave \times i \rfloor, i = 2, 3, \dots, (m-1) \quad (6)$$

where $\text{Value}(Q(i))$ denotes the $Q(i)$ -th sorted value of training data, i denotes the i th cut point, and $Q(i)$ is calculated by Equation (6).

3. PROPOSED IFTS MODEL

Most FTS models have two major limitations [2,4]: (1) they lack persuasiveness in determining the universe of discourse and the lengths of intervals and (2) they do not appropriately consider the weights of fuzzy relations. Above limitations are also existed in the IFTS models. The degrees of nonmembership and uncertainty are not considered when making forecasts. Nevertheless, some IFTS approaches can be used to make forecasts according to the degrees of membership, nonmembership, and uncertainty. However, they set the same degrees of uncertainty for each set. In addition, the degrees of uncertainty are zero values or decide it subjectively.

To overcome these limitations, a weighted intuitionistic fuzzy relations time series model using the NQDA method is proposed to improve membership and non-membership functions. However, most of previous FTS and IFTS models did not suggest a feasible number of linguistic values. The logarithm value of the number of data is calculated to determine the number of linguistic values on the basis of the number of training data. The concept of weighted intuitionistic fuzzy relations is also integrated into the proposed procedure to improve forecasting accuracy levels. The weight of each intuitionistic fuzzy relation is calculated according to the sum of cardinalities in the training data set.

The steps of this paper can be divided into four steps: (1) obtain the number and intervals of linguistic values, (2) intuitionistic fuzzify the historical data, (3) develop the weighting intuitionistic fuzzy rules, and (4) defuzzification and making the forecasts. The output of the first step is mainly executed by NQDA. In the second step, a transformation equation is proposed in this research to obtain membership and nonmembership functions. The nonmembership value of each data points is the inverse of its membership value; thus, the maximal degree of membership function is calculated by the averaged membership values of data with positive membership values in the interval of each linguistic value.

Step 1. To obtain the number and intervals of linguistic values.

Step 1.1. Define the universe of discourse U . Let $U = [D_{min} - D_1, D_{max} + D_2]$ where D_{min} and D_{max} denote the minimum and maximum values of the historical data, respectively, and D_1 and D_2 denote two feasible values.

Step 1.2. Determine the number of linguistic values by Equation (7). The Lagrange function is adopted to make the number of linguistic values reasonable and at least one data fuzzified in each interval. If the base of this Lagrange function is larger, the length of interval will be larger and the number of linguistic values will be less. After some pretest experiments, we

found that the number of linguistic values and the root mean square errors (RMSEs) for several data sets are converged and stable when the bases approximate $\frac{\epsilon}{2}$. Hence, we suggest to set the base to $\frac{\epsilon}{2}$.

$$l = \left\lceil \log_{\frac{\epsilon}{2}} n \right\rceil \pm 2 \quad (7)$$

where l is the number of interval and n is the number of training data.

- Step 1.3. Assume we want to partition the universe of discourse into m linguistic values. Then, we need to calculate $m - 1$ intervals for these linguistic values. The average observation quantity of the each interval (Ave) is computed by Equation (4). Then, the cut point of each linguistic value can be computed by Equations (5) and (6).

Before calculating the degree of nonmembership, the average degree of membership and the highest degree of membership for each interval must first be calculated. The operation steps are as follows:

- Step 2. To develop the weighting intuitionistic fuzzy rules

- Step 2.1. Calculate the average degree of membership for linguistic value i (i.e., $height_mem(i)$). Adding and averaging the membership of the same interval, the purpose of which is to calculate the highest value of the membership function for IFS by Equation (8). Then, the membership function of IFS can be defined as

$$\mu_A(x) = \quad (8)$$

$$\begin{cases} 0 & , x < a \\ \frac{x-a}{b-a} \times height_mem(i) & , a \leq x < b \\ height_mem(i) & , x = b \\ \frac{c-x}{c-b} \times height_mem(i) & , b < x \leq c \\ 0 & , x > c \end{cases}$$

where a , b , and c are left, highest, and right of IFS, and $height_mem(i)$ is the highest value of the membership function for IFS, which is calculated by the average degree of FS for training data.

- Step 2.2. Calculate the height of membership function for IFS. The average degrees of membership values for each linguistic value are calculated.

- Step 2.3. Calculate the degree of uncertainty of IFS. To determine the uncertainty of each IFS, we determine the uncertainty (i.e., π_A) by the minimum nonzero membership value of the training data in Step 2.1. Then, according to definition, the height of nonmembership of each intuitionistic fuzzy linguistic value can be calculated by $V_A(x) = 1 - \pi_A(x) - \mu_A(x)$.

However, the value of $\pi_A(x) + \mu_A(x)$ might be larger than “1,” which is conflicted with the definition of 2.1, because $V_A(x)$ will be less than 0. For example, if there is just one training data in linguistic value i , and its membership value of FS is 0.6. Then the minimum (i.e., $\min(mem(x))$) and average membership (i.e., $height_mem(i)$) values are both equal to 0.6. Hence, the function of uncertainty is calculated by Equation (9) to avoid this circumstance. Then, the nonmembership of IFS can be calculated using Equation (10).

$$\pi(i) = \begin{cases} [\min(mem(i)) | \forall mem > 0] \\ \text{if } (\min[mem(i)] + height_mem(i)) \leq 1 \\ 1 - \mu \\ \text{if } (\min[mem(i)] + height_mem(i)) > 1 \end{cases} \quad (9)$$

where i denotes the i th linguistic value and $\min(mem(i))$ is the minimum nonzero membership value for training data in linguistic value i .

$$nomem(i) = 1 - mem(i) - \pi(i) \quad (10)$$

According to the outputs of Steps 2.1 to 2.3, each training data can be intuitionistic fuzzified, and its degree of membership and nonmembership of each linguistic value can be expressed as (11) and (12). Hence, the intuitionistic fuzzy relationship can be obtained, and then the weighted intuitionistic fuzzy rules are established. The steps are as follows:

$$mem(x) = \begin{matrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{matrix} \begin{bmatrix} \mu \\ \mu \\ \vdots \\ \mu \end{bmatrix}_{n \times 1} \quad (11)$$

$$1 - nomem(x) = \begin{matrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{matrix} \begin{bmatrix} 1 - nomem_{11} \\ 1 - nomem_{21} \\ \vdots \\ 1 - nomem_{n1} \end{bmatrix}_{n \times 1} \quad (12)$$

where μ is the intuitionistic membership value and γ is the intuitionistic nonmembership value. Assuming that a fuzzy semantic is fuzzification to two intervals of L_2 and L_3 then the second and third positions in the matrix of $mem(x)$ will be placed in the ambiguous meaning of the two intervals, and the rest of the positions will be zero.

Step 3. To develop the weighting intuitionistic fuzzy rules

Step 3.1. Establishing an intuitionistic fuzzy relationship. The output values of each training data for Equations (11) and (12) are calculated. The intuitionistic fuzzy relationship will be obtained according to Definition 2.4.

Step 3.2. Calculating intuitionistic cardinality weights for membership and nonmembership relations. The outputs can be shown as Equations (13) and (14) by calculating the sum of cardinalities for each intuitionistic fuzzy relationship.

$$R_{mem(x)} = \begin{matrix} & L_1 & L_2 & \cdots & L_n \\ \begin{matrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{matrix} & \begin{bmatrix} mem_{11} & mem_{12} & \cdots & mem_{1n} \\ mem_{21} & mem_{22} & & \\ \vdots & & \ddots & \vdots \\ mem_{n1} & & \cdots & mem_{nn} \end{bmatrix} & L_{n \times n} \end{matrix} \quad (13)$$

$$R_{1-nomem(x)} = \begin{matrix} & L_1 & L_2 & \cdots & L_n \\ \begin{matrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{matrix} & \begin{bmatrix} 1 - nomem_{11} & 1 - nomem_{12} & \cdots & 1 - nomem_{1n} \\ 1 - nomem_{21} & 1 - nomem_{22} & & \\ \vdots & & \ddots & \vdots \\ 1 - nomem_{n1} & & \cdots & 1 - nomem_{nn} \end{bmatrix} & L_{n \times n} \end{matrix} \quad (14)$$

where $R_{mem(x)}$ and $R_{1-nomem(x)}$ are the matrices representing the belonging weight and the investing weight.

Step 3.3. Establishing weighted intuitionistic fuzzy rule matrix. After the integration for all the intuitionistic fuzzy relation matrix is finished, the weighted intuitionistic fuzzy rule matrix is made.

Step 4. To intuitionistically defuzzification and make the forecasts. This step calculates the intuitionistic defuzzified values of linguistic values using Equation (15). Then, the forecasting value of $t + 1$ (i.e., $F(t + 1)$) can be calculated

by Equation (17) according to Equation (16).

$$D(L_j) = \frac{a_{L_j} + b_{L_j} + c_{L_j}}{3} \quad (15)$$

$$C = \begin{matrix} & C_1 \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} & \begin{bmatrix} C_{11} \\ C_{21} \\ \vdots \\ C_{n1} \end{bmatrix}_{n \times 1} \end{matrix}, \quad \text{where } C_{i1} = \begin{cases} 1 & \text{if } mem(x_i) > 0 \\ 0 & \text{if } mem(x_i) = 0 \end{cases} \quad (16)$$

$$F(t+1) = \frac{\frac{[D(L_j)]_{1 \times n} \times [R_{mem}^T]_{n \times n} \times [mem(x)]_{n \times 1}}{[1, 1, \dots, 1]_{1 \times n} \times [R_{mem}^T]_{n \times n} \times [C]_{n \times 1}} - \frac{[D(L_j)]_{1 \times n} \times [R_{1-nomem}^T]_{n \times n} \times [1-nomem(x)]_{n \times 1}}{[1, 1, \dots, 1]_{1 \times n} \times [R_{1-nomem}^T]_{n \times n} \times [C]_{n \times 1}}}{[1, 1, \dots, 1]_{1 \times n} \times ([mem(x)]_{n \times 1} - [1-nomem(x)]_{n \times 1})} \quad (17)$$

where C_i represents a conditional matrix. If the membership and the nonmembership values are larger than zero in the matrix of Equations (11) and (12), the corresponding value of C_i will be assigned as 1, otherwise, it will be assigned as zero.

4. COMPARISON AND VERIFICATION

Experiments were performed on four data sets in this study by using the proposed models. The results and comparisons are provided in Sections 4.1, 4.2, 4.3, and 4.4.

4.1. Data Set One: Enrollments at the University of Alabama

The numerical data is the yearly data on enrollments at the University of Alabama, which has been adopted in many FTS models [2,4,25,28]. The original data are listed in Table 1.

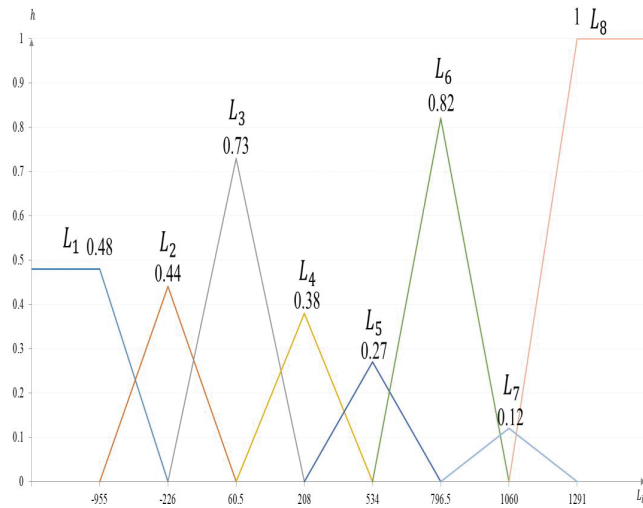
Eight IFSs were obtained after data transformation, and their heights of linguistic values are listed in Table 2 and shown in Figure 2. Then, following Step 2.3, the training data are subjected to intuitionistic fuzzification and listed in Table 3. The intuitionistic fuzzy relations can be obtained according to Table 3 and some of them are listed in Table 4.

Table 1 The yearly enrollments of the University of Alabama [25,28].

Year	Enrollments	Year	Enrollments
1971	13055	1982	15433
1972	13563	1983	15497
1973	13867	1984	15145
1974	14696	1985	15163
1975	15460	1986	15984
1976	15311	1987	16859
1977	15603	1988	18150
1978	15861	1989	18970
1979	16807	1990	19328
1980	16919	1991	19337
1981	16388	1992	18876

Table 2 The height for each intuitionistic fuzzy set (IFS).

Interval	Height (L_i)
L_1	0.48
L_2	0.44
L_3	0.73
L_4	0.38
L_5	0.27
L_6	0.82
L_7	0.12
L_8	1.00

**Figure 2** The graph of height for each intuitionistic fuzzy set (IFS).

Then, the membership and nonmembership value can be calculated by Equations (8–10). The intuitionistic fuzzy relations can be obtained according to Table 3 and Definition 2.4, and some of them are listed in Table 4. In Table 4, the number of intuitionistic fuzzy relations for each nearby years is four ($2 * 2 = 4$), because the data in Table 3 have two linguistic values.

The intuitionistic cardinality weights for membership and nonmembership relations in Table 4 can be calculated using Step 3.2. The outputs of Step 3.2 are shown in Equations (18) and (19), which are the intuitionistic fuzzy rules for forecasting.

Then, the forecasted results of proposed model for data set one are calculated, which are provided to make comparisons with Tian and Wang's model [29]. In Tian and Wang's approach, a method of transferring FS to IFS is proposed, and then several previous

Table 3 The outputs of intuitionistic fuzzification for training data.

Date	Momentum	Linguistic Value ($L_i, mem(x), nomem(x)$) \cup ($L_j, mem(x), nomem(x)$)
1971	*	*
1972	508	($L_4, 0.03, 0.96$) \cup ($L_5, 0.25, 0.72$)
1973	304	($L_4, 0.27, 0.72$) \cup ($L_5, 0.08, 0.89$)
1974	829	($L_6, 0.72, 0.00$) \cup ($L_7, 0.01, 0.99$)
1975	764	($L_5, 0.03, 0.94$) \cup ($L_7, 0.72, 0.00$)
1976	-149	($L_2, 0.32, 0.68$) \cup ($L_3, 0.20, 0.60$)
1977	292	($L_4, 0.28, 0.71$) \cup ($L_5, 0.07, 0.90$)
1978	258	($L_4, 0.32, 0.67$) \cup ($L_5, 0.04, 0.93$)
1979	946	($L_6, 0.35, 0.30$) \cup ($L_7, 0.07, 0.93$)
1980	112	($L_3, 0.48, 0.32$) \cup ($L_4, 0.13, 0.86$)
1981	-531	($L_1, 0.20, 0.72$) \cup ($L_2, 0.26, 0.74$)
1982	-955	($L_1, 0.48, 0.44$) \cup ($L_2, 0.00, 1.00$)
1983	64	($L_3, 0.71, 0.09$) \cup ($L_4, 0.01, 0.98$)
1984	-352	($L_1, 0.08, 0.84$) \cup ($L_2, 0.36, 0.64$)
1985	18	($L_2, 0.07, 0.93$) \cup ($L_3, 0.62, 0.18$)
1986	821	($L_6, 0.74, 0.00$) \cup ($L_7, 0.01, 0.99$)
1987	875	($L_6, 0.58, 0.07$) \cup ($L_7, 0.04, 0.96$)
1988	1291	($L_7, 0.00, 1.00$) \cup ($L_8, 1.00, 0.00$)
1989	820	($L_6, 0.75, 0.00$) \cup ($L_7, 0.01, 0.99$)
1990	358	($L_4, 0.21, 0.78$) \cup ($L_5, 0.12, 0.85$)
1991	9	($L_2, 0.08, 0.92$) \cup ($L_3, 0.60, 0.20$)
1992	-461	($L_1, 0.15, 0.77$) \cup ($L_2, 0.30, 0.70$)

*: doesn't exist

FTS models are transferred to IFTS models (e.g., Chen [27], Cheng *et al.* [4], and Chang and Huang [32]). Besides, the results of one recent FTS [33] and one IFTS [17] approaches are also adopted for comparison in Table 5. The direction accuracy (DA) and RMSE obtained with these models of and the forecasts obtained with the proposed model are listed in Table 5. The proposed models with $\pi \geq 0$ or $\pi > 0$ are both better than other models in Table 5.

$$R_{mem} = \begin{matrix} & L_1 & L_2 & L_3 & L_4 & L_5 & L_6 & L_7 & L_8 \\ \begin{matrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \end{matrix} & \begin{bmatrix} 0.2 & 0.07 & 0.56 & 0.01 & 0 & 0 & 0 & 0 \\ 0.34 & 0.15 & 0.36 & 0.28 & 0.07 & 0.07 & 0.01 & 0 \\ 0.43 & 0.92 & 0 & 0.2 & 0.07 & 0.62 & 0.01 & 0 \\ 0.14 & 0.22 & 0.21 & 0.31 & 0.07 & 0.59 & 0.08 & 0 \\ 0 & 0.11 & 0.15 & 0.32 & 0.12 & 0.12 & 0.05 & 0 \\ 0 & 0.32 & 0.55 & 0.34 & 0.15 & 1.3 & 0.04 & 0.58 \\ 0 & 0 & 0.07 & 0.08 & 0.02 & 0.02 & 0.01 & 0.04 \\ 0 & 0 & 0 & 0 & 0 & 0.75 & 0.01 & 0 \end{bmatrix} \end{matrix} \quad L_{8 \times 8} \quad (18)$$

$$R_{1-nomem} = \begin{matrix} & L_1 & L_2 & L_3 & L_4 & L_5 & L_6 & L_7 & L_8 \\ \begin{matrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \end{matrix} & \begin{bmatrix} 0.28 & 0.07 & 0.72 & 0.02 & 0 & 0 & 0 & 0 \\ 0.34 & 0.15 & 0.36 & 0.29 & 0.1 & 0.07 & 0.01 & 0 \\ 0.67 & 0.92 & 0 & 0.29 & 0.1 & 0.82 & 0.01 & 0 \\ 0.16 & 0.24 & 0.22 & 0.33 & 0.11 & 0.61 & 0.08 & 0 \\ 0 & 0.14 & 0.21 & 0.38 & 0.18 & 0.18 & 0.08 & 0 \\ 0 & 0.32 & 1.08 & 0.36 & 0.21 & 1.93 & 0.04 & 0.93 \\ 0 & 0 & 0.07 & 0.08 & 0.02 & 0.02 & 0.01 & 0.04 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.01 & 0 \end{bmatrix} \end{matrix} \quad L_{8 \times 8} \quad (19)$$

Table 4 | The one period intuitionistic fuzzy relation by proposed model.

Year ($t - 1$) \rightarrow Year (t)	Momentum(t)	Relation No.	One Period Intuitionistic Fuzzy Relation
			$(L_i, mem(x), 1 - nomem(x)) \rightarrow (L_j, mem(x), 1 - nomem(x))$
1972 \rightarrow 1973	508 \rightarrow 304	1	$(L_4, 0.03, 0.04) \rightarrow (L_4, 0.27, 0.28)$
		2	$(L_4, 0.03, 0.04) \rightarrow (L_5, 0.08, 0.11)$
		3	$(L_5, 0.25, 0.28) \rightarrow (L_4, 0.27, 0.28)$
		4	$(L_5, 0.25, 0.28) \rightarrow (L_5, 0.08, 0.11)$
1973 \rightarrow 1974	304 \rightarrow 829	5	$(L_4, 0.27, 0.28) \rightarrow (L_6, 0.72, 1.00)$
		6	$(L_4, 0.27, 0.28) \rightarrow (L_7, 0.01, 0.01)$
		7	$(L_5, 0.08, 0.11) \rightarrow (L_6, 0.72, 1.00)$
		8	$(L_5, 0.08, 0.11) \rightarrow (L_7, 0.01, 0.01)$
1974 \rightarrow 1975	829 \rightarrow 764	9	$(L_6, 0.72, 1.00) \rightarrow (L_5, 0.03, 0.06)$
		10	$(L_6, 0.72, 1.00) \rightarrow (L_6, 0.72, 1.00)$
		11	$(L_7, 0.01, 0.01) \rightarrow (L_5, 0.03, 0.06)$
		12	$(L_7, 0.01, 0.01) \rightarrow (L_6, 0.72, 1.00)$
\vdots	\vdots	\vdots	\vdots
1991 \rightarrow 1992	9 \rightarrow -461	77	$(L_2, 0.08, 0.08) \rightarrow (L_1, 0.15, 0.23)$
		78	$(L_2, 0.08, 0.08) \rightarrow (L_2, 0.30, 0.30)$
		79	$(L_3, 0.60, 0.80) \rightarrow (L_1, 0.15, 0.23)$
		80	$(L_3, 0.6, 0.80) \rightarrow (L_2, 0.30, 0.30)$

Table 5 | Comparisons of forecasting performance (enrollments at the University of Alabama).

Methods	Discretization Method		RMSE	DA (%)
Tian and Wang [29]	Song and Chissom [25]	IFTS	638.91	76.19
		Weighted IFTS	521.95	80.95
	Chen [27]	IFTS	634.75	71.43
		Weighted IFTS	516.94	76.19
Jiang et al. [33]	Lee et al. [34]	IFTS	621.78	71.43
		Weighted IFTS	505.45	76.19
		Harmony search intelligence algorithm	395.88	77.78
Abhishekh et al. [17]	Maximum degree of score function		382.03	66.67
Proposed model	NQDA (7 intervals) [8]	$\pi \geq 0$	346.60	95.24
		$\pi > 0$	348.56	95.24

DA, direction accuracy; RMSE, root mean square error; IFTS, intuitionistic fuzzy time series; NQDA, Nth quantile discretization approach.

4.2. Data Set Two: Seasonal Total Electricity Consumption in Taiwan

The second experimental data in this study are the seasonal total electricity consumption in Taiwan from 1996 to 2017, which are introduced by Chang et al. [35]. The NQDA is firstly adapted to performed fuzzification (i.e., Step 1). Then, the degrees of membership and nonmembership are calculated according to Step 3.

The NQDA-based FTS model is included for comparisons to verify the model proposed in this study. The original data and their momentums are presented in Table 6.

The experimental results are shown in Table 7. The RMSE and prediction of the correct direction rate (DA) with 5, 7, and 9 linguistic values are calculated and compared with those obtained by Chang et al. [35]. This study has lower RMSE and higher DA than the FTS model under the same number of linguistic values. The results in

Table 6 | The seasonal total electricity consumption in Taiwan [35].

Year(Season)	Total Electricity Consumption	Momentum
1996 (one)	28909.04	*
1996 (two)	32,354.14	3,445.10
1996 (three)	37,954.53	5,600.39
1996 (four)	35,089.06	-2,865.46
1997 (one)	30,993.07	-4,096.00
\vdots	\vdots	\vdots
2017 (one)	58,118.15	-7,249.80
2017 (two)	63,514.35	5,396.20
2017 (three)	72,377.75	8,863.40
2017 (four)	67,382.55	-4,995.20

*: doesn't exist

Table 7 show that proposed IFTS model is mostly superior to the FTS model under the same number of intervals (fuzzify by NQDA).

Table 7 Comparison of Taiwan's total electricity consumption performance.

Methods	No. of Linguistic Values	RMSE	DA (%)
Chang <i>et al.</i> [35]	5	8382.72	33
	7	8614.59	61
	9	8334.11	41
Proposed IFTS model	5	5956.24	78
	7	6168.62	63
	9	6306.73	49

DA, direction accuracy; RMSE, root mean square error; IFTS, intuitionistic fuzzy time series.

4.3. Data Set Three: TAIEX

The third data set is the daily closing price of TAIEX from 2000 to 2004. Considering that some previous FTS methods have been applied to make forecasts for data set three, this paper also make forecasts for comparing the performance via RMSE and DA. The main objectives of this experiment include the testing of forecasting performance and the feasibility of the number of linguistic values calculated using Step 1.2 and Equation (7). Thus, the closing price of TAIEX will be intuitionistically fuzzified to 16 linguistic values. Finally, the forecasting results of RMSE from 2000 to 2004 obtained by the proposed model and some previous FTA models are listed in Table 8. In Table 8, the best RMSE can be obtained under 16 IFSs when forecasting the data of different years. The proposed model has better forecasting performance in 2001 and 2003, and its average RMSE is lower than those of the other models, while the DA is an important judgement for the stock trading strategy.

4.4. Data Set Four: IT Project Expenditure Data

The data set four was illustrated by Cheng *et al.* [4], which is expenditure in information technology maintenance by an optoelectronics company involved in the production and marketing of fiber optic communication devices. The monthly data of project expenditures on information technology maintenance are presented in Table 10. The number of intervals is 11, which is calculating by Equation (7). After executing proposed forecasting process, the RMSE and DA by different models are listed in Table 11

5. CONCLUSION

Relative objective fuzzification was adopted to improve persuasiveness in determining the lengths of intervals and MFs and non-MFs. The step-by-step procedures of the proposed weighted ITFS model are provided in Section 3. The main differences between previous models and the proposed model is that the proposed model integrates the concepts of average fuzzified values to determine the membership and nonmembership and calculates the number of linguistic values on the basis of the number of training data. The main characteristic is that the transformation equation from fuzzy sets to IFSs is developed based on the training data for the forecasting of time series data. NQDA calculates the cut point by the

observation value of each interval and thus is more objective than the conventional method. The forecasting results of three data sets indicate that the proposed model provides accuracy levels superior to those of other methods.

In the experimental section, the Tables 3 and 4 illustrates the outputs of dataset one according to Steps 2.1–2.3. Then, the forecasting performances of four datasets are listed in Tables 5, 7–9, and 11. In Tables 5, 7, and 11, the RMSE and DA are both better than comparison models. It may reveals that the proposed model has outstanding forecasting ability in the small sample prediction. Besides, from Tables 8 and 9, the proposed model has lower average RMSE, and the DA is mostly higher than previous models for TAIEX forecasting. The forecasting results of dataset three indicate that the proposed models provide accuracy levels superior to those of other methods. Besides, we suppose that the higher DA can help people have better profit for stock trading.

In future studies, the proposed IFTS model can be applied to obtain forecasts for other time-related data sets, such as project expenditures, stock indexes, exchange rates, number of tourists, or number of hospital outpatients. The integration of discretization approaches with the proposed models is an interesting research topic. According to Liu *et al.* [41], the development of high-order IFTS model and integration with other aggregation operators is also an interesting direction. Furthermore, determining the degree of nonmembership in more reasonable ways is still an unsolved problem.

CONFLICTS OF INTEREST

Long-Sheng Chen declares that he has no conflict of interest. Mu-Yen Chen declares that he has no conflict of interest. Jing-Rong Chang declares that he has no conflict of interest. Pei-Yu Yu declares that she has no conflict of interest.

AUTHORS' CONTRIBUTIONS

L.-S. Chen contributed substantially to the conception and design of this study, supervised the research, and co-wrote the paper. M.-Y. Chen developed the theory, analyzed data and drafted this study. J.-R. Chang designed experiments, supervised the findings of this work, and provided critical revision and final approval of the version to publish. P.-Y. Yu performed the analytic calculations and performed the numerical simulations, and co-wrote the paper.

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ETHICAL APPROVAL

This article does not contain any studies with human participants or animals performed by any of the authors.

Table 8 | The RMSE of TAIEX from 2000 to 2004 by different models.

Methods	Parameter	Years					
		2000	2001	2002	2003	2004	Avg.
Chang and Liu [8]	1-order	129.70	120.62	70.84	60.39	56.71	87.65
	2-order	123.19	119.05	67.94	57.43	56.25	84.77
	3-order	120.03	124.73	63.83	51.97	54.19	82.95
Cai et al. [36]		131.53	112.59	60.33	51.54	50.33	81.26
Ye et al. [37]		125.42	113.22	63.99	52.99	52.40	81.60
Su and Cheng [38]		132.19	113.23	65.83	57.62	54.33	84.64
Proposed model	16 interval $\text{ans}\pi \geq 0$	127.96	106.57	64.29	49.05	54.52	80.48

RMSE, root mean square error; TAIEX, Taiwan Stock Exchange Capitalization Weighted Stock Index.

Table 9 | The DA of TAIEX from 2000 to 2004 by different models.

Methods	Parameter	Years					
		2000 (%)	2001 (%)	2002 (%)	2003 (%)	2004 (%)	Avg.
Ismail and Efendi [39]		45.65	50.00	50.00	38.10	52.27	47.20
Joshi and Kumar [40]		45.65	54.76	50.00	40.48	36.36	45.45
Chang and Liu [8]	1-order	68.89	43.90	51.22	46.34	51.16	52.30
	2-order	70.45	42.50	55.00	47.50	47.62	52.61
	3-order	72.10	43.59	53.85	46.15	51.22	53.38
Proposed model	16 intervals and $\pi \geq 0$	63.83	65.12	69.77	58.14	60.00	63.37

TAIEX, Taiwan Stock Exchange Capitalization Weighted Stock Index; DA, direction accuracy.

Table 10 | Monthly data of project expenditures [4].

Month	Expenditure	Month	Expenditure
2000/6	184000	2001/9	230000
2000/7	195000	2001/10	227000
2000/8	191000	2001/11	225000
2000/9	210000	2001/12	223000
2000/10	209000	2002/1	218000
2000/11	210000	2002/2	205000
2000/12	223000	2002/3	215000
2001/1	238000	2002/4	196000
2001/2	229000	2002/5	188000
2001/3	250000	2002/6	176000
2001/4	248000	2002/7	189000
2001/5	235000	2002/8	193000
2001/6	248000	2002/9	172000
2001/7	244000	2002/10	179000
2001/8	237000		

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Table 11 | Comparisons of forecasting performance (IT project expenditure).

Methods	Discretization method	RMSE	DA (%)
Cheng et al. [4]	Trapezoid Fuzzification Approach (TFA)	10612.06	71.43
	Minimize Entropy Principle Approach (MEPA)	10099.35	71.43
	Weighted Cumulative Probability Distribution	10678.43	60.71
Chang and Yu [9]	Approach (CPDA)		
	Weighted Minimize Entropy Principle Approach (MEPA)	9850.63	67.86
Proposed model	NQDA (11 intervals) and $\pi \geq 0$	9787.56	77.78

DA, direction accuracy; RMSE, root mean square error; IFTS, intuitionistic fuzzy time series; NQDA, Nth quantile discretization approach.

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