

Constructing a Multi-Production Function for Investment in Agricultural Organizations

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Abstract—The development of intelligent systems of agricultural management increases the requirements for models that describe the process of agricultural production. To date, there is a need to develop a new class of production functions that describe simultaneous delivery of several types of products by one agricultural organization. The article analyzes the results of various studies devoted to the construction of production functions in relation to the economy, examines the theoretical foundations of constructing a multi-production function, explores various aspects of such work, in particular, the list of available and used economic indicators, the methods used to estimate parameters. Based on the data on organizations of the Nizhny Novgorod region, an algorithm was proposed for constructing a production function that describes the production of grain and potatoes, based on the application of the stochastic gradient method and combining elements of production possibility frontier and the Cobb-Douglas function. The quantity of grain and potato produced (in centners) was used as a dependent indicator, capital (the value of the capacity of fixed assets, in hp), labor (the number of employees) and the number of sown areas of both types of crops were used as independent variables. The author's approach to measuring errors and the degree of accuracy of the approximation of multi-production functions was proposed. It is concluded that the presented algorithm has prospects for further application, further research prospects are described, and it is noted that multi-production functions can become a more precise tool for identifying agricultural production patterns.

Keywords—*innovation, agricultural production, production functions*

I. INTRODUCTION

Production functions are a classic and long-used tool for identifying production patterns. They are both a purely theoretical tool, that makes it possible to determine the objective laws of the interplay between production factors in

the production process, and the basis for developing measures to regulate the activities of economic actors.

Production functions have been used by various researchers to assess the performance of rural banks in Indonesia [1], the characteristics of the Romanian economy [2], the U.S. economy [3], the Norwegian forest sector [4], the high-tech industry and other sectors of the Chinese economy [5–8], the food industry of Chile [9], to estimate the income of organizations in various sectors of the economy in the United States, Brazil, Germany, England [1,10], and as a tool for performing a number of other tasks.

The study [11], which resulted in the assessment of the existing strategy in the area of technological changes in the energy sector of Pakistan and the formed proposals for its modernization, as well as the work [12], which has investigated the impact of the energy policy of Portugal on the economic growth, prove the possibility of using the results of the study of the obtained production functions as a basis for developing practical recommendations for the management of various economy sectors.

The literature on the application of production functions in agriculture is quite diverse:

- the patterns of changes in the laws of the agricultural sector performance are investigated through the dynamics of the production function coefficients [13];

- the process of agricultural production itself is analyzed [14], for example, with regard to barley [15] and other crops [16].

A separate area today is the use of water as a separate factor of production (hydroeconomic modelling). In fact, in this case, using a production function, the process of growing crop products itself is modelled. Examples are works which

study the impact of water as a resource on crop yields [17] (including with respect to various types and properties of soil [18]), on grain production in the Czech Republic [19], on fish production [20, 21], construct the dependence of the influence of water salinity on the amount of spring wheat in Iran [22].

II. PROBLEM STATEMENT

A. Composition of independent variables

At the current time, however, there are gaps in ways of describing the processes of production of agricultural products of several types simultaneously by means of production functions. Meeting that challenge requires the use of the multi-production PF class. Examples of new approaches are represented in the works [23–27], the equation of the production function for the size of 4 different species of the caught fish is constructed in the study [28]. However, since other crops are taken as the object of our research, the same approach is not applicable.

In addition, when constructing a multi-production PF, as indicated in the study [28], a significant part of existing methods and techniques of PF parameters estimation is not applicable to the case of several produced products – accordingly, there is a problem of estimation of the coefficients of the function under consideration.

Thereafter follows a discussion of some issues concerning the construction of a multi-production PF in the case of production of only two crops – grain and potato.

The practice of using production functions in research goes back over a century; during this time, researchers have used different types of production functions, different algorithms determining coefficients of models (especially this issue is relevant today), and different indicators as independent variables. All these issues require a detailed discussion.

The following indicators were used as independent variables in different studies:

- fixed assets and labor costs [10];
- the amount of labor, fixed capital of infrastructure facilities, fleet, ICT [29];
- the number of employees, sown areas, the amount of applied fertilizers of various types, the total capacity of agricultural machinery [13];
- labor input, the amount of fertilizers, pesticides, the size of harvesting equipment, the quantity of consumed fuel, and the volume of harvested area [19];
- labor costs, capital input, material and energy costs [9, 30];
- the amount of capital, energy and the number of employees [3, 31].

It is obvious that, according to the researchers, the presence of indicators of fixed capital and labor is obligatory in the model. The presence and composition of the remaining indicators depend on the specific characteristics of work and investigated hypotheses.

In our case, the level of the object under consideration (agricultural organizations) and the data accumulated on the object make it possible to use physical indicators – for

example, the amount of produced grain and potatoes (in centners) is used as a dependent indicator, it is proposed to use capital (the value of the capacity of fixed assets, in hp), labor (the number of employees) and the number of sown areas of both types of crops. Labor, capital and land are the main resources of agricultural production, and their use is reasonable in the model when there is a lack of other information.

B. The question of production function form and methods of parameter estimation

The issue of the choice of the form of a production function is more complicated. Currently, research on the development of new forms of PF is actively continuing [32] (for example, quasi-Cobb-Douglas production functions [33], quasi-sum and quasi-homogeneous production functions [34, 35], quasi-concave functions [36]. However, today, the most widespread studies are those which are based on the use of either CES-models and its modifications [11, 37–41], or on the use of Cobb-Douglas models [9, 12, 19, 29, 30], or on the use of various modifications of the model of production frontiers [1, 25].

As a result of studying the literature, we decided to use the classical form of the production function of the Cobb-Douglas type, since it permits a more detailed analysis of the elasticity of production factors [2].

The issue of the choice of an effective method for determining coefficients of a production function is also important. Under conditions of multicollinearity of independent variables [36] (this effect is also expected for our data, since, with an increase in the area of agricultural lands, both the value of fixed assets and the number of employees increase) and the distribution of variables other than in case of a normal one and the presence of heavy tails in it [42], the classical least squares method does not provide optimal and efficient estimates.

Various researchers propose:

- the use of coefficients of collinear refraction and parameters estimation by means of the Kmenta method [41];
- parameters estimation by means of swarm optimization algorithms [38, 43, 44];
- the use of multivariate distributions [45];
- the determination of indicators of factors (characterizing the economy state) elasticity directly;
- the application of the LSM to linearized equations of CES functions [37];
- the use of various hidden variables [9, 20, 46–48];
- the use of neural networks as a parameter estimation tool [22];
- the use of the maximum likelihood method [9];
- the use of ridge regression [11];
- the use of the method of moments [31];
- the use of different modified LSMs [31];
- the use of various cointegrating algorithms to identify the presence of a cause-and-effect relationship [13];

– the use of spline regression [46].

Each of the methods presented has its advantages and disadvantages. However, in the scientific literature, there is not yet a large number of studies on the applicability of the LSM and its analogues to determine coefficients of a multi-production function. That is the reason for the choice of numerical optimization methods to determine model parameters, and, in particular, we have chosen the stochastic gradient method.

III. MATERIALS AND METHODS

The information base of the study was made up of the data on agricultural organizations of the Nizhny Novgorod region, that produce only grain and potato, for 2014–2017. The aim of the study is to construct a multi-production PF describing the output of grain and potato at agricultural organizations in the region under study.

To construct an equation of production function explicitly, the following assumptions were accepted:

1. The presence of a certain level of production resources at an organization leads to the presence of different combinations of crop yields, and the yield choice depends on the a priori unknown distribution of resources within the organization.

2. These combinations are described by the equation of an ellipse – thus, assuming that *x* is the amount of grain produced, *y* is the amount of potato produced, the following equation is obtained:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = f(K, L, T) \tag{1}$$

where *f*(*K*, *L*, *T*) is a production function.

If, however, the yield of relevant crops are taken as the parameters *a* and *b*, the following valid equation is obtained:

$$\frac{x^2}{a_x^2} + \frac{y^2}{b_x^2} = T \tag{2}$$

where *T* is a total sown area at an organization;

a_x – grain yield;

b_x – potato yield.

Grain and potato yields also depend on the amount of resources and can be described by production functions.

To determine the parameters of these equations, from among the entire set of organizations, only those ones were selected that produced either grain or potato. On their basis, models of grain and potato yield were constructed:

$$a_x = 4.45L^{0.135}T^{0.152} \tag{3}$$

$$a_y = 24.175T^{0.445} \tag{4}$$

where *L* is a number of employees;

T is a sown area for a particular crop.

Both equations are statistically significant, and all their coefficients are statistically significant as well.

Thus, knowing crop yields of each organization and the number of sown areas for a particular crop, it is possible to calculate the left side of the equation (1), and then to obtain data for calculating the right side of the model. As a result, we obtain:

$$\frac{0.05x^2}{L^{0.27}T^{0.3}} + \frac{0.0017y^2}{T^{0.89}} = K^{0.579}T^{1.12} \tag{5}$$

where *K* is a total machinery capacity, hp.

IV. RESULTS

Knowing the amount of resources for an organization, it is possible to construct an explicit production possibility frontier for it (more precisely, this will be the most likely production possibility frontier for a given amount of resources) and to evaluate the performance of an agricultural organization regarding its PPF.

However, for such models, it is extremely difficult to apply regular methods for determining errors. As a way to determine errors, we propose to use the distance from the point characterizing the real crop yield to the production possibility frontier along a straight line connecting the origin of coordinates and the point *R*.

The straight line from point (0; 0) to the point *R* (*x*₀; *y*₀) of the real output reflects some distribution of resources within the organization – accordingly, the error metric will be the distance from the output point to the production possibility frontier along a straight line. We determine the formula for finding the length of the *RS* segment for a particular organization explicitly.

Since when an error is found, the right side of the formula (5) is a numerical value, it can be rewritten as follows $\frac{x^2}{a^2} + \frac{y^2}{b^2} = c$. The line *OR* is described by the equation $y = \frac{y_0}{x_0} \cdot x$. Then, equating them (and they intersect at the point *S*), we can analytically obtain the abscissas and ordinates of the point *S* and find the distance between points *S* and *R* using the segment length formula.

The sum of these distances regarding all the points will be a value characterizing the error of the constructed model.

Assuming that the equation of the left side is not entirely correct (since the selected form of the left side may not reflect real dependencies), using the stochastic gradient method, minimizing the overall error, we obtain the final equation of the production function for organizations that produce potato and grain:

$$\frac{0.05x^2}{L^{0.27}T^{0.3}} + \frac{0.0017y^2}{T^{0.89}} + 11.39x + 4.16y = K^{0.579}(T_1 + T_2)^{1.12} \tag{6}$$

where *T*₁ is the area of grain cultivation, ha;

*T*₂ the area under of potato cultivation, ha.

The equation (6) allows us to draw the following main conclusions:

1. There are effects that shift the equation of production possibility frontier from its classical form.

2. Considering the coefficients of the right side of the model (6), we note that there is an effect of a positive return to scale – an increase in the amount of existing resources by 1% leads to an increase in output by more than 1%.

3. Comparing the output function elasticity in different resources, we note that the efficiency of investments in land resources significantly exceeds the efficiency of investments in capital, which is in some contradiction with today's thesis about the need for intensive rather than extensive development.

This article proves the possibility of describing the production process of several types of agricultural products by one economic entity using production function. Further research should be focused on:

- the search for the optimal functional form of the left and right parts of the model;
- attempts to describe production of different combinations of product types;
- development of algorithms for description and analysis of multi-production functions;
- the study of estimates obtained using various algorithms of parameter estimates;
- development of models with coefficients that change over time [49];
- the use of these functions for solving optimization problems of investment in various economic resources [50, 51];
- their application for a more detailed analysis of the agricultural production process [52].

V. CONCLUSION

Multi-production functions can become a more precise tool for identifying patterns of agricultural production, which will provide information for the development of appropriate measures for agricultural production regulation.

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